Oldřich John; Jan Malý; Jana Stará

Nowhere continuous solutions to elliptic systems

Commentationes Mathematicae Universitatis Carolinae, Vol. 29 (1988), No. 2, 397--398

Persistent URL: http://dml.cz/dmlcz/106652

ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

TWO RESULTS CONCERNING STRING GRAPHS

Jan Kratochvíl (MFF UK, Sokolovská 83, 18600 Praha 8, Czechoslovakia, received 10.3. 1988)

String graphs are defined as intersection graphs of curves in the plane. Two problems on string graphs have long been open: whether the string graphs can be characterized by a finite set of forbidden induced minors and whether recognizing string graphs is NP-hard. Here the answers to these questions are stated.

It is easy to see that every induced minor (i.e. graph obtained by vertex deletions and edge contractions) of a string graph is a string one as well. This fact leads to introducing the class of critical (with respect to the induced-minor order) nonstring graphs, and to the question of its finiteness. We claim:

Theorem 1. The class of critical nonstring graphs is infinite.

It follows from the works of Robertson-Seymour that every minor closed class of graphs is polynomially recognizable. It is also known that recognizing and induced-minor closed class of graphs may be NP-hard or even undecidable. However, no natural induced-minor closed class recognizing that would be NP-hard was known. We claim:

Theorem 2. Recognizing string graphs is NP-hard.

Note that every string graph has a string representation in which any two strings share a finite number of common intersecting points. It seemed that recognizing realizability of graphs by intersection graphs might be easier, if one puts constraints on the number of common intersecting points that two strings may share. A bit surprisingly, the method of our proof of Theorem 2 yields the following:

Theorem 3. Determining whether a given string graph has a string representation in which any two strings share at most one common intersecting point is NP-complete.

The proofs in detail are supposed to appear elsewhere.

Acknowledgements. The author would like to thank R. Thomas and M. Fellows for discussions and correspondence, respectively, on string graphs, and especially to J. Nešetřil for introducing the problem and for many encouraging remarks.

References:

G. Ehrlich, S. Even, T. Tarjan: Intersection graphs of curves in the plane, J. Combin. Th. B 21(1976), 8-20.

J. Kratochvíl, M. Goljan, P. Kučera: String graphs, Academia, Prague, 1986.

F.W. Sinden: Topology of thin film RC circuits, Bell System Tech. J.(1966), 1639-1662.

NOWHERE CONTINUOUS SOLUTIONS TO ELLIPTIC SYSTEMS

O. John, J. Malý, J. Stará (MFFUK, Sokolovská 83, 18600 Praha 8, Czechoslovakia, received 30.3. 1988) Submitted to Boll. UMI.

For each given F_σ set M in $\mathbf{R}^3$ there is a linear elliptic system with measurable bounded coefficients in $\mathbf{R}^3$ which has a weak solution u with the following properties: (i) u is bounded, (ii) u is essentially discontinuous in each point of M, (iii) u is continuous in each point of $\mathbf{R}^3 \setminus M$.

The construction of the system is given together with the proofs of all properties mentioned above. In particular, the system with nowhere continuous solution can be constructed.

A NOTE ON THE REGULARITY OF AUTONOMOUS QUASILINEAR ELLIPTIC AND PARABOLIC SYSTEMS

O. John, J. Malý, J. Stará (MFF UK, Sokolovská 83, 18600 Praha 8, Czechoslovakia, received 30.3. 1988) Submitted to Communications in Partial Differential Equations.

In this note we obtain as a main result the fact that if all BMO solutions in $\mathbf{R}^n$ of the system

$$D_\alpha (A_{ij}^{\alpha\beta}(u) D_\beta u^j) = 0$$

are continuous then the system has Liouville property and so it is regular. (The system is said to be regular if its every BMO weak solution on each domain $\Omega \subset \mathbf{R}^n$ is locally Hölder continuous.)

We give an example of an autonomous quasilinear elliptic system having equibounded solutions on $\mathbf{R}^n$ which are Hölder continuous but their Hölder norms blow up as the solutions tend to a singular solution.

The results are also modified for parabolic systems.