Commentationes Mathematicae Universitatis Carolinae

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Commentationes Mathematicae Universitatis Carolinae, Vol. 28 (1987), No. 4, 784--785

Persistent URL: http://dml.cz/dmlcz/106589

IDENTITIES FOR DIRECT DECOMPOSABILITY OF CONGRUENCES CAN BE WRITTEN IN TWO VARIABLES

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A variety V has directly decomposable congruences if every congruence relation on the product $A\times B$ of algebras $A,B\in V$ is uniquely determined by its projections onto A and B.

Varieties with directly decomposable congruences form a Mal´cev class. The known identities contain at least three variables. We state that two variables are enough.

Theorem. For a variety V the following conditions are equivalent:

(1) V has directly decomposable congruences;

(2) There exist binary terms $r_1,\dots,r_m,s_1,\dots,s_m,t_1,\dots,t_m$ and $(2+m)$-ary terms $d_1,\dots,d_n$ such that V satisfies

$$x=d_i(y,y,r_1(x,y),\dots,r_m(x,y)),\ 1\le i\le n,$$
$$x=d_1(x,y,s_1(x,y),\dots,s_m(x,y)),$$
$$y=d_1(x,y,t_1(x,y),\dots,t_m(x,y)),$$
$$d_i(y,x,s_1(x,y),\dots,s_m(x,y))=d_{i+1}(x,y,s_1(x,y),\dots,s_m(x,y)),\ 1\le i<n,$$
$$d_i(y,x,t_1(x,y),\dots,t_m(x,y))=d_{i+1}(x,y,t_1(x,y),\dots,t_m(x,y)),\ 1\le i<n,$$
$$y=d_n(y,x,s_1(x,y),\dots,s_m(x,y)),$$
$$y=d_n(y,x,t_1(x,y),\dots,t_m(x,y)).$$

LANDESMAN-LAZER CONDITION FOR PERIODIC PROBLEMS WITH JUMPING NONLINEARITIES

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Let us consider the periodic boundary value problem at resonance

(1) $x''(t)+m^2x(t)+g(t,x(t))=e(t),\ x(0)-x(2\pi)=x'(0)-x'(2\pi)=0,$

$m\ge 0$ is an integer, $e\in L^1(0,2\pi)$. We assume that g is a Carathéodory´s function satisfying the growth restriction

$$|g(t,x)|\le p(t)+c|x|;$$

for a.e. $t\in[0,2\pi]$, all $x\in R$ with $c>0$ and $p\in L^1(0,2\pi)$. Moreover, assume that $g_+(t)=\liminf_{x\to+\infty} g(t,x)$ and $g_-(t)=\limsup_{x\to-\infty} g(t,x)$. We impose the following restriction on the growth of g. Let for a.e. $t\in[0,2\pi]$,

$0\le\limsup_{x\to+\infty} x^{-1}g(t,x)\le a-m^2$ and $0\le\limsup_{x\to-\infty} x^{-1}g(t,x)\le b-m^2$

with strict inequality on the set of positive measure in $[0,2\pi]$, where

$$a^{-1/2}+b^{-1/2}=2(m+1)^{-1}.$$

Theorem. Assume that g satisfies all the assumptions stated above. Then the periodic problem (1) has at least one solution provided that

$$\int_0^{2\pi}e(t)v(t)dt<\int_{v>0}g_+(t)v(t)dt+\int_{v<0}g_-(t)v(t)dt$$

for all $v\in\operatorname{Span}\{\sin mt,\cos mt\}\setminus\{0\}$.

Remark 1. Note that our assumptions laid on g are satisfied also in the case when g is "jumping" over eigenvalues different from m^2. In this direction our results generalize the previous ones (see [1]).

Remark 2. By the same approach used for the periodic problem we can prove analogous existence results for the two-point boundary value problem.

Reference

[1] R. Iannacci and M.N. Nkashama, Unbounded perturbations of forced second order ordinary differential equations at resonance, J. Differential Equations 69(1987), 289-309.