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Abstract: We introduce a concept of $\mathcal{C}$-paracompactness as a generalization of various types of paracompactness in uniform spaces. We prove some basic properties and give some characterizations. As an application, we give a characterization of uniformly $\mathcal{m}$-paracompact spaces analogous to Morita theorem on $\mathcal{m}$-paracompact spaces.

Key words: $\mathcal{C}$-paracompact, uniformly $\mathcal{m}$-paracompact.

Classification: 54E15

Introduction. Various properties of paracompact type have been studied in the literature (e.g. [2],[6],[7],[11]). We introduce a common generalization of these concepts, the concept of $\mathcal{C}$-paracompact space. We show that some basic properties are due to this general setting. As an application, we give a characterization of $\mathcal{m}$-uniformly paracompact spaces analogical to the famous Morita theorem [10] concerning $\mathcal{m}$-paracompact topological spaces.

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1. $\mathcal{C}$-paracompact spaces. Saying "space" we always mean, if not stated otherwise, a uniform Hausdorff space. Compact topological spaces are, of course, understood as spaces with the

natural uniformity. A compactification of a space is just some Hausdorff compactification of the underlying topological space.

Let the function $\mathscr{C}$ assign to each space X the collection $\mathscr{C}_X$ of open covers of X in such a manner that the two following conditions are satisfied:

(i) let $\mathcal{W}$ be an open cover of X. Then $\mathcal{W} \in \mathscr{C}_X$, provided $\mathcal{W}$ is refined by some cover belonging to $\mathscr{C}_X$;

(ii) let $f: Y \longrightarrow X$ be a uniform map, let $\mathcal{V} \in \mathscr{C}_X$. Then $f_{-1}(\mathcal{V}) \in \mathscr{C}_Y$.

<u>Definition 1.1.</u> a) A space X is $\mathscr{C}$-paracompact if for any open cover $\mathcal{V}$ of X, the cover $\mathcal{V}^{<\omega}$ (consisting of unions of finite subfamilies of $\mathcal{V}$) belongs to $\mathscr{C}_X$.

b) Let $\mathfrak{m}$ be an infinite cardinal number. A space X is $\mathfrak{m}$-$\mathscr{C}$-paracompact, if for any open cover $\mathcal{V}$ of X, $|\mathcal{V}| \leq \mathfrak{m}$, the cover $\mathcal{V}^{<\omega}$ belongs to $\mathscr{C}_X$.

<u>Remark.</u> Obviously, a space is $\mathscr{C}$-paracompact, if and only if it is $\mathfrak{m}$-$\mathscr{C}$-paracompact for any $\mathfrak{m}$.

<u>Proposition 1.2.</u> Let $f: X \longrightarrow Y$ be a perfect map onto. Let $f_{-1}(\mathscr{C}_Y) \subset \mathscr{C}_X$. Then, X is $\mathfrak{m}$-$\mathscr{C}$-paracompact, provided Y is $\mathfrak{m}$-$\mathscr{C}$-paracompact.

<u>Corollary 1.</u> $X \times K$ is $\mathfrak{m}$-$\mathscr{C}$-paracompact (resp. $\mathscr{C}$-paracompact), provided X is $\mathfrak{m}$-$\mathscr{C}$-paracompact (resp. $\mathscr{C}$-paracompact) and K is a compact space.

<u>Corollary 2.</u> Let X and X be topologically equivalent spaces, let X' be finer than X. Then X' is $\mathfrak{m}$-$\mathscr{C}$-paracompact, provided X is $\mathfrak{m}$-$\mathscr{C}$-paracompact.

Corollary 1 follows from the fact that the projection $X \times K \to X$ is a perfect map. Corollary 2 is a trivial observation, which can be proved immediately.

Proof of the proposition: let $\mathcal{V}$ be an open cover of X, $|\mathcal{V}| \leq m$. Since f is perfect, $\mathcal{V}^{<\omega} \succ \{f_{-1}(y) \mid y \in Y\}$ and we can take for any y an open neighborhood U_y of y in such a way that

$$f_{-1}(\{U_y \mid y \in Y\}) \prec \mathcal{V}^{<\omega}.$$ For $W \in \mathcal{V}^{<\omega}$ defi

$$U_W = \bigcup \{U_y \mid f_{-1}(U_y) \subset W\}.$$

Since Y is m-$\mathcal{C}$-paracompact, the cover $\{U_W \mid W \in \mathcal{V}^{<\omega}\}^{<\omega}$ belongs to $\mathcal{C}_Y$. Therefore, $\mathcal{V}^{<\omega}$ belongs to $\mathcal{C}_X$, because

$$f_{-1}(\{U_W \mid W \in \mathcal{V}^{<\omega}\}^{<\omega}) \prec \mathcal{V}^{<\omega}.$$

Paracompactness can be also described by means of some completeness condition. This was done by Corson in [1]. The immediate generalization of his weakly Cauchy filters works for $\mathcal{C}$-paracompact spaces.

<u>Definition 1.3</u>. A filter $\mathcal{F}$ of subsets of a space X is weakly $\mathcal{C}$-Cauchy if for any cover $\mathcal{V} \in \mathcal{C}_X$ there is $V \in \mathcal{V}$ such that $\{V\} \cup \mathcal{F}$ is a centered system.

<u>Theorem 1.4</u>. A space is m-$\mathcal{C}$-paracompact iff any weakly $\mathcal{C}$-Cauchy filter with a base of cardinality at most m has a limit point.

<u>Corollary</u>. A space is $\mathcal{C}$-paracompact iff any weakly $\mathcal{C}$-Cauchy filter has a limit point.

Proof of the theorem: a) Let $\mathcal{F}$ be a base of weakly $\mathcal{C}$-Cauchy filter, $|\mathcal{F}| \leq m$. Let the filter have no limit point.

Then, for any $x \in X$ there are an open neighborhood U_x of x and $F_x \in \mathcal{F}$ such that $U_x \cap F_x = \emptyset$. For $F \in \mathcal{F}$ define

$$U_F = \bigcup \{ U_x \mid F_x = F \}.$$

Then the cover $\mathcal{W} = \{U_F \mid F \in \mathcal{F}\}^{<\omega}$ belongs to $\mathcal{C}_X$. But obviously $\{W\} \cup \mathcal{F}$ is centered for no $W \in \mathcal{W}$. Contradiction.

b) Let $\mathcal{U}$ be an open cover of X, $|\mathcal{U}| \le m$. Let $\mathcal{U}^{<\omega} \notin \mathcal{C}_X$. It means that for any $\mathcal{W} \in \mathcal{C}_X$ there is $W \in \mathcal{W}$ such that W cannot be covered by finitely many sets from $\mathcal{U}$. Therefore, the filter generated by the base

$$\{X - \bigcup \mathcal{U}' \mid \mathcal{U}' \in [\mathcal{U}]^{<\omega}\}$$

is a weakly $\mathcal{C}$-Cauchy filter without a limit point. Contradiction.

The classical theorem of Tamano can be easily modified to

<u>Theorem 1.5</u>. Let K be a compactification of a space X. Then, X is $\mathcal{C}$-paracompact iff for any compact $C \subset K - X$ there exists a cover $\mathcal{V} \in \mathcal{C}_X$ such that for any $V \in \mathcal{V}$ $\overline{V}^K \cap C = \emptyset$.

Proof: a) Let $C \subset K - X$ be a compact set. Then for any $x \in X$ there is an open neighborhood V_x of x such that $\overline{V}_x^K \cap C = \emptyset$. The desired cover is, of course,

$$\{V_x \mid x \in X\}^{<\omega}.$$

b) Let $\mathcal{W}$ be an open cover of X. Take for any $W \in \mathcal{W}$ a set $\widetilde{W}$ open in K such that $W = \widetilde{W} \cap X$. Take the compact set $C = K - \bigcup \{\widetilde{W} \mid W \in \mathcal{W}\}$. Let $\mathcal{V} \in \mathcal{C}_X$ be such a cover that for any $V \in \mathcal{V}$ $\overline{V}^K \cap C = \emptyset$. Since $\overline{V}^K$ is a compact subset of $K - C$, it can be covered by finitely many $\widetilde{W}$'s. Hence, $\mathcal{V} \prec \mathcal{W}^{<\omega}$.

<u>Remark</u>. $\mathcal{C}$-paracompact spaces need not have paracompact topology. In fact, taking for $\mathcal{C}_X$ all open covers of a space X, we have all spaces $\mathcal{C}$-paracompact.

Lemma 1.6. $\mathcal{C}$-paracompact space X has a paracompact topology iff any cover belonging to $\mathcal{C}_X$ admits a locally finite open refinement.

Proof: a) "only if" is trivial.

b) Let $\mathcal{V}$ be an open cover of X. Let $\mathcal{W}$ be an open locally finite refinement of $\mathcal{V}^{<\omega}$. Take for any $W \in \mathcal{W}$ a family $\mathcal{V}_W \in [\mathcal{V}]^{<\omega}$ such that $W \subset \bigcup \mathcal{V}_W$. Thus,

$\{W \cap V \mid W \in \mathcal{W}, V \in \mathcal{V}_W\}$ is the desired refinement of $\mathcal{V}$.

The case that $\mathcal{C}$-paracompact spaces are always topologically paracompact is the most important. The lemma 1.6 says that we must restrict ourselves to mappings $\mathcal{C}$ with the additional property:

for any space X all covers from $\mathcal{C}_X$ have locally finite open refinement.

We will call such a mapping good.

Definition 1.7. A collection $\mathcal{A}$ of subsets of a space X is $\mathcal{C}$-locally finite if there exists $\mathcal{V} \in \mathcal{C}_X$ such that for each $V \in \mathcal{V}$ the set

$V(\mathcal{A}) = \{A \mid A \in \mathcal{A}, A \cap V \neq \emptyset\}$ is finite.

Proposition 1.8. Let $\mathcal{C}$ be good. Then the following properties of a space X are equivalent:

(i) X is $\mathcal{C}$-paracompact,

(ii) X has a paracompact topology and locally finite collections coincide with $\mathcal{C}$-locally finite collections,

(iii) any open cover of X admits a $\mathcal{C}$-locally finite open refinement.

Proof is straightforward.

It is also possible to give a characterization of $\mathfrak{m}$-$\mathcal{C}$-paracompactness by means of some properties of products with compact spaces. The characterization is not very nice in the general case, but useful for applications.

We call the subsets of the form $X \times \{k\}$ of the product $X \times K$ slices.

Proposition 1.9. Let X be an $\mathfrak{m}$-$\mathcal{C}$-paracompact space. Let K be a compact space, let $C \subset K$ be a closed set, $\chi(K) \leq \mathfrak{m}$. Then for any closed $F \subset X \times K$ disjoint to $X \times C$ there exists an open cover $\mathcal{W}$ of $X \times K$ of the form

(+) $\mathcal{W} = \{U \times V \mid U \in \mathcal{U}, V \in \mathcal{V}_U\}$.

$\mathcal{U}$ belonging to $\mathcal{C}_X$, each $\mathcal{V}_U$ being an open cover of K, such that

$St(X \times C, \mathcal{W}) \cap F = \emptyset$.

Proof: let $\mathcal{H} = \{H_\alpha \mid \alpha < \mathfrak{m}\}$ be a basis of open neighborhoods of C. Let F be a closed subset of $X \times K$ disjoint to $X \times C$. Then for any $x \in X$ there exist an open neighborhood V_x of x and $H_x \in \mathcal{H}$ such that $\overline{V}_x \times \overline{H}_x \cap F = \emptyset$. Define $V_\alpha = \cup\{V_x \mid H_x = H_\alpha\}$. Take $\mathcal{U} = \{V_\alpha \mid \alpha < \mathfrak{m}\}^{<\omega}$. Then $\mathcal{U} \in \mathcal{C}_X$ and for any $U \in \mathcal{U}$ there is an $H_U \in \mathcal{H}$ such that $U \times \overline{H}_U \cap F = \emptyset$. From the normality of K there exists an open cover $\{L_1^U, L_2^U\}$ such that $C \subset L_1^U$, $C \cap L_2^U = \emptyset$, $L_1^U \subset H_U$, $L_2^U \supset K - H_U$.

The desired cover is

$\mathcal{W} = \{U \times L_i^U \mid U \in \mathcal{U}, \cancel{i \in}\{1,2\}\}$.

If the statement of the proposition holds, for any slice, we say for the brevity that slices can be separated from closed sets. The symbol I denotes the unit interval.

Theorem 1.10. The following properties of a space X are equivalent:

(i) X is $\mathfrak{m}$-$\mathcal{C}$-paracompact,

(ii) slices in $X \times I^{\mathfrak{m}}$ can be separated from closed sets,

(iii) slices in $X \times \{0,1\}^{\mathfrak{m}}$ can be separated from closed sets.

Proof: (i)$\rightarrow$(ii) Proposition 1.9.

(ii)$\rightarrow$(iii) it is obvious, since $I^{\mathfrak{m}}$ contains $\{0,1\}^{\mathfrak{m}}$.

(iii)$\rightarrow$(i). Let $\mathcal{V} = \{V_\alpha \mid \alpha < \mathfrak{m}\}$ be an open cover of X. Denote π_α the α-th projection $\{0,1\}^{\mathfrak{m}} \rightarrow \{0,1\}$. Denote $U_\alpha = V_\alpha \times \pi_\alpha^{-1}(0)$. Then $F = X \times \{0,1\}^{\mathfrak{m}} - \bigcup_{\alpha < \mathfrak{m}} U_\alpha$ is a closed set, disjoint to the slice $X \times \{\underline{0}\}$ ($\underline{0}$ denotes here a point with all coordinates equal to zero). Let $\mathcal{W} = \{U \times V \mid U \in \mathcal{U}, V \in \mathcal{V}_U\}$ be a separating cover of the form (+). Take for any $U \in \mathcal{U}$ some $V_U \in \mathcal{V}_U$ such that $\underline{0} \in V_U$. Then there is a finite set $Z_U \subset \mathfrak{m}$ such that

$$A_U = \bigcap_{\alpha \in Z_U} \pi_\alpha^{-1}(0) \subset V_U$$

Take $t \in A_U$ such that $\pi_\alpha(t) = 0$ iff $\alpha \in Z_U$. Since $U \times \{t\} \subset \subset \bigcup V_\alpha \times \pi_\alpha^{-1}(0)$, clearly

$$U \times \{t\} \subset \bigcup_{\alpha \in Z_U} V_\alpha \times \pi_\alpha^{-1}(0) \text{ and } U \subset \bigcup_{\alpha \in Z_U} V_\alpha .$$

Thus, $\mathcal{U} \prec \mathcal{V}^{<\omega}$.

Analogously, one can give a characterization of $\mathcal{C}$-paracompact spaces.

Theorem 1.11. The following properties of a space X are equivalent:

(i) X is $\mathcal{C}$-paracompact,

(ii) for some (and then any) compactification K of X it is possible to separate any set of the form $X \times C$, C being a compact subset of K - X, from the diagonal $\Delta_X = \{\langle x,x\rangle \mid x \in X\}$.

Proof: (i)$\longrightarrow$(ii). Proposition 1.9 (Δ_X is closed).

(ii)$\longrightarrow$(i). Let $C \subset K - X$ be compact, let $\mathcal{W}$ be a separating cover of the form (+), $\mathcal{W} = \{U \times V \mid U \in \mathcal{U}, V \in \mathcal{V}_U\}$. Let for some $U \in \mathcal{U}$ there is $y \in \bar{U}^K \cap C$. Take $V \in \mathcal{V}_U$ such that $y \in V$. Thus, $U \times V \cap X \times C \neq \emptyset \neq U \times V \cap \Delta_X$, which is impossible. X is, therefore, $\mathcal{C}$-paracompact according to Theorem 1.5.

The last remark in this section concerns the case of locally compact spaces.

Observation 1.12. Let X be a locally compact space. Then, X is $\mathcal{C}$-paracompact iff there exists $\mathcal{V} \in \mathcal{C}_X$ such that $\bar{V}$ is compact for any $V \in \mathcal{V}$.

Proof: routine.

2. Examples. We give here a short list of some concepts of paracompactness and of corresponding mappings $\mathcal{C}$.

concept	$\mathcal{C}_X$ = covers with refinement
paracompact uniform spaces [2],[5]	open σ-uniformly discrete
uniformly paracompact spaces [11]	open uniform
uniformly para-Lindelöf spaces [7]	open uniformly locally countable
uniformly hypocompact [7] spaces	open star-finite uniform
supercomplete spaces [8]	open, uniform in locally fine coreflection of X
compact spaces	$\{X\}$

spaces with uniformly paracompact metric-fine coreflection [6]	open, uniform in metric-fine coreflection

It is not very difficult to find $\mathcal{C}$ for these examples. In fact, any combinatorial property, preserved by uniform preimages, generates in an obvious way some mapping $\mathcal{C}$.

3. Uniformly m-paracompact spaces. It is not surprising that inner properties of $\mathcal{C}$ can provide us more interesting theorems (see e.g. [5],[6]). We turn our attention to m - $\mathcal{C}$-paracompact spaces for one special $\mathcal{C}$. In this section $\mathcal{C}_X$ is always formed by open uniform covers. m - $\mathcal{C}$-paracompact spaces are called uniformly m-paracompact. We shall prove theorems analogous to the famous theorems of Morita [10] and Tamano [12].

We shall use the semi-uniform product $*$ of J.R. Isbell [8]. The important role of $*$ in this context was pointed out by Z. Frolík [6]. We recall briefly the basic properties of the product $*$.

The product $X * Y$ of the spaces X and Y is fully described by the natural equivalence

$U(X * Y,-) \simeq U(X,U(Y,-))$;

$*$ is the tensor product of the category of uniform spaces. ($U(X,Y)$ is, of course, the set of uniform maps from X to Y endowed with the uniformity of uniform convergence.)

We shall need the following properties (see [8]): Let Y be a compact space. Then

1) $X * Y$ is topologically equivalent to $X \times Y$,

2) any uniform cover of $X * Y$ has a refinement of the form (+) (see section 1),

3) if X has a basis consisting of point-finite uniform covers, then all covers of the form (+) are uniform in $X * Y$,

4) $X * Y$ is a fine space, provided X is fine.

We need the following

<u>Lemma 3.1.</u> Let X be uniformly $\mathfrak{m}$-paracompact. Then

a) uniform covers of X of cardinality $\leq \mathfrak{m}$ form a uniformity,

b) this uniformity has a point-finite basis.

<u>Remark.</u> The part a) is due to Jan Pelant, the part b) is a slight modification of one observation of A. Hohti [7].

Proof: a) it is enough to show that any uniform cover $\mathcal{U} = \{U_\alpha \mid \alpha < \mathfrak{m}\}$ has a uniform refinement $\mathcal{V}$, $|\mathcal{V}| \leq \mathfrak{m}$, such that $\{St(x, \mathcal{V}) \mid x \in X\} \prec \mathcal{U}$.

Take $\mathcal{U}_1$ an uniform cover such that $\mathcal{U}_1 \overset{*}{\prec} \mathcal{U}$. Take for any $V \in \mathcal{U}_1$ such $U_V \in \mathcal{U}$ that $St(V, \mathcal{U}_1) \subset U_V$. Define for $\alpha < \mathfrak{m}$ $U'_\alpha = \bigcup\{V \mid V \in \mathcal{U}_1, U_V = U_\alpha\}$. Thus, $|\{U'_\alpha\}| \leq \mathfrak{m}$ and $\{St(U', \mathcal{U}_1) \mid U \in \mathcal{U}\} \prec \mathcal{U}$. Let $\mathcal{W}$ be an open refinement of $\{U'_\alpha\}$, uniformly locally finite with respect to the cover $\{St(W, \mathcal{W}_1) \mid W \in \mathcal{W}_1\}$, $\mathcal{W}_1$ being a uniform cover refining $\mathcal{U}_1$. We may and shall suppose that $\mathcal{W} = \{W_\alpha \mid \alpha < \mathfrak{m}\}$, $W_\alpha \subset U'_\alpha$. Define for $W \in \mathcal{W}_1$ the set $L(W) = \{\alpha \mid St(W, \mathcal{W}_1) \cap W_\alpha \neq \emptyset\}$. Define the equivalence $\sim$ on $\mathcal{W}_1$: $W \sim W'$ iff $L(W) = L(W')$. Take the cover

$\mathcal{V} = \{\bigcup[W]_\sim \mid W \in \mathcal{W}_1\}$. Then $|\mathcal{V}| \leq \mathfrak{m}$.

Take $x \in X$. Then x belongs exactly to the sets $W_{\alpha_1}, \dots, W_{\alpha_n} \in \mathcal{W}$. Let $x \in \bigcup[W]_\sim$. Then $L(W) \supset \{\alpha_1, \dots, \alpha_n\}$. Since any $W' \sim W$ intersects W_{α_1},

$$\bigcup[W]_\sim \subset St(W_{\alpha_1}, \mathcal{W}_1) \subset St(U'_{\alpha_1}, \mathcal{U}_1) \subset U_{\alpha_1}.$$

Hence, $\{St(x,\mathcal{V}) \mid x \in X\} \prec \mathcal{U}$.

b) Let $\mathcal{V}$, $\mathcal{U}$ be uniform covers of X, $|\mathcal{U}| \cdot |\mathcal{V}| \leq m$, $\mathcal{V} \overset{*}{\prec} \mathcal{U}$. Take an open refinement $\mathcal{H}$ of $\mathcal{V}$, uniformly locally finite with respect to the uniform cover $\{St(W,\mathcal{W}) \mid W \in \mathcal{W}\}$, $\mathcal{W}$ being a uniform cover, $\mathcal{W} \prec \mathcal{V}$. We may and shall suppose that $|\mathcal{H}| \leq m$. Then $\{St(H,\mathcal{W}) \mid H \in \mathcal{H}\}$ is the desired point-finite uniform refinement of $\mathcal{U}$.

Theorem 3.2. The following properties of a space X are equivalent:

(i) X is uniformly m-paracompact,

(ii) slices in $X * I^{m}$ can be separated by means of uniformly continuous functions,

(iii) slices in $X * \{0,1\}^{m}$ can be separated by means of uniformly continuous functions.

(Separation means that for any closed set F disjoint to a slice $X \times C$ there is a uniformly continuous function f such that $f \restriction F = 0$ and $f \restriction X \times C = 1$.)

Proof: it follows immediately from the preceding lemma, Theorem 1.10 and properties 2 and 3 of the product $*$.

Analogously, one gets from the theorem 1.11:

Theorem 3.3([4]). The following properties of a space X are equivalent:

(i) X is uniformly paracompact,

(ii) for some (and then any) compactification K of X it is possible to separate any set $X \times C$, C being a compact subset of K - X, from Δ_X by means of uniformly continuous function.

Remark. For locally fine spaces uniform m-paracompactness is equivalent to the condition:

(iv) slices in $X * T(m + 1)$ can be separated by means of uniformly continuous functions ($T(m + 1)$ is the set of ordinals less or equal to m with the order topology).

The proof uses a simple induction. In general, (iv) is not equivalent to (i):

<u>Example</u>: Take a set X, $|X| = \aleph_\omega$. Let the uniformity on X be generated by all partitions of X into less than $\aleph_\omega$ pieces. Then, slices in $X * T(\aleph_\omega + 1)$ can be separated (Proposition 1.9), but X is not uniformly $\aleph_\omega$-paracompact - the cover consisting of singletons has no uniformly locally finite open refinement.

J.E. Mack studied [9] topological spaces whose countable directed open covers are normal - so called cb-spaces (topological spaces are supposed to be T_1-completely regular). Generalizing this concept in an obvious way to m-cb-spaces we can state

<u>Theorem 3.4</u>. The following properties of a T_1-completely regular topological space are equivalent:

(i) X is an m-cb-space,

(ii) slices in $X \times I^{m}$ can be separated by means of continuous functions,

(iii) slices in $X \times \{0,1\}^{m}$ can be separated by means of continuous functions,

(iv) slices in $X \times T(m + 1)$ can be separated by means of continuous functions.

Proof: follows easily from the theorems of this section and the property 4 of the semi-uniform product.

<u>Concluding remark</u>. Most of the material of this note is contained in the thesis [3].

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