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COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINAE
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UNIFORMLY NORMAL STRUCTURE AND FIXED POINTS OF UNIFORMLY LIPSCHITZIAN MAPPINGS

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Abstract: Every Banach space E with the uniformly normal structure, i.e. $\tilde{N}(E)<1$, has the following property: if C is a nonempty, bounded, closed and convex subset of E, $A\subset N$ is a subset with Banach measure $\mu(A)=1$, and if $T:C\to C$ has the property that its iterates T^i for $i\in A$ are Liptschitzian with the Lipschitz constant $k<[\tilde{N}(E)]^{-1/2}$, then T has a fixed point in C.

This result generalizes fixed point theorems proved by E. Casini and E. Maluta [4] and the result proved by M. Krüppel and the present author [9].

Key words and phrases: Chebyshev center, uniformly normal structure, Banach measure, μ-uniformly k-Lipschitzian mappings, μ-center of $\{x_n\}$ with respect to C, James spaces, fixed points.

Classification: 47H09, 47H10

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1. Introduction. Our aim is to study an open problem on normal type structures and the fixed point theory which follows from the known results by K. Goebel, W.A. Kirk and R.L. Thele [7],[8]. The question is whether, in a Banach space E, reflexivity and normal structure are sufficient to assure, for suitable $k>1$, the fixed point property (F.P.P. for short) for μ-uniformly k-Lipschitzian self-mapping, i.e. to assure that for every nonempty, bounded, closed, convex subset C of E and every map $T:C\to C$, such that $\|T^nx-T^ny\|\le k\cdot\|x-y\|$ for any $x,y\in C$ and $n\in A$ for some $A\subset N$ with $\mu(A)=1$, T has a fixed point in C.

2. Notation. In this paper, E will always denote an infinite dimensional real or complex Banach space. For a subset C of E, we write diam(C) for the diameter of C, cl(C) for the closure of C and co(C) for the convex hull of C. To simplify the notation we state the following rules: $\{x_n\}$ will always denote a bounded sequence in E, and $\{x_n\}_i^j$ will denote the set of elements of $\{x_n\}$ with $i\le n\le j$. Finally we denote by l_p^n the n-dimensional space with p-norm.

3. Uniformly normal structure. We recall the concept of a Chebyshev center. Let B and C be subsets of a Banach space E and let B be bounded.

For each $x \in C$ define $r(x)=\sup\{\|x-y\|: y \in B\}$ and put $r_0(C,B)=\inf\{r(x): x \in C\}$. Then the possibly empty set $\{x \in C: r(x)=r_0(C,B)\}$ is called the Chebyshev center of B with respect to C and $r_0(C,B)$ the radius of B with respect to C. It is well known that if C is weakly compact and convex then Chebyshev centers with respect to C are nonempty, weakly compact and convex.

We now recall that a normed space (or a convex subset) E is said to have the normal structure if for every nonempty, bounded, convex, non-singleton subset C of E, the Chebyshev radius of C relative to C, $r_0(C,C)$, is strictly smaller than the diameter of C, i.e. there exists at least one point $x \in C$ with $\sup\{\|x-y\|: y \in C\} < \mathrm{diam}(C)$. Such a point x is called *non-diametral*. This concept was introduced by M.S. Brodskij and D.P. Mil'man (1948).

Let E be a Banach space and C a (nonempty) weakly compact, convex subset of E. A mapping $T: C \to C$ is said to be nonexpansive if

$$\|Tx-Ty\| \leq \|x-y\| \quad \text{for all } x,y \in C.$$

It is now known (see D.E. Alspach, A fixed point free nonexpansive map, Proc. Amer. Math. Soc. 82(1981), 423-424) that, in the absence of further assumptions, such a mapping need not have a fixed point. On the other hand, the classical result of W.A. Kirk (1965) say the following: let C be a nonempty, weakly compact, convex subset of a Banach space E, and suppose also that C has the normal structure. Then every nonexpansive mapping $T: C \to C$ has a fixed point. (For normal type structures and their applications to the fixed point theory, we refer to the exhaustive survey of W.A. Kirk [10] and S. Swaminathan [15].)

The concept of uniformly normal structure is due to A.A. Gillespie and B.B. Williams [6]. A Banach space E is said to have the uniformly normal structure if there exists $h \in (0,1)$ such that every bounded, closed, convex subset C of E contains a point z such that $\sup\{\|z-x\|: x \in C\} \leq h \cdot \mathrm{diam}(C)$. It is known that every uniformly convex space has the uniformly normal structure, and it was shown in [6] that if C is a nonempty, bounded, closed and convex subset of a Banach space with the uniformly normal structure, then every nonexpansive self-mapping of C has a fixed point. This result is noteworthy in that it does not require any weak compactness assumption. E. Maluta in [14] showed that the uniformly normal structure implies reflexivity and defined the constant of the uniformity of normal structure in the following way:

Definition 3.1. We set

$$\tilde{N}(E) := \sup\left\{\frac{r_0(C,C)}{\mathrm{diam}(C)} : \begin{array}{l} C \text{ a nonempty, bounded, non-singleton, convex} \\ \text{subset of } E \end{array}\right\}.$$

Hence $\tilde{N}(E)<1$ characterizes the uniformly normal structure. Of course $\tilde{N}(E)\not\leq 1$ and $\tilde{N}(E)=\frac{1}{2}$ if $E=(R^2, \|\cdot\|_\infty)$, but $\tilde{N}(E)>\frac{1}{2}$ if the dimension of E is greater. In any Hilbert space and in any two-dimensional Banach space $\tilde{N}(E)=\frac{1}{2}\cdot J(E)$, where J(E) is the Jung constant of E. As a consequence we obtain $\tilde{N}(l_2^n)= \left(\frac{n}{2n+2}\right)^{1/2}$ for Euclidean spaces l_2^n, and $\tilde{N}(l^2)=2^{-1/2}$.

We now define a class of James spaces which has recently been the object of a very intensive study.

Definition 3.2. Let $\beta>1$ and let E_β be the real Hilbert space l^2 renormed according to $|x|_\beta=\max\{\|x\|_2, \beta\cdot\|x\|_\infty\}$ where $\|\cdot\|_2$ denotes the l^2-norm and $\|\cdot\|_\infty$ the sup-norm of l^∞ space.

Since $\|x\|_2\leq|x|_\beta\leq(1+\beta)\cdot\|x\|_2$, the space E_β is not only reflexive but also superreflexive, and moreover it is known that for $\beta\geq 2^{1/2}$, E_β fails to have a normal structure (E_β has a normal structure iff $\beta<2^{1/2}$). Indeed, consider the set

$$C=\{x=(x_1,x_2,\dots)\in E_\beta : x_j\geq 0 \text{ for all je and } \|x\|_2\leq 1\}.$$

For $\beta\geq 2^{1/2}$, diam(C)= β . Now let e_n be the n-th unit vector in L^2. Since $\lim_n |x-e_n|_\beta=\beta$ for each x in C, we see that all points of C are diametral.

J.B. Baillon and R. Schöneberg [2] proved that James spaces E_β for $\beta\leq 2$ have the F.P.P. for nonexpansive self-mapping. For $\beta>2$, this problem is still open. E. Casini and E. Maluta [4] proved

Theorem 3.3. For $1\leq\beta\leq 2^{1/2}$, $\tilde{N}(E_\beta)=\beta\cdot 2^{-1/2}$ and, as a consequence, for $\beta<2^{1/2}$ the James space E_β has the uniformly normal structure.

4. Short history of uniformly Lipschitz mappings

Example 4.1. Let $B=\{x\in l^2 : \|x\|\leq 1\}$ and $k>1$. The mapping $T:B\to B$ defined by $T(x_1,x_2,\dots)=(t(1-\|x\|),x_1,x_2,\dots)$, where t is a constant such that $t<1$ and $0<t\leq(k^2-1)^{1/2}$, satisfies $\|Tx-Ty\|\leq k\cdot\|x-y\|$ for all $x,y\in B$, but it is fixed point free.

This example shows that the Kirk's theorem may fail to hold for the class of mappings T having a Lipschitz constant $k>1$, no matter how near to 1 we choose k. A class intermediate between these and the nonexpansive mappings. is provided by the following. A mapping $T:C\to C$, $C\subset E$, is said to be uniformly k-Lipschitz ($k>1$) if for each $x,y\in C$

$$\|T^n x-T^n y\|\leq k\cdot\|x-y\|,\quad n=1,2,\dots .$$

In [8], there is proved

Theorem 4.2. (Goebel, Kirk, Thale, 1974.) Let E be a Banach space with the characteristic of convexity of E, $\varepsilon_0(E)=\sup\{\varepsilon\in[0,2]: \delta_E(\varepsilon)=0\}<1$. Then there exists a constant $\gamma>1$ such that the uniformly k-Lipschitz mappings have the F.P.P. if $k<\gamma$.

The constant γ is derived from the modulus of convexity of E, that is, the function $\delta_E:[0,2]\to[0,2]$ defined as follows:

$$\delta_E(\varepsilon)=\inf\{1-\tfrac{1}{2}\cdot\|x+y\| : \|x\|\le 1,\ \|y\|\le 1,\ \|x-y\|\ge\varepsilon\}.$$

It was shown in [8] that Theorem 4.2 holds if γ is taken to be the solution of the equation $x\cdot(1-\delta_E(x^{-1}))=1$. In a Hilbert space H this yields $\gamma_H= =\frac{1}{2}\cdot 5^{1/2}$ and in L^p, $p\ge 2$, $\gamma_p=(1+2^{-p})^{1/p}$. Recently, T.C. Lim [13] has defined the extended constant $\overline{\gamma}_p$ in L^p spaces for $p>2$. Let α be the unique solution of the equation

$$(p-2)x^{p-1}+(p-1)x^{p-2}-1=0,\quad 0\le x\le 1,$$

then the extended constant $\overline{\gamma}_p$ in L^p is $\overline{\gamma}_p=\left(1+\dfrac{1+\alpha^{p-1}}{(1+\alpha)^{p-1}}\right)^{1/p}$, $p>2$. For example, for p=3 and 4, we have $\alpha_3=2^{1/2}-1$ and $\alpha_4=\frac{1}{2}$ and

$$\overline{\gamma}_3=(3-2^{1/2})^{1/3} > \gamma_3=(1+2^{-3})^{1/3},$$

$$\overline{\gamma}_4=(\tfrac{4}{3})^{1/4} > \gamma_4=(1+2^{-4})^{1/4}.$$

In a subsequent development, E.A. Lifschitz [12] initiated a more topological approach and considered uniformly Lipschitz mappings in metric spaces. Instead of using the modulus of convexity Lifschitz associated, with each metric space (M,d), a constant $\varkappa(M)$ defined as follows:

$$\varkappa(M)=\sup\{b>0: \bigvee_{a>1}\ \bigwedge_{x,y\in M}\ \bigvee_{r>0} [d(x,y)>r \Longrightarrow \Longrightarrow \bigvee_{z\in M} B(x,br)\cap B(y,ar)\subset B)z,r)]\},$$

where B(x,r) denotes the closed ball of radius r centered at x.

In general, $\varkappa(M)\ge 1$. If E is a Banach space, then

$$\varkappa_0(E)=\inf\{\varkappa(C): C\subset E \text{ is nonempty, bounded, closed, convex}\}>1$$

iff $\varepsilon_0(E)<1$ [5]. In any Hilbert space $\varkappa_0(H)\ge 2^{1/2}$, Lifschitz proved that: if (M,d) is a bounded, complete metric space and if $T:M\to M$ is uniformly k-Lipschitz with $k<\varkappa(M)$, then T has a fixed point in M. Lifschitz theorem combined with Theorem 4.2 implies

Theorem 4.3. (Lifschitz, 1975.) Every Hilbert space has the F.P.P. for uniformly k-Lipschitz mappings with $k<2^{1/2}$.

The first example of a fixed point free uniformly Lipschitz mapping in l^2 (with k=2) was given in [8]. Lifschitz gave an example of a uniformly $\frac{\pi}{2}$-Lipschitz self-mapping of the unit ball of l^2 which is fixed point free (cf. [1]).

Example 4.4. (Lifschitz, 1975.) Let $E=l^2$ with the usual norm $\|x\| = (\sum_{i=1}^{\infty} |x_i|^2)^{1/2}$ and $B=\{x \in l^2 : \|x\| \leq 1\}$. Define $T:B \to B$ by

$$Tx = \begin{cases} (\cos \frac{\pi \cdot \|x\|}{2})\, e_1 + \left(\frac{1}{\|x\|} \sin \frac{\pi \cdot \|x\|}{2}\right) \cdot Px, & x \neq 0, \\ e_1 & , \ x=0, \end{cases}$$

where P is the right shift: $P(x_1,x_2,\ldots)=(0,x_1,x_2,\ldots)$.

For any two points $x \neq y$ in B, consider the curve $p(t)=T((1-t)y+tx)$, $0 \leq t \leq 1$. A computation shows that $\|p'(t)\| < \frac{\pi}{2} \cdot \|x-y\|$ for all t. Therefore $\|Tx-Ty\| \leq \int_0^1 \|p'(t)\|dt < \frac{\pi}{2} \cdot \|x-y\|$. Since P is an isometry and $T^n = P^{n-1} \circ T$; T is uniformly Lipschitz with Lipschitz constant $\frac{\pi}{2}$. But it is fixed point free: if $x=Tx$, then $\|x\|=1$ and $x=Px$, which is im possible.

5. A Banach measure and further results. Let $A \subset N$. A number of elements of the set A will be denoted by $|A|$ and $N_n := \{1,2,\ldots,n\}$, $s_n := \frac{|A \cap N_n|}{n}$ for $n=1,2,\ldots$. The sequence $\{s_n\}$ is bounded.

Definition 5.1 [11]. A Banach measure of a set $A \subset N$ is a number

$$\mu(A) := \underset{n}{\mathrm{LIM}}\, s_n .$$

This measure has the following properties [9]:

1) $0 \leq \mu(A) \leq 1$,
2) $\mu(N)=1$,
3) $(A \cap B = \emptyset) \Rightarrow (\mu(A \cup B) = \mu(A) + \mu(B)$,
4) $(\mu(A)=1) \Rightarrow (\mu(A \cap B) = \mu(B))$,
5) $\mu(A+1) = \mu(A)$, where $A+1 := \{x+1 : x \in A\}$,
6) $\mu(sA) = \frac{1}{s} \cdot \mu(A)$, where $sA := \{sx : x \in A\}$, $s \in N$,
7) $(C=\{c_1,c_2,\ldots\} \subset N \wedge \lim_k \frac{k}{c_k} = r) \Rightarrow (\mu(C)=r)$, where $A,B \subset N$.,

Definition 5.2. Let $A \subset N$ and a Banach measure $\mu(A)=1$. We shall say that $T:C \to C$ is μ-uniformly k-Lipschitz if there exists a constant $k>1$ such that for all $n \in A$ we have

$$\|T^n x - T^n y\| \leq k \cdot \|x-y\| \text{ for each } x,y \in C.$$

Recently M. Krüppel and the present author proved [9]:

Theorem 5.3. Let E be a uniformly convex Banach space. Then there exists a constant $\gamma > 1$ satisfying $\gamma(1-\delta_E(\gamma^{-1}))=1$ such that μ-uniformly k-Lipschitz mappings have the F.P.P. if $k < \gamma$.

If H is a Hilb ert space then Theorem 5.3 is true for $k < \gamma_H = 2^{1/2}$ [11].

6. Main result on fixed point theory. Let $\{x_n\}$ be a bounded sequence in a Banach space $(E, \|\cdot\|)$ and let C be a closed, convex subset of E. Consider the functional $r_\mu : E \to [0,+\infty)$ defined by

$$r_\mu(x,\{x_n\})=\inf\{S\in R: \bigwedge_{\varepsilon>0} \mu[n:\|x_n-x\|< S+\varepsilon]=1\}$$

and call it the μ-radius of $\{x_n\}$ in x. Let

$$r_\mu(C,\{x_n\})=\inf\{r_\mu(x,\{x_n\}):x\in C\},$$

$$\mathscr{C}_\mu(C,\{x_n\})=\{x\in C: r_\mu(x,\{x_n\})=r_\mu(C,\{x_n\})\}$$

and call them: μ-radius of $\{x_n\}$ with respect to C, μ-center of $\{x_n\}$ with respect to C, respectively.

Lemma 6.1. Let E be a Banach space with $\widetilde{N}(E)<1$. Then, for every bounded sequence $\{x_n\}$, there exists a point $z\in \mathrm{clco}\,\{x_n\}$, such that

(i) $r_\mu(z,\{x_n\})\le \widetilde{N}(E)\cdot d(\{x_n\})$,

(ii) $\bigwedge_{y\in E} \|z-y\|\le r_\mu(y,\{x_n\})$,

where $d(\{x_n\})=\lim_k(\sup\{\|x_n-x_m\|:n,m\ge k\})$.

Proof. For each $p\ge 1$, set $A_p=\mathrm{clco}\,\{x_n\}_p^\infty$ and set $A=\bigcap_{p=1}^\infty A_p$. Since each A_p is weakly compact (because E is reflexive) then the set $\mathscr{C}_\mu(A_p,\{x_n\})$, A, $\mathscr{C}_\mu(A,\{x_n\})$ is nonempty. For each p, choose z_p in $\mathscr{C}_\mu(A_p,\{x_n\})$ and consider a weakly convergent subsequence of $\{z_p\}$, say $\{z_{p_n}\}$. Call z its weak limit. Taking into account the monotonicity of the sequence $\{A_p\}$, we obtain that $z\in A$. To prove that (i) holds for z, we observe that $r_\mu(z_p,\{x_n\})$ is a monotone decreasing sequence which has $r_\mu(A,\{x_n\})$ as an upper bound. Moreover, since r_μ is weakly lower semicontinuous, we have

$$\lim_p r_\mu(z_p,\{x_n\})=\lim_j r_\mu(z_{p_j},\{x_n\})\ge r_\mu(z,\{x_n\})\ge r_\mu(A,\{x_n\}).$$

Hence

$$\lim_p r_\mu(z_p,\{x_n\})=r_\mu(z,\{x_n\})=r_\mu(A,\{x_n\}).$$

Since, for any p,

$$r_\mu(z_p,\{x_n\})=r_\mu(z_p,\{x_n\}_p^\infty)\le\widetilde{N}(E)\cdot d(\{x_n\}_p^\infty)=\widetilde{N}(E)\cdot d(\{x_n\}),$$

we obtain

$$r_\mu(z,\{x_n\})\le\widetilde{N}(E)\cdot d(\{x_n\}).$$

Observe that any point $z \in A$ satisfies (ii). In fact z belongs to A_p for each p, hence

$$\|z-y\| \leq \lim_{p}(\sup\{\|x-y\|: x \in A_p\}) \leq r_{\mu}(y,\{x_n\}). \qquad \text{Q.E.D.}$$

Theorem 6.2. Every Banach space E which has the uniformly normal structure has the F.P.P. for μ-uniformly k-Lipschitz mappings with $k<[\tilde{N}(E)]^{-1/2}$.

Proof. Let C be a nonempty, closed, convex, bounded subset of E. Let $T: C \rightarrow C$ be μ-uniformly k-Lipschitz with $k<[\tilde{N}(E)]^{-1/2}$. For any $x \in C$, consider the sequence $\{T^n x\}$ and let $z(x)$ be the point z of Lemma 6.1 corresponding to the sequence $\{T^n x\}$. Set $p(x)=r_{\mu}(x,\{T^n x\})$. By the condition (i) of the Lemma 6.1 we have

$$(1) \quad r_{\mu}(z,\{T^n x\}) \leq \tilde{N}(E)\cdot d(\{T^n x\}) \leq \tilde{N}(E)\cdot \sup\{\|T^n x - T^n x\|: n,m>0\} \leq$$
$$\leq \tilde{N}(E)\cdot k \cdot \sup\{\|T^i x - x\|: i>0\} \leq$$
$$\leq \tilde{N}(E)\cdot k \cdot p(x).$$

Moreover, for $N>1$ we have

$$(2) \quad r_{\mu}(T^N z,\{T^n x\}) = \inf\{S \in R: \bigwedge_{\varepsilon>0} \mu[n: \|T^n x - T^N z\| < S+\varepsilon] = 1\} \leq$$
$$\leq k\cdot \inf\{S \in R: \bigwedge_{\varepsilon>0} \mu[n: \|T^{n-N} x - z\| < S+\varepsilon] = 1\} =$$
$$= k\cdot r_{\mu}(z,\{T^n x\}).$$

Condition (ii) of Lemma 6.1 yields, by (1) and (2),

$$(3) \quad p(z) \leq k^2 \cdot \tilde{N}(E)\cdot p(x) = \xi \cdot p(x) \text{ with } \xi < 1.$$

Define a sequence $\{x_n\}$ in the following way: x_1 is any point of C, $x_{n+1}=z(x_n)$. Then $\{x_n\}$ is a Cauchy sequence. In fact, we have

$$\|x_{n+1}-x_n\| \leq \|x_{n+1}-T^m x_n\| + \|T^m x_n - x_n\| \leq$$
$$\leq \|x_{n+1}-T^m x_n\| + p(x_n) \leq r_{\mu}(x_{n+1},\{T^j x_n\}) + p(x_n) \leq$$
$$\leq k\cdot\tilde{N}(E)\cdot p(x_n) + p(x_n) = (1+k\cdot\tilde{N}(E))\cdot p(x_n).$$

Hence, by (3),

$$\|x_{n+1}-x_n\| \leq (1+k\cdot\tilde{N}(E))\cdot p(x_n) \leq (1+k\cdot\tilde{N}(E))\cdot \xi^n \cdot p(x_1).$$

Let $y=\lim_{n} x_n$. Then $Ty=y$, because

$$\|Ty-y\| \leq \|y-x_n\| + \|x_n - Tx_n\| + \|Tx_n - Ty\| \leq$$
$$\leq \|y-x_n\| + \|x_n - Tx_n\| + k\cdot\|x_n - y\| \leq (1+k)\cdot\|x_n-y\| + p(x_n) \leq$$
$$\leq (1+k)\cdot\|x_n - y\| + \xi^n \cdot p(x_1) \longrightarrow 0 \text{ as } n \longrightarrow \infty. \qquad \text{Q.E.D.}$$

7. Remarks and consequences.

I. It was proved in [3] that $\tilde{N}(E) \leq 1-\delta_E(1)$, thus $\varepsilon_0(E)<1$ implies the uniformly normal structure, therefore Theorem 6.2 generalizes Theorem 5.3 and Theorem 4.2.

II. Combining Theorems 3.3 and 6.2, we have

Corollary 7.1. For $\beta < 2^{1/2}$, E_β has the F.P.P. for μ-uniformly k-Lipschitz mappings with $k < 2^{1/4} \cdot \beta^{-1/2}$.

III. Lemma 7.2. Let E be a Banach space with the uniformly normal structure, i.e. $\tilde{N}(E) < 1$, and let $\gamma > 1$ satisfy $\gamma(1-\delta_E(\gamma^{-1}))=1$. If $\tilde{N}(R) < \frac{4}{5}$ then $\gamma < \gamma_0 = [\tilde{N}(E)]^{-1/2}$.

Proof. Since $f(x)=x\cdot(1-\delta_E(x^{-1}))$ is an increasing function in $[1,2]$, it is enough to show that $f([\tilde{N}(E)]^{-1/2}) > 1$ for $\tilde{N}(E) < \frac{4}{5}$. From the Nordlander inequality $\delta_E(\varepsilon) \leq 1-(1-\frac{1}{4}\cdot\varepsilon^2)^{1/2}$ we obtain

$$f([\tilde{N}(E)]^{-1/2}) \geq ([\tilde{N}(E)]^{-1} - \tfrac{1}{4})^{1/2} > 1 \text{ for } \tilde{N}(E) < \tfrac{4}{5}. \quad \text{Q.E.D.}$$

Corollary 7.3. For $1 \leq \beta < \frac{4}{5}\cdot 2^{1/2}$, the condition $k < [\tilde{N}(E)]^{-1/2}$ is weaker than $k < \gamma$, where γ is the unique solution of $x(1-\ _E(x^{-1}))=1$.

8. Open problems

1. Is the Lifschitz theorem true for $2^{1/2} \leq k < \frac{\pi}{2}$?
2. Do James spaces E_β have the F.P.P. for $\beta > 2$?
3. $\tilde{N}(L^P)$ = ?

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