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ANNOUNCEMENTS OF NEW RESULTS

DOUBLE LAYER POTENTIALS ON BOUNDARIES WITH CORNERS AND EDGES

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We say that a bounded open set $D \subset R^3$ is rectangular if each point z in the boundary ∂D of D has a neighbourhood $U \subset \partial D$ homeomorphic with R^2 such that U is contained in the three planes passing through z parallel to the coordinate planes. If $z \in \partial D$ is not situated on an edge then n(z) denotes the unit exterior normal to D at z; otherwise n(z)=0 (= the zero vector in R^3). $C(\partial D)$ stands for the space of all continuous functions on ∂D and H_2 denotes the surface measure on ∂D. For each $f \in C(\partial D)$ the double layer potential

$$Wf(x) = (4\pi)^{-1} \int_{\partial D} f(z)n(z)(z-x)/|z-x|^3 \, dH_2(z)$$

is a harmonic function of the variable $x \in R^3 \setminus \partial D$ admitting a continuous extension from D to ∂D: $W_i f(z) = \lim_{\substack{x \to z \\ x \in D}} Wf(x)$, $z \in \partial D$. $W_i : f \mapsto W_i f$ is a bounded linear operator acting on $C(\partial D)$. Let us denote by $\| \cdot \|$ the usual maximum norm, by I the identity operator, by Q the space of all compact linear operators on $C(\partial D)$. As shown in [1] J. Král and W. Wendland: Some examples concerning applicability of the Fredholm-Radon method in potential theory, Aplikace matematiky 31(1986).
it may happen for simple rectangular sets that

$$\inf \{ \| W_i - \alpha I - T \| ; \ T \in Q \} / |\alpha| \geq 1$$

for each value of the parameter $\alpha \neq 0$. The ideas described in [1] together with some geometrical considerations permit to establish the following result.

Theorem. For each rectangular set D there is a norm p inducing the topology of uniform convergence on $C(\partial D)$ such that

$$2 \inf \{ p(W_i - \tfrac{1}{2} I - T); \ T \in Q \} < 1.$$

This result has applications in connection with potential-theoretic boundary value problems; it implies, in particular, that for each rectangular set D with a connected complement the corresponding operator W_i is invertible on $C(\partial D)$ (cf. [1]).