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COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINAE
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BOUNDARY VALUE PROBLEMS FOR EVOLUTION INCLUSIONS

Nikolaos S. PAPAGEORGIOU x), xx)

Abstract: This paper examines boundary value problems for evolution inclusions, with nonlinear boundary conditions. Two existence theorems are proved. One for convex multivalued perturbations and the other for nonconvex ones. Finally an example from partial differential equations is presented.

Key words: Evolution operator, measurable multifunction, upper semicontinuity, mild solution, Rädstrom embedding.

Classification: 35K35

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1) Introduction. In this paper we study boundary value problems for evolution inclusions. Our work was motivated by the papers of Anichini [1], Kartsatos [6] and Zecca-Zezza [13]. Anichini [1] considered quasilinear differential equations in R^n, with nonlinear boundary conditions and using a fixed point theorem due to Eilenberg-Montgomery, established the existence of solutions. Kartsatos [6] also considers boundary value problems for R^n-valued differential equations, but over an unbounded time interval. Finally Zecca-Zezza [13], extend the work of Kartsatos to differential equations in Banach spaces.

In this note, the differential inclusion is defined on a compact time interval and this allows us to weaken considerably the hypotheses on the orientor field $F(t,x)$. Furthermore, to the contrary to Zecca-Zezza [13], here the linear operator is in general unbounded, covering this way the very important case of partial differential operators. Also we establish the existence of solutions for problems with nonconvex multivalued perturbations, a case which is not addressed in the paper of Zecca-Zezza [13]. Finally we

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present an application to partial differential equations.

2) Preliminaries. Let (Ω,Σ) be a measurable space and X a separable Banach space. Throughout this paper we shall be using the following notations:

$P_{f(c)}(X)=\{A\subseteq X$:nonempty, closed, (convex)$\}$ and

$P_{(w)k(c)}(X)=\{A\subseteq X$:nonempty, (weakly-)compact, (convex)$\}$.

A multifunction $F:\Omega\to P_f(X)$ is said to be measurable, if for all $z\in X$ $\omega\to d(z,F(\omega))=\inf\{\|z-x\|: x\in F(\omega)\}$ is measurable. Other equivalent definitions of measurability of multifunctions can be found in Wagner [12]. By S_F^1 we denote the set of $L^1(X)$ selectors of $F(\cdot)$ i.e. $S_F^1=\{f\in L^1(X):f(\omega)\in F(\omega)\ \mu$-a.e.$\}$. This set may be empty. It is nonempty if and only if $\omega\to\inf\{\|x\|:x\in F(\omega)\}$ belongs in L^1_+. Using S_F^1 we can define a set valued integral for $F(\cdot)$, by setting $\int_\Omega F=\{\int_\Omega f:f\in S_F^1$.

Next let Y, Z be Hausdorff topological spaces. Let $F:Y\to 2^Z\setminus\{\emptyset\}$ be a multifunction. We say that $F(\cdot)$ is upper semicontinuous (u.s.c.) (resp. lower semicontinuous (l.s.c.)), if for all $U\subseteq Z$ open $F^+(U)=\{y\in Y:F(y)\subseteq U\}$ is open in Y (resp. $F^-(U)=\{y\in Y:F(y)\cap U\neq\emptyset\}$ is open in Y).

3) Existence result: convex case. Let $T=[0,b]$ and X a separable Banach space. The multivalued boundary value problem under consideration is the following:

$$(*)\qquad \left\{\begin{array}{l}\dot{x}(t)\in A(t)x(t)+F(t,x(t))\\ Lx=Mx\end{array}\right\}$$

We shall assume that the family of linear operators $\{A(t):t\in T\}$ generates a strongly continuous evolution operator $S(t,s)$, $0\le s\le t\le b$. So by a solution of $(*)$, we shall understand a mild solution. Thus we say that $x(\cdot)\in C(T,X)$ solves $(*)$ if and only if

$$x(t)=S(t,0)x(0)+\int_0^t S(t,s)f(s)ds,\ \text{for some } f\in S^1_{F(\cdot,x(\cdot))} \text{ and } Lx=Mx.$$

The full set of hypotheses on the data of the problem $(*)$ is the following:

H(A): The family $\{A(t):t\in[0,b]\}$, generates a strongly continuous evolution operator $S:\Delta=\{0\le s\le t\le b\}\to\mathcal{L}(X)$ which is compact for $t-s>0$.

H(F): $F:T\times X\to P_{wkc}(X)$ is a multifunction s.t.

(1) $(t,x)\to F(t,x)$ is measurable,

(2) $x\to F(t,x)$ is u.s.c. from X into X_w,

(3) $\lim\limits_{n\to\infty}\frac{1}{n}\int_0^b \sup\limits_{\|x\|\leq n} |F(t,x)|dt=0$.

H(L): $L:C(T,X)\longrightarrow X$ is continuous, linear. Also if $V:C(T,X)\longrightarrow C(T,X)$ is defined by $(Vx)(\cdot)=x(\cdot)-S(\cdot,0)x(0)$, then there exists $K:X\longrightarrow \ker V$ continuous, linear s.t. $(I-L_0K)(Mx-L\int_0^t S(t,s)f(s)ds)=0$ for all $f\in S^1_{F(\cdot,x(\cdot))}$ and all $x(\cdot)\in C(T,X)$, with $L_0=L|_{\ker V}$.

H(M): $M:C(T,X)\longrightarrow X$ is a generally nonlinear, completely continuous operator s.t.

$$\lim_{\|x\|\to\infty}\frac{\|Mx\|}{\|x\|}=0.$$

Having these hypotheses, we can now state our first existence result concerning (*).

Theorem 1: If the hypotheses H(A), H(F), H(L) and H(M) hold, then (*) admits a mild solution.

Proof: For some $x_0\in\ker L_0$, consider the multifunction $R:C(T,X)\longrightarrow 2^{C(T,X)}\setminus\{\emptyset\}$ defined by:

$$R(x)=\{y\in C(T,X):y(t)=x_0(t)+KMx-KL\int_0^t S(t,s)f(s)ds+\int_0^t S(t,s)f(s)ds, t\in T,$$
$$f\in S^1_{F(\cdot,x(\cdot))}\}.$$

Because of the hypothesis H(L), it is easy to check that a fixed point of $R(\cdot)$ is the desired mild solution of (*).

From the definition of $R(\cdot)$ and the convexity of the values of $F(\cdot,\cdot)$ (and so of $S^1_{F(\cdot,x(\cdot))}$), we see that $R(\cdot)$ is convex valued. We claim that the values of $R(\cdot)$ are also closed. So let $y_n\in R(x)$, $y_n\longrightarrow y$ in $C(T,X)$. We have:

$$y_n(t)=x_0(t)+KMx-KL\int_0^t S(t,s)f_n(s)ds+\int_0^t S(t,s)f_n(s)ds$$

with $f_n\in S^1_{F(\cdot,x(\cdot))}$. But from Proposition 3.1 of [9], we know that $S^1_{F(\cdot,x(\cdot))}$ is weakly compact in $L^1(X)$ and by the Eberlein-Smulian theorem is weakly sequentially compact. Thus by passing to a subsequence if necessary, we may assume that $f_n\xrightarrow{w} f\in S^1_{F(\cdot,x(\cdot))}$ in $L^1(X)$. Then exploiting the fact that a continuous, linear operator is also weakly continuous and that

$\int_0^t S(t,s)f_n(s)ds\xrightarrow{w}\int_0^t S(t,s)f(s)ds$, we get that

$y_n(t)\xrightarrow{w} x_0(t)+KMx-KL\int_0^t S(t,s)f(s)ds+\int_0^t S(t,s)f(s)ds, t\in T$ and

$f \in S^1_{F(\cdot,x(\cdot))}$. So $y(t)=x_0(t)+KMx-KL\int_0^t S(t,s)f(s)ds+\int_0^t S(t,s)f(s)ds \Longrightarrow$ $\Longrightarrow y \in R(x)$. Hence we conclude $R(x) \in P_{fc}(C(T,X))$.

Now we shall show that $R(\cdot)$ has a closed graph ($GrR=\{(x,y) \in C(T,X)\times C(T,X): y \in R(x)\}$ = graph of $R(\cdot)$). To this end let $(x_n,y_n) \in GrR$, $(x_n,y_n) \longrightarrow$ $\longrightarrow (x,y)$ in $C(T,X)\times C(T,X)$. Then we have:

$y_n(t)=x_0(t)+KMx_n-KL\int_0^t S(t,s)f_n(s)ds+\int_0^t S(t,s)f_n(s)ds$, with $f_n \in S^1_{F(\cdot,x_n(\cdot))}$.

Let $G(t)=\overline{conv} \bigcup_{n\geq 1} F(t,x_n(t))$. Since by the hypothesis H(F)(2), $F(t,\cdot)$ is u.s.c. from X into X_w, it maps compact sets in X into w-compact sets. Therefore $\overline{\bigcup_{n\geq 1} F(t,x_n(t))}^w$ is w-compact and by the Krein-Smulian theorem we have that $\overline{conv} \bigcup_{n\geq 1} F(t,x_n(t))$ is w-compact. So for all $t \in T$, $G(t) \in P_{wkc}(X)$. Finally from the hypothesis H(F)(3), we see that $G(\cdot)$ is integrably bounded (i.e. $t \longrightarrow |G(t)|=\sup\{\|z\|: z \in G(t)\} \in L^1_+$). Hence once again Proposition 3.1 of [9] tells us that S^1_G is w-compact in $L^1(X)$. So by passing to a subsequence if necessary, we may assume that $f_n \xrightarrow{w} f$ in $L^1(X)$. From Theorem 3.1 of [10], we have that:

$$f(t) \in \overline{conv}\ w\text{-}\overline{\lim}\ f_n(t) \subseteq \overline{conv}\ w\text{-}\overline{\lim}\ F(t,x_n)) \subseteq F(t,x_n(t)) \text{ a.e.,}$$

the last inclusion following from the hypothesis H(F)(2). So $f \in S^1_{F(\cdot,x(\cdot))}$. Also note that $x_0(t)+KMx_n-KL\int_0^t S(t,s)f_n(s)ds+\int_0^t S(t,s)f_n(s)ds$ converges weakly to

$x_0(t)+KMx-KL\int_0^t S(t,s)f(s)ds+\int_0^t S(t,s)f(s)ds=y(t) \Longrightarrow y \in R(x) \Longrightarrow GrR$ is closed.

Next, we claim that there exists $r>0$ s.t. for $\|x\|_\infty \leq r \Longrightarrow |R(x)|=$ $=\sup\{\|y\|_\infty : y \in R(x)\} \leq r$. Suppose not. Then, we can find $\{x_n\}_{n\geq 1} \subseteq C(T,X)$ s.t. $\|x_n\|_\infty \leq n$ and $|R(x_n)|>n$. So we have $1<\frac{|R(x_n)|}{n}$. But note that for $y_n \in$ $\in R(x_n)$ we have:

$$\|y_n(t)\| < \|x_0\|_\infty+\|KMx_n\| + \|KL\int_0^t S(t,s)f_n(s)ds\|+\|\int_0^t S(t,s)f_n(s)ds\|$$

$$\leq \|x_0\|_\infty+\|K\|\cdot\|Mx_n\| + \|KL\|\cdot N\int_0^t |F(s,x_n(s))|ds+N\int_0^t |F(s,x_n(s))|ds$$

where $\|S(t,s)\| \leq N$. So we have:

$$\frac{|R(x_n)|}{n} \leq \frac{\|x_0\|_\infty}{n} + \|K\|\frac{\|Mx_n\|}{n} + N(\|KL\|+1)\int_0^t \frac{|F(s,x_n(s))|}{n}ds$$

$$\leq \frac{\|x_0\|_\infty}{n} + \|K\|\frac{\|Mx_n\|}{\|x_n\|} + N(\|KL\|+1)\int_0^t \frac{|F(s,x_n(s)|}{n}ds.$$

Using the hypotheses H(F)(3) and H(M), as well as the above inequality, we get that $1\leq\underline{\lim}_{n\to\infty}\frac{R(x_n)}{n}=0$, a contradiction. So indeed $R:B_r\to P_{fc}(B_r)$, where $B_r=\{x\in C(T,X):\|x\|_\infty\leq r\}$.

Now we claim that $\overline{R(B_r)}$ is compact in C(T,X). First note that for every $t\in T$, we have:

$$R(B_r)(t)\subseteq x_0(t)+KM(B_r)-KL\int_0^t S(t,s)P(s)ds+\int_0^t S(t,s)P(s)ds,$$

where $P(s)=\{x\in X:\|x\|=\sup(|F(s,x)|:\|x\|\leq r)=u_r(s)\}$. But recall that $M(\cdot)$ is completely continuous, So $\overline{M(B_r)}$ is compact $\Rightarrow\overline{KM(B_r)}$ is compact. Also since by the hypothesis H(A), S(t,s) is compact for $t-s>0$, we have that $S(t,s)P(s)\in P_{kc}(X)$ and clearly $s\to S(t,s)P(s)$ is measurable and integrably bounded. Hence using the Rädstrom embedding theorem (see Hiai-Umegaki [5], Theorem 4.5), we have that $\int_0^t S(t,s)P(s)ds\in P_{kc}(X)$ (note that in the above mentioned result of Hiai-Umegaki [5], the R.N.P.-hypothesis on X is superfluous, since by the corollary to Proposition 3.1 of [9], $\int_0^t S(t,s)P(s)ds$ is closed). So for all $t\in T$, $\overline{R(B_r)(t)}\in P_k(X)$. Now, let t', $t\in T$, $t<t'$. For $y\in R(B_r)$, we have:

$$\|y(t')-y(t)\|\leq\|S(t',0)x_0-S(t,0)x_0\|+\|KL\|\cdot\|\int_0^{t'}S(t',s)f(s)ds-\int_0^t S(t,s)f(s)ds\|+\|\int_0^{t'}S(t',s)f(s)ds-\int_0^t S(t,s)f(s)ds.$$

Since S(t,s) is a strongly continuous evolution operator, given $\varepsilon>0$, there exists $\delta_1(\varepsilon)>0$ s.t. if $\|t'-t\|<\delta_1$, $\|S(t',0)x_0-S(t,0)x_0\|<\varepsilon/3$. Also note that:

$$\|\int_0^{t'}S(t',s)f(s)ds-\int_0^t S(t,s)f(s)ds)\|\leq\|\int_0^{t-\delta_2}(S(t',s)f(s)-S(t,s)f(s)ds\|+$$
$$+\|\int_{t-\delta_2}^t(S(t',s)-S(t,s))f(s)ds+\int_t^{t'}S(t',s)f(s)ds\|.$$

Because of H(A), from Proposition 2.1 of [11], we have that $t\to S(t,s)$ is continuous in the uniform operator topology, uniformly in s, for t-s bounded away from 0. So by choosing $\delta_2(\varepsilon)>0$ appropriately small, we have:

$$\|\int_0^{t-\delta_2}(S(t',s)-S(t,s))f(s)ds\|+\|\int_{t-\delta_2}^t(S(t',s)-S(t,s))f(s)ds\|+$$
$$\|\int_0^{t'}S(t',s)f(s)ds\|\leq\int_0^{t-\delta_2}\|S(t',s)-S(t,s)\|u_r(s)ds+2N\int_{t-\delta_2}^t u_r(s)ds+$$

$+N\int_t^{t'} u_r(s)ds < \varepsilon/3\alpha$, where $\alpha=\max(1,\|KL\|)$. Thus for $\delta=\min(\delta_1,\delta_2)$, we have for $|t-t'|<\delta$

$\|y(t')-y(t)\| < \varepsilon$ for all $y(\cdot)\in R(B_r) \Rightarrow R(B_r)$ is equicontinuous.

Invoking the Arzela-Ascoli theorem, we deduce that $\overline{R(B_r)}$ is compact in $C(T,X)$. Since $R(\cdot)$ has a closed graph and compact range when restricted to B_r, from Theorem 7.1.16 of Klein-Thompson [7] $R(\cdot)$ is u.s.c. and so we can apply the Kakutani-KyFan fixed point theorem to get $x\in B_r$ s.t. $x\in R(x)$. As we already indicated, $x(\cdot)$ is a mild solution of $(*)$. <u>Q.E.D.</u>

Remark. If L_0 has a continuous, linear inverse, then $H(L)$ is satisfied.

4) Existence result: nonconvex case. We also have an existence result for the case where the multivalued perturbation $F(t,x)$ is nonconvex valued. In this case the hypothesis about $F(\cdot,\cdot)$ takes the following form:

<u>$H(F)'$</u>: $F:T\times X\to P_f(X)$ is a multifunction s.t.

(1) $(t,x)\to F(t,x)$ is measurable,

(2) $x\to F(t,x)$ is l.s.c. from X into X,

(3) $\underline{\lim}_{n\to\infty} \frac{1}{n}\int_0^b \sup_{\|x\|\le n} |F(t,x)|dt=0$.

Theorem 2: <u>If</u> the hypotheses $H(A)$, $H(F)'$, $H(L)$, $H(M)$ hold with M linear, <u>then</u> $(*)$ admits a mild solution.

Proof: We have already seen in the proof of Theorem 1 that $R(\cdot)$ maps the ball B_r into itself and furthermore $\hat{W}=\overline{\text{conv}}\, R(B_r)$ is compact in $C(T,X)$. Let $H:\hat{W}\to P_f(L^1(X))$ be the multifunction defined by $H(x)=S^1_{F(\cdot,x(\cdot))}$. Let $x_n\to x$ in $C(T,X)$. Then because of the hypotheses $H(F)'(1)$ and (3) we can apply Theorem 4.1 of [10] and get that $H(x)\subseteq s\text{-}\underline{\lim}\, H(x_n) \Rightarrow H(\cdot)$ is l.s.c. (see Delahaye-Denel [3]). Apply Fryszkowski's selection theorem [4], to get $h:\hat{W}\to L^1(X)$ continuous s.t. $h(x)\in H(x)$, for all $x\in\hat{W}$. Then consider the following problem:

$$(*)(y)\qquad \left\{\begin{array}{l} \dot{x}(t)=A(t)x(t)+h(y)(t) \\ Lx=Mx \end{array}\right\}$$

Let $Q:\hat{W}\to P_{fc}(W)$ be the multifunction defined by $Q(y)=\{\text{Set of mild solutions of } (*)(y)\}$. It has nonempty values by Theorem 1. Let $(y_n,x_n)\in \text{Gr}\, Q$ s.t. $(y_n,x_n)\to(y,x)$ in $C(T,X)\times C(T,X)$. We have: $x_n(t)=S(t,0)x_n(0)+$

$+\int_0^t S(t,s)h(y_n)(s)ds$, $Lx_n=Mx_n$. Passing to the limit as $n\to\infty$ and exploiting the continuity of $h(\cdot)$, we get that:

$x(t)=S(t,0)x(0)+\int_0^t S(t,s)h(y)(s)ds,\ Lx=Mx \Longrightarrow x\in Q(y)$.

$\longrightarrow$ GrQ is closed in C(T,X)xC(T,X).

Since $\hat{W}$ is compact in C(T,X), we conclude that Q(·) is u.s.c. Apply the Kakutani-KyFan fixed point to get $y\in Q(y)$. Clearly y(·) solves (*). Q.E.D.

5) Application: We consider the following multivalued boundary value problem.

$$(**)\left\{\begin{array}{l}\frac{\partial u(t,y)}{\partial t}\in\sum_{k=1}^{n}\frac{\partial}{\partial y_k}p(y)\frac{\partial}{\partial y_k}u(t,y)+f(t,y,u(t,y))\text{ on }TxG\\ u(t,y)=0\ \ (t,y)\in Tx\partial G\\ u(0,y)-u(b,y)=\int_G\int_0^b g(t,y,z,u(t,z))dt\,dz\end{array}\right\}$$

Here $G\subseteq R^n$ is an open, bounded domain with a smooth boundary ∂G. Also $T=[0,b]$. We assume that $g:TxGxGxR\longrightarrow R$ is a function satisfying the Carathéodory conditions, i.e. $z\longrightarrow g(t,y,z,r)$ is measurable and $r\longrightarrow g(t,y,z,r)$ is continuous. Moreover for each $k>0$ there exist measurable functions

$\beta_k:TxGxG\longrightarrow R_+$ and $\gamma_k:TxGxGxR\longrightarrow R_+$ s.t.

$|g(t,y,z,r)\le\beta_k(t,y,z)$ for $|r|\le k$ and $\int_G\int_0^b\beta_k(t,y,z)dt\ d\le M_r$ and

$|g(t,y,z,r)-g(t,y',z,r)|\le\gamma_k(t,y,y',z)$ for $|r|\le k$,

$\lim_{y\to y'}\int_G\int_0^b\gamma_k(t,y,y',z)dt\ dz=0$ uniformly in y'.

Finally, there exist $p\ge1$ and $\beta<2$ s.t. $|g(t,y,z,r)|\le p(1+|x|^\beta)$. Also assume that $f:TxGxR\longrightarrow P_{fc}(R)$ is a multifunction s.t.

(a) $(t,y)\longrightarrow f(t,y,r)$ is measurable,

(b) $r\longrightarrow f(t,y,r)$ is d-continuous (i.e. for every $z\in R$ $r\longrightarrow$ $\longrightarrow d(z,f(t,y,r))$, is continuous),

(c) $|f(t,y,r)|\le k(t)(1+|r|^\alpha)$ $0<\alpha<1$.

Set $X=L^2(G)$, $D(A)=W_0^2(G)$ and on D(A) consider the operator

$$Au=\sum_{k=1}^{n}\frac{\partial}{\partial y_k}p(y)\frac{\partial}{\partial y_k}u(y).$$

So A(·) is densely defined and it is well known (see for example Martin [8]) that it generates a compact semigroup S(t), $t\in T$.

Also let $F:TxX\longrightarrow P_{fc}(X)$ be defined by $F(t,u)=S^2_{f(t,\cdot,u(\cdot))}$. Clearly because of the reflexivity of $X=L^2(G)$, $F(t,u)\in P_{wkc}(X)$. Furthermore note that for all $v\in X$ we have $d(v,F(t,u))=\int_F d(v(z),f(t,z,u(z)))dz$. From the hypotheses on $f(\cdot,\cdot,\cdot)$, $(t,u)\longrightarrow d(v,F(t,u))$ is measurable in t, continuous in u, hence jointly measurable and so $(t,u)\longrightarrow F(t,u)$ is measurable. Also from Theorem 4.2 of [10], we have that $F(t,\cdot)$ is u.s.c. from X into X_w. Next let $M:C(T,X)\longrightarrow X$ be defined by $(Mu)(y)=\int_G\int_0^b g(t,y,z,u(t)(z))dt\,dz$. From Proposition 4.2, p. 175 of Martin [8], we have that $M(\cdot)$ is completely continuous. Also using the growth condition on g, we have

$$\lim_{\|u\|\to 0}\frac{Mu_2}{\|u\|_{C(T,X)}}=0.$$

Let $L:C(T,X)\longrightarrow X$ be defined by $Lu=u(0,\cdot)-u(b,\cdot)$. Clearly this is continuous, linear. Furthermore the only solution of $\dot{u}=Au$, $u(0)=u(b)$, is $u\equiv 0$. Thus if $\hat{L}x=L(S(\cdot)x)$, $\hat{L}:X\longrightarrow X$ is continuous, linear and $\hat{L}x=(Id-S(b))x=0$ has zero as its only solution. So $\hat{L}^{-1}$ exists (Fredholm alternative) $\Longrightarrow$ H(L) is satisfied. So if we rewrite (**) as the following evolution equation

$$(**)'\qquad \begin{aligned}&\dot{u}(t)\in Au(t)+F(t,u(t)),\\&Lu=Mu,\end{aligned}$$

we see that all hypotheses of Theorem 1 are satisfied and so we conclude the existence of a solution belonging in $C(T,L^2(G))$.

It is clear that the general existence results proved here, can give us periodic solutions for the problems of evolution inclusions, extending this way the work of Aubin-Cellina [2].

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