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IDENTITIES FOR DIRECT DECOMPOSABILITY OF CONGRUENCES CAN BE WRITTEN IN TWO VARIABLES

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A variety V has directly decomposable congruences if every congruence relation on the product $A \times B$ of algebras $A, B \in V$ is uniquely determined by its projections onto A and B .

Varieties with directly decomposable congruences form a Mal'cev class. The known identities contain at least three variables. We state that two variables are enough.

Theorem. For a variety V the following conditions are equivalent:

- (1) V has directly decomposable congruences;
- (2) There exist binary terms $r_1, \dots, r_m, s_1, \dots, s_m, t_1, \dots, t_m$ and $(2+m)$ -ary terms d_1, \dots, d_n such that V satisfies

$$\begin{aligned} x &= d_1(y, y, r_1(x, y), \dots, r_m(x, y)), \quad 1 \leq i \leq n, \\ x &= d_1(x, y, s_1(x, y), \dots, s_m(x, y)), \\ y &= d_1(x, y, t_1(x, y), \dots, t_m(x, y)), \\ d_1(y, x, s_1(x, y), \dots, s_m(x, y)) &= d_{i+1}(x, y, s_1(x, y), \dots, s_m(x, y)), \quad 1 \leq i < n, \\ d_1(y, x, t_1(x, y), \dots, t_m(x, y)) &= d_{i+1}(x, y, t_1(x, y), \dots, t_m(x, y)), \quad 1 \leq i < n, \\ y &= d_n(y, x, s_1(x, y), \dots, s_m(x, y)), \\ y &= d_n(y, x, t_1(x, y), \dots, t_m(x, y)). \end{aligned}$$

LANDESMAN-LAZER CONDITION FOR PERIODIC PROBLEMS WITH JUMPING NONLINEARITIES

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Let us consider the periodic boundary value problem at resonance

$$(1) \quad x''(t) + m^2 x(t) + g(t, x(t)) = e(t), \quad x(0) - x(2\pi) = x'(0) - x'(2\pi) = 0,$$

$m \neq 0$ is an integer, $e \in L^1(0, 2\pi)$. We assume that g is a Carathéodory's function satisfying the growth restriction

$$|g(t, x)| \leq p(t) + c|x|;$$

for a.e. $t \in [0, 2\pi]$, all $x \in \mathbb{R}$ with $c > 0$ and $p \in L^1(0, 2\pi)$. Moreover, assume that $g_+(t) = \liminf_{x \rightarrow +\infty} g(t, x)$ and $g_-(t) = \limsup_{x \rightarrow -\infty} g(t, x)$. We impose the following restriction on the growth of g . Let for a.e. $t \in [0, 2\pi]$,

$$0 \leq \limsup_{x \rightarrow +\infty} x^{-1} g(t, x) \leq a - m^2 \quad \text{and} \quad 0 \leq \limsup_{x \rightarrow -\infty} x^{-1} g(t, x) \leq b - m^2$$

with strict inequality on the set of positive measure in $[0, 2\pi]$, where $a^{-1/2} + b^{-1/2} = 2(m+1)^{-1}$.

Theorem. Assume that g satisfies all the assumptions stated above. Then the periodic problem (1) has at least one solution provided that

$$\int_0^{2\pi} e(t)v(t)dt \leq \int_{v > 0} g_+(t)v(t)dt + \int_{v < 0} g_-(t)v(t)dt$$

for all $v \in \text{Span} \{ \sin mt, \cos mt \} \setminus \{0\}$.

Remark 1. Note that our assumptions laid on g are satisfied also in the case when g is "jumping" over eigenvalues different from m^2 . In this direction