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Extensions of mappings from products

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## ANNOUNCEMENT OF NEW RESULTS

### EXTENSIONS OF MAPPINGS FROM PRODUCTS

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In the following results,  $\{X_i\}$  is a family of metric spaces,  $X$  is a subset of  $\prod X_i$  such that  $\bar{X}$  is regularly closed.

Proposition. For every locally finite cover  $\mathcal{U}$  of  $X$  composed of sets regularly open in  $X$  there exists a  $\sigma$ -discrete (in  $\prod X_i$ ) locally finite (in the  $G_\sigma$ -closure of  $X \cup (\prod X_i - \bar{X})$ ) collection  $\mathcal{V}$  composed of basic open sets in  $\prod X_i$  such that the trace of  $\mathcal{V}$  on  $X$  refines  $\mathcal{U}$ .

Corollaries: 1. The fine uniformity of  $X$  is the restriction of the fine uniformity of the  $G_\sigma$ -closure of  $X \cup (\prod X_i - \bar{X})$ .

2 (Štěpín). Every regularly closed subset of  $\prod X_i$  is a zero set.

3. Every continuous mapping on  $X$  into a Banach space (normed space if  $\bar{X}$  is closed) can be continuously extended onto the  $G_\sigma$ -closure of  $X \cup (\prod X_i - \bar{X})$ , in particular, onto  $\prod X_i$  if  $\bar{X} - X$  contains no nonvoid  $G_\sigma$ -subset of  $\prod X_i$ .

4. Every continuous mapping on  $X$  into a topologically complete space (e.g. into a paracompact or realcompact space) can be continuously extended onto the  $G_\sigma$ -closure of  $X$ .

5 (Pelant). Locally fine spaces are subfine.

The above results can be applied e.g. when  $X$  contains a  $\Sigma$ -product of  $\{X_i\}$  or is regularly closed, or as the description of the fine uniformity on  $\prod X_i$ .

In the case that  $\text{pr}_J X = \prod_J X_i$  for all countable  $J$ , we can prove an analogy of the Proposition also for paracompact  $p$ -spaces  $X_i$ .