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Extensions of mappings from products

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ANNOUNCEMENT OF NEW RESULTS

EXTENSIONS OF MAPPINGS FROM PRODUCTS

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In the following results, $\{X_i\}$ is a family of metric spaces, X is a subset of $\prod X_i$ such that $\bar{X}$ is regularly closed.

Proposition. For every locally finite cover $\mathcal{U}$ of X composed of sets regularly open in X there exists a σ-discrete (in $\prod X_i$) locally finite (in the G_δ-closure of $X \cup (\prod X_i - \bar{X})$) collection $\mathcal{V}$ composed of basic open sets in $\prod X_i$ such that the trace of $\mathcal{V}$ on X refines $\mathcal{U}$.

Corollaries: 1. The fine uniformity of X is the restriction of the fine uniformity of the G_δ-closure of $X \cup (\prod X_i - \bar{X})$.

2 (Ščepin). Every regularly closed subset of $\prod X_i$ is a zero set.

3. Every continuous mapping on X into a Banach space (normed space if X is closed) can be continuously extended onto the G_δ-closure of $X \cup (\prod X_i - \bar{X})$, in particular, onto $\prod X_i$ if $\bar{X} - X$ contains no nonvoid G_δ-subset of $\prod X_i$.

4. Every continuous mapping on X into a topologically complete space (e.g. into a paracompact or realcompact space) can be continuously extended onto the G_δ-closure of X.

5 (Pelant). Locally fine spaces are subfine.

The above results can be applied e.g. when X contains a Σ-product of $\{X_i\}$ or is regularly closed; or as the description of the fine uniformity on $\prod X_i$.

In the case that $pr_J X = \prod_J X_i$ for all countable J, we can prove an analogy of the Proposition also for paracompact p-spaces X_i.