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Petr Simon

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A CLOSED SEPARABLE SUBSPACE NOT BEING A RETRACT OF $\beta\mathbb{N}$

Petr Simon (Mathematical Institute of Charles University, Sokolovská 83, 18600 Praha, Czechoslovakia), oblatum 17.4. 1984.

D. Maharam [M] proved that the following are equivalent:

(a) For each ideal $I \subseteq \mathcal{P}(\mathbb{N})$, if there is a one-to-one homomorphism from $\mathcal{P}(\mathbb{N})/I$ to $\mathcal{P}(\mathbb{N})$, then there is a lifting from $\mathcal{P}(\mathbb{N})/I$ to $\mathcal{P}(\mathbb{N})$, too;

(b) every non-void closed separable subspace of $\beta\mathbb{N}$ is a retract of $\beta\mathbb{N}$,

and has raised the question, whether (a) or (b) is a true statement.

The answer to the Maharam's problem is in negative. We can prove the two theorems below.

Theorem 1. There exists a subspace $X \subseteq \beta\mathbb{N} - \mathbb{N}$ satisfying the following:

(1) $X = \bigcup_{n\in\omega} X_n$, where $|X_0| = 1$ and for each $n \in \omega$, the set X_n is countable discrete;

(2) for each $n < m < \omega$, $X_n \subseteq \overline{X}_m - X_m$;

(3) for each $n < \omega$ and for each $x \in X_n$, x is a $\mathfrak{c}$ - OK point in $\overline{X}_{n+1} - X_{n+1}$;

(4) suppose $\{U_k : k \in \omega\} \subseteq \mathcal{P}(\mathbb{N})$ to be a family of sets such that for some $n_0 < \omega$, $U_0^* \cap X_{n_0}$ is finite and for each $i < k < \omega$, $U_i^* \cap X_{n_0+i} \subseteq U_k^*$. Then there is a family $\{V_\alpha : \alpha \in \mathfrak{c}\} \subseteq \mathcal{P}(\mathbb{N})$ such that for each $\alpha \in \mathfrak{c}$, $V_\alpha^* \supseteq X \cap \bigcap_{k\in\omega} U_k^*$ and for each $k < \omega$ and for each finite set $\alpha_0 < \alpha_1 < \dots < \alpha_k < \mathfrak{c}$, $\bigcap_{i\le k} V_{\alpha_i}^* \subseteq \bigcap_{i\le k} U_i^*$;

(5) for each mapping $f: \mathbb{N} \to X$ there is a set $T \subseteq \mathbb{N}$ and an integer $n_1 < \omega$ such that $T^* \cap X \neq \emptyset$ and for each $n > n_1$, $X_n \cap \overline{f[T]} \cap X_{n+1} = \emptyset$.

Theorem 2. If a subspace $X \subseteq \beta\mathbb{N}$ satisfies (1) - (5) from Theorem 1, then $\overline{X}$ is not a retract of $\beta\mathbb{N}$.

It should be noted that the first example of a closed separable subspace of $\beta\mathbb{N}$ which is not a retract of $\beta\mathbb{N}$ was given by M. Talagrand under CH in [T] and the second one by A. Szymański under MA in [S].

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SHORT BRANCHES IN RUDIN-FROLÍK ORDER

Eva Butkovičová (MÚ SAV, Jesenná 5, 04154 Košice,Československo),
oblatum 27.4. 1984.

Rudin-Frolík order of types of ultrafilters in βN has the following properties:

(1) each type of ultrafilters has at most $2^{\aleph_0}$ predecessors, [2],

(2) the cardinality of each branch is at least $2^{\aleph_0}$.

Thus, in Rudin-Frolík order the cardinality of branches can be only $2^{\aleph_0}$ or $(2^{\aleph_0})^+$. It was shown in [1] that there exists a chain order - isomorphic to $(2^{\aleph_0})^+$. Hence, the existence of a branch of cardinality $(2^{\aleph_0})^+$ is proved.

The following result solves the problem of the existence of a branch having smaller cardinality.

Theorem. In Rudin-Frolík order there exists an unbounded chain order-isomorphic to ω_1.

By the properties (1) and (2) the branch containing this chain has cardinality $2^{\aleph_0}$.

References: [1] E. Butkovičová: Long chains in Rudin-Frolík order, Comment. Math. Univ. Carolinae 24(1983), 563-570.
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RESULTS ON DISJOINT COVERING SYSTEMS ON THE RING OF INTEGERS

Ivan Korec, Department of Algebra, Faculty of Mathematics and Physics of Comenius University, 84215 Bratislava, Czechoslovakia
oblatum 12.4. 1984.

A system of congruence classes

(1) $a_1 (\mathrm{mod}\ n_1),\ a_2 (\mathrm{mod}\ n_2),\ \ldots,\ a_k (\mathrm{mod}\ n_k)$

will be called a disjoint covering system (DCS) if for every integer x there is exactly one $i \in \{1, 2, \ldots, k\}$ such that $x \equiv a_i (\mathrm{mod}\ n_i)$. The integers $n_1, n_2, \ldots, n_k$ will be called moduli of (1) and their least common multiple will be called the common modulus of (1).

If $k > 1$ then no two moduli of (1) are relatively prime. This condition can be expressed in the form

(2) $$\bigwedge_{i=1}^{k} \bigwedge_{j=1}^{k} \varphi(n_i, n_j)$$

where $\varphi(x, y)$ is the formula

$$\exists z\, \exists u\, \exists v\, (z \neq 1 \wedge z.u = x \wedge z.v = y)$$

Consider more generally the formulae of the form