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ON INJECTIVE HOLOMORPHIC FREDHOLM MAPPINGS OF INDEX 0 IN COMPLEX BANACH SPACES

Dietmar ABTS[1])

Abstract: We prove that an injective holomorphic Fredholm mapping of index 0 defined on an open subset G of a complex Banach space maps G biholomorphically onto the open set f(G). This is the infinite-dimensional version of a deep theorem in $\mathbb{C}^n$ due to Osgood. There are counter-examples which show that the assertion does not hold for arbitrary holomorphic functions in infinite-dimensional spaces. We also establish a criterion for the injectivity of a holomorphic map which can be approximated by injective holomorphic maps. In the finite-dimensional case this theorem is due to Carathéodory.

Key words: Complex Banach space, holomorphic mapping, linear Fredholm operator of index 0, analytic set, measure of non-compactness, strict set contraction.

Classification: 46G20

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1. Introduction. Let X and Y be complex Banach spaces and let G be an open subset of X. A function $f:G \longrightarrow Y$ is called holomorphic if f has a complex-linear Fréchet derivative $f'(x)$ at each point x of G (cf. Hille, Phillips [8]). The map f is called biholomorphic if f is injective, f(G) is open and the inverse f^{-1} is holomorphic.

It is a known result that in $\mathbb{C}^n$ an injective holomor-

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1) This paper is based on part of the author's dissertation research at RWTH Aachen under the supervision of Prof. Dr. J. Reinermann, cf.[1].

phic map is biholomorphic. The corresponding result does not seem to be known in infinite dimensions even assuming the range is an open set (cf. Suffridge [13]).
The following example shows that we cannot omit the assumption that the range f(G) is an open set.

Let c_o be the space of complex null sequences $x = (x_n)$ with the norm $\|x\| := \sup|x_n|$. Define $f: c_o \longrightarrow c_o$ by

$$f((x_1, x_2, \dots)) := (x_1^2, x_1^3, x_2^2, x_2^3, \dots).$$

Then f is an injective holomorphic map. But $f'(o) = 0$, hence f^{-1} fails to be holomorphic.

Now we present a special class of holomorphic maps in complex Banach spaces for which the problem raised above has a positive solution.

Definition: Let X and Y be complex Banach spaces and let G be an open subset of X. A map $f: G \longrightarrow Y$ is called holomorphic Fredholm mapping of index 0 if f is holomorphic and $f'(x)$ is a linear Fredholm operator of index 0 for each $x \in G$, i.e. $\dim f'(x)^{-1}(o) = \operatorname{codim} f'(x)(X) < \infty$ (cf. Hirzebruch, Scharlau [9]).
Obviously all holomorphic functions mapping an open set in $\mathbb{C}^n$ into $\mathbb{C}^n$ belong to this class of operators.
If $g: G \longrightarrow X$ is holomorphic and a strict set contraction with respect to the Kuratowski-measure of noncompactness, then Id-g is a holomorphic Fredholm mapping of index 0 (cf. Nussbaum [12]; Eisenack, Fenske [7]).

We note that g+h is a strict set contraction provided that $g: G \longrightarrow X$ is compact and $h: G \longrightarrow X$ is a Lipschitz map with constant $k < 1$.

2. <u>Main results</u>. A function $f:G \to Y$ is said to be <u>locally injective</u> if for each $x \in G$ there is a neighborhood U of x in G such that $f|_U$ is injective.

<u>Theorem 1</u>: Let X and Y be complex Banach spaces, let G be an open subset of X and $f:G \to Y$ a holomorphic Fredholm mapping of index 0.
Then f is locally injective if and only if $f'(x)$ is a homeomorphism onto Y for each $x \in G$.
The example in the introduction shows that the Fredholm property of f is essential.

<u>Corollary</u>: Let X and Y be complex Banach spaces, G an open subset of X and let $f:G \to Y$ be an injective holomorphic Fredholm mapping of index 0.
Then f maps G biholomorphically onto the open set $f(G)$.
This is an easy consequence of theorem 1 and the implicit function theorem for holomorphic maps yielding the holomorphy of the inverse f^{-1} (cf. Dieudonné [6]).

<u>Theorem 2</u>: Let $G \subset X$ be open and connected and let $f: G \to Y$ be a holomorphic Fredholm mapping of index 0 such that there is some $x_0 \in G$ with $f'(x_0)$ injective.
Then the set $\{x \in G \mid f'(x)$ is a homeomorphism onto $Y\}$ is open, connected and dense in G.
Our next theorem gives a criterion for the injectivity of a holomorphic map which can be approximated by injective holomorphic maps.

There is a well-known theorem in complex function theory which says that a complex-valued holomorphic function f defined on a region G in $\mathbb{C}$ is constant or injective provided that f can be approximated uniformly on compact subsets of G by a sequence of injective holomorphic functions (cf. Diederich, Remmert [5]).

The analogous statement does not hold in the higher-dimensional case. Let $f\colon \mathbb{C}^2 \to \mathbb{C}^2$ and $f_n\colon \mathbb{C}^2 \to \mathbb{C}^2$ be defined by $f(x,y):=(x,o)$ and $f_n(x,y):=(x,\frac{y}{n})$ respectively. Then f is neither constant nor injective.

<u>Theorem 3</u>: Let X and Y be complex Banach spaces, G an open and connected subset of X and let $f:G \to Y$ be a holomorphic Fredholm mapping of index 0 such that there is a sequence $(f_n)_{n\in\mathbb{N}}$ of injective holomorphic mappings $f_n:G \to Y$ which converges locally uniformly in G to f.

Then f is injective if and only if there is some $x \in G$ such that $f'(x)$ is injective.

In the case $X = Y = \mathbb{C}^n$ the theorem is due to Carathéodory [4]. The proofs of the theorems are given in section 4.

3. <u>Auxiliary lemmas</u>. Throughout the following let X and Y be complex Banach spaces. $L(X,Y)$ denotes the space of linear and continuous operators $T:X \to Y$ equipped with the corresponding operator norm. For $x \in X$ and $r > o$ $B(x,r)$ denotes the open ball with radius r and center x, $\overline{B}(x,r)$ denotes the closed ball.

<u>Lemma 1</u>: Let $G \subset X$ be open and connected, U an open and nonempty subset of G and let $f:G \to Y$ be holomorphic such that $f|_U = o$.

Then $f = o$.

<u>Lemma 2</u>: Let $G \subset X$ be open and $f:G \to Y$ holomorphic.

Then the derivative $f':G \longrightarrow L(X,Y)$ is holomorphic.

<u>Lemma 3</u>: Let $G \subset X$ be open and let $(f_n)_{n\in\mathbb{N}}$ be a sequence of holomorphic functions $f_n:G \to Y$ which converges locally uniformly in G to the function $f:G \longrightarrow Y$.
Then f is holomorphic and the sequence (f_n') converges locally uniformly to the derivative f' with respect to the operator norm.

Lemma 1, 2 and 3 can be easily deduced from the theory given in Bochnak, Siciak [2],[3].

<u>Lemma 4</u>: Let $G \subset X$ be open and connected, $f:G \to \mathbb{C}$ holomorphic and let $(f_n)_{n\in\mathbb{N}}$ be a sequence of holomorphic functions $f_n:G \to \mathbb{C}$ which converges locally uniformly to f.
Suppose that f_n has no zeroes in G for $n \in \mathbb{N}$.
Then either $f = o$ or f has no zeroes in G.

<u>Proof</u>: Suppose $f \neq o$ and let $x_o \in G$. By lemma 1 there is $h \in X$ and $r > o$ such that $g(z) := f(x_o+zh)$ ($z \in \mathbb{C}$ and $|z| < r$) is nonconstant. Define $g_n(z) := f_n(x_o+zh)$ for $n \in \mathbb{N}$ and $|z| < r$. Then by assumption g_n has no zeroes and (g_n) converges locally uniformly to g. Hence by a well-known result g has no zeroes, too (cf. Diederich, Remmert [5]). In particular $g(o) \neq o$, i.e. $f(x_o) \neq o$.

<u>Lemma 5</u>: Let $G \subset \mathbb{C}^n$ be open and $f:G \longrightarrow \mathbb{C}^n$ be holomorphic and injective.
Then $\det f'(x) \neq o$ for all $x \in G$.

For a proof see Narasimhan [11], chapt. 5, Th. 5.

<u>Lemma 6</u>: Let $T:X \longrightarrow Y$ be a linear Fredholm operator of index 0. Then there is a linear continuous operator $F:X \longrightarrow Y$ such that F(X) is finite-dimensional and T+F is a homeomor-

phism onto Y.

Lemma 7: Let $G \subset X$ be open, $f:G \longrightarrow Y$ be a holomorphic Fredholm mapping of index 0 and $x_0 \in G$.
Then there is a linear continuous operator $F:X \longrightarrow Y$ such that $F(X)$ is finite-dimensional and $S := f'(x_0)+F$ is a linear homeomorphism onto Y, and there exists an open neighborhood U of x_0 in G such that the mappings defined by

$R(x) := f(x)-f(x_0)-f'(x_0)(x-x_0) \quad (x \in U)$,

$g(x) := -S^{-1} \circ R(x) \quad (x \in U)$ and

$k(x) := x_0-S^{-1}(f(x_0))+S^{-1} \circ F(x-x_0) \quad (x \in X)$

have the following properties:

k(X) is contained in a finite-dimensional subspace of X,

g is Lipschitz function with constant less than 1 and

$f(x)=S(x-k(x)-g(x))$ for all $x \in U$.

Proof: By lemma 6 we may choose a linear continuous operator $F:X \longrightarrow Y$ such that $F(X)$ is finite-dimensional and $S:=f'(x_0)+F$ is a linear homeomorphism onto Y. Let $\varepsilon > 0$ with $\varepsilon \| S^{-1} \| < 1$. By the mean-value theorem there exists an open neighborhood U of x_0 in G such that $R|_U$ is Lipschitz function with constant ε. For $x \in U$ we have

$$f(x) = S(x-x_0-S^{-1} \circ F(x-x_0)+S^{-1} \circ R(x)+S^{-1}(f(x_0))),$$

hence $f(x) = S(x-k(x)-g(x))$.

Lemma 8: Let $U \subset X$ be open, $\lambda \in [0,1)$ and $g:U \longrightarrow X$ such that g is holomorphic and $\| g(x)-g(y) \| \leq \lambda \cdot \| x-y \|$ for all $x,y \in U$. Then Id-g maps U biholomorphically onto the open set (Id-g)(U).

Proof: By an easy application of Banach's fixed point theorem Id-g is injective and (Id-g)(U) is open.

Since $\|g'(x)\| \leq \lambda$ for all $x \in U$, $(Id-g)'(x)$ is invertible and the implicit function theorem (cf. Dieudonné [6]) yields the holomorphy of $(Id-g)^{-1}$.

Lemma 9: Let $x_o \in X$, $r > o$, $f: B(x_o,r) \longrightarrow Y$ be holomorphic and $f'(x_o)$ a linear homeomorphism onto Y. Let $(f_n)_{n \in \mathbb{N}}$ be a sequence of holomorphic maps $f_n: B(x_o,r) \longrightarrow Y$ converging uniformly to f on $B(x_o,r)$.
Then there exists a neighborhood V of $f(x_o)$ in Y and $n_o \in \mathbb{N}$ such that $V \subset \bigcap_{n \geq n_0} f_n(B(x_o,r))$.

Proof: Let $f_o := f$ and $y_o := f_o(x_o)$. By lemma 3 $f_n'(x_o) \longrightarrow f_o'(x_o)$ in $L(X,Y)$. Since $f_o'(x_o)$ is a homeomorphism, there is $n_1 \in \mathbb{N}$ such that $f_n'(x_o)$ is a homeomorphism for $n \geq n_1$.
Define $S_n(x,y) := f_n'(x_o)^{-1}(y-f_n(x))+x$ for $x \in B(x_o,r)$, $y \in Y$ and $n \in \{o\} \cup \{k \mid k \geq n_1\}$. We have $S_n(x,y) = o$ iff $f_n(x) = y$, and $(S_n)_x(x,y) = -f_n'(x_o)^{-1} \circ f_n'(x) + Id$.
$\|(S_n)_x(x,y)\| \leq \| f_n'(x_o)^{-1}\| \, (\| f_n'(x) - f_o'(x)\| + \| f_o'(x) - f_o'(x_o)\| + \| f_o'(x_o) - f_n'(x_o)\|)$. Since $(f_n'(x_o)^{-1}) \longrightarrow f_o'(x_o)^{-1}$, $(\| f_n'(x_o)^{-1}\|)_{n \in \mathbb{N}}$ is bounded. Lemma 2 and 3 imply that there is $o < \delta < r$, $\lambda \in [0,1)$ and $n_2 \geq n_1$ such that $\|(S_n)_x(x,y)\| \leq \lambda$ for $\|x-x_o\| \leq \delta$, $y \in Y$ and $n \in \{o\} \cup \{k \mid k \geq n_2\}$.
$\| S_n(x,y) - S_n(\tilde{x},y)\| = \| \int_0^1 (S_n)_x(x+t(\tilde{x}-x))(x-x)dt \| \leq \lambda \| x-\tilde{x}\|$.
Let $o < \varepsilon < (1-\lambda)\delta$. There is $\rho > o$, $n_3 \geq n_2$ such that
$\| S_n(x_o,y) - x_o \| \leq \| f_n'(x_o)^{-1}\| \, (\| f_n(x_o) - f_o(x_o)\| + \| f_o(x_o) - y\|) < \varepsilon$ for $n \in \{o\} \cup \{k \mid k \geq n_3\}$ and $\|y-y_o\| \leq \rho$.
Hence $\|S_n(x,y) - x_o\| \leq \lambda\delta + \varepsilon < \lambda\delta + (1-\lambda)\delta = \delta$ for $\| x-x_o\| \leq \delta$, $\| y-y_o\| \leq \rho$ and $n \in \{o\} \cup \{k \mid k \geq n_3\}$.
By Banach's fixed point theorem there is exactly one function

$\varphi_n : B(y_o, \rho) \longrightarrow B(x_o, \sigma)$ such that $S_n(\varphi_n(y), y) = \varphi_n(y)$, i.e. $f_n(\varphi_n(y)) = y$.

The estimation $\|\varphi_o(y) - \varphi_o(\tilde{y})\| \leq \|S_o(\varphi_o(y), y) - S_o(\varphi_o(\tilde{y}), y)\| + \|S_o(\varphi_o(\tilde{y}), y) - S_o(\varphi_o(\tilde{y}), \tilde{y})\| \leq \lambda \|\varphi_o(y) - \varphi_o(\tilde{y})\| + \|f_o'(x_o)^{-1}\| \|y - \tilde{y}\|$ shows that φ_o is continuous. In rather the same way it is shown that $(\varphi_n) \longrightarrow \varphi_o$ uniformly on $B(y_o, \rho)$.

Because of $\|\varphi_n(y) - x_o\| \leq \|\varphi_n(y) - \varphi_o(y)\| + \|\varphi_o(y) - \varphi_o(y_o)\|$ there is $o < \eta < \rho$ and $n_4 \in \mathbb{N}$ such that $\varphi_n(B(y_o, \eta)) \subset B(x_o, r)$ for $n \geq n_4$. Now let $V := B(y_o, \eta)$, then $V \subset \varphi_n^{-1}(B(x_o, r)) \subset f_n(B(x_o, r))$ for $n \geq n_4$ and we are done.

Definition: Let $G \subset X$ be open. A subset $A \subset G$ is called an analytic set if for each $x \in G$ there exists an open neighborhood U of x in G and finitely many holomorphic functions $f_1, \ldots, f_n : U \longrightarrow \mathbb{C}$ such that

$$A \cap U = \{ z \in U \mid f_1(z) = \ldots = f_n(z) = o \}.$$

Lemma 10: Let $G \subset X$ be open and connected and let A be an analytic subset of G such that $A \neq G$.
Then $G \setminus A$ is open, connected and dense in G.

The proof may be carried out along the lines of the finite-dimensional version of lemma 10 given in Narasimhan [11], chapt. 4, Prop. 1.

Lemma 11: Let $x_o \in X$, $r > o$, $f : B(x_o, r) \longrightarrow Y$ be a holomorphic Fredholm mapping of index 0. Let $F \in L(X,Y)$ such that $F(X)$ is finite-dimensional and $f'(x) - F$ is invertible for all $x \in B(x_o, r)$. Let $P : Y \longrightarrow F(X)$ be a linear continuous projection onto $F(X)$. Define $S : B(x_o, r) \longrightarrow L(Y,Y)$ by $S(z) := F \circ (f'(z) - F)^{-1}$. Then S is holomorphic and for all $z \in B(x_o, r)$

$f'(z)$ is invertible if and only if $\det\,(P\circ(Id+Z))|_{F(X)})\neq o$.

Proof: By lemma 2 S is holomorphic. Let $z\in B(x_o,r)$ and $Q:=Id-P$. It is easy to see that $Id+P\circ S(z)\circ Q$ is invertible. $f'(z) = (Id+S(z))\circ(f'(z)-F) =$

$$(Id+P\circ S(z)\circ Q)\circ(Id+P\circ S(z)\circ P)\circ(f'(z)-F).$$

Hence $f'(z)$ is invertible iff $Id+P\circ S(z)\circ P$ is invertible. Since $Id+P\circ S(z)\circ P = Q+P\circ(Id+S(z))\circ P$, $Id+P\circ S(z)\circ P$ is invertible iff $P\circ(Id+S(z))|_{F(X)}\in L(F(X),F(X))$ is invertible. Hence $f'(z)$ is invertible iff $\det\,(P\circ(Id+S(z))|_{F(X)})\neq o$.

4. Proof of the theorems

Proof of theorem 1: If $f'(x)$ is a homeomorphism onto Y, then by the implicit function theorem (cf. Dieudonné [6]) there exists a neighborhood U of x in G such that $f|_U$ is injective. Now we suppose that f is locally injective. Let $x_o\in G$. We choose F, S, U, R, g and k according to lemma 7 and such that $f|_U$ is injective. By lemma 8 $Id-k\circ(Id-g)^{-1}$: $(Id-g)(U)\longrightarrow X$ is holomorphic. The identity $f(x) =$ $= S(x-k(x)-g(x))$ for $x\in U$ implies that $Id-k\circ(Id-g)^{-1}$ is injective.

We assume that $f'(x_o)$ is not invertible. Then there is $h\in X\setminus\{o\}$ such that $f'(x_o)(h) = o$. We have $R'(x_o) = o$, $g'(x_o) =-S^{-1}\circ R'(x_o) = 0$, $(Id-g)'(x_o) = Id-g'(x_o) = Id$, $k'(x_o) = S^{-1}\circ F$, therefore $(Id-k\circ(Id-g)^{-1})'((Id-g)(x_o)) =$ $= Id-S^{-1}\circ F$.

Let E be a finite-dimensional subspace of X such that $k(X)\subset$ $\subset E$, $(Id-g)(x_o)\in E$ and $h\in E$. Then the function $Id-k\circ(Id-g)^{-1}|_{(Id-g)(U)\cap E}:(Id-g)(U)\cap E\longrightarrow E$ is holomorphic and injective.

Now by lemma 5

$Id - S^{-1} \circ F|_E = (Id - k \circ (Id-g)^{-1}|_{(Id-g)(U) \cap E})'((Id-g)(x_o))$ is invertible in $L(E,E)$.

But $F(h) = f'(x_o)(h) + F(h) = S(h)$, and $S^{-1} \circ F(h) = h$, $h \neq o$, a contradiction. Hence $f'(x_o)$ is invertible.

Proof of theorem 2: Let $A := \{x \in G \mid f'(x)$ is not injective$\}$. By assumption $A \neq G$. We show that A is an analytic subset of G. Then lemma 10 yields the assertion.

Let $\hat{x} \in G$. By lemma 6 there is a finite-dimensional linear operator $F \in L(X,Y)$ such that $f'(\hat{x}) - F$ is invertible. Since $x \longmapsto f'(x)$ is continuous, we find $r > o$ with $B(\hat{x},r) \subset G$ such that $f'(x) - F$ is invertible for all $x \in B(\hat{x},r)$.

Define $\varphi : B(\hat{x},r) \longrightarrow \mathbb{C}$ by $\varphi(z) : \det(P \circ (Id + S(z))|_{F(X)})$ according to lemma 11. Then φ is holomorphic and $A \cap B(\hat{x},r) =$ $= \{x \in B(\hat{x},r) \mid \varphi(x) = o\}$.

Proof of theorem 3: If f is injective, then by theorem 1 $f'(x)$ is injective for all $x \in G$.

Now suppose that there is $\hat{x} \in G$ such that $f'(\hat{x})$ is injective. We first show that $f'(x)$ is injective for all $x \in G$. Let $x_o \in G$. By lemma 6 there exists a finite-dimensional linear operator $F \in L(X,Y)$ such that $f'(x_o) - F$ is invertible. By lemma 2 and 3 there is $r > o$ and $n_o \in \mathbb{N}$ such that $B(x_o,r) \subset G$, $f_n'(x) - F$ and $f'(x) - F$ are invertible and $f_n'(x)$ is a Fredholm operator of index 0 for $n \geq n_o$, $x \in B(x_o,r)$.

Define $S(z) := F \circ (f'(z) - F)^{-1}$, $S_n(z) := F \circ (f_n'(z) - F)^{-1}$ for

$z \in B(x_o,r)$ and $n \geq n_o$. Let P be a linear continuous projection from Y onto F(X). Define $\varphi : B(x_o,r) \to \mathbb{C}$ by $\varphi(z) :=$
$:= \det(P \circ (Id+S(z))|_{F(X)})$ and $\varphi_n : B(x_o,r) \to \mathbb{C}$ for $n \geq n_o$ by $\varphi_n(z) := \det(P \circ (Id+S_n(z))|_{F(X)})$. We verify the assumptions of lemma 4.

By lemma 11 φ and φ_n are holomorphic. By theorem 1 $f_n'(x)$ is invertible for $x \in B(x_o,r)$ and $n \in \mathbb{N}$. $\varphi_n(z) \neq o$ for $z \in$ $\in B(x_o,r)$, $n \geq n_o$ by lemma 11. Since $(f_n'(x)) \to f'(x)$ locally uniformly in G, $(S_n(z)) \to S(z)$ locally uniformly in $B(x_o,r)$ (note that $\|A^{-1}-B^{-1}\| \leq \dfrac{\|A-B\| \cdot \|A^{-1}\|^2}{1 - \|A-B\| \cdot \|A^{-1}\|}$ provided that $A,B \in L(X,Y)$ and $\|A-B\| \leq \dfrac{1}{\|A^{-1}\|}$, cf. Kato [10], chapt. I, § 4). Hence $(\varphi_n) \to \varphi$ locally uniformly in $B(x_o,r)$.

By theorem 2 there is $x \in B(x_o,r)$ such that $f'(x)$ is invertible, hence $\varphi(x) \neq o$ by lemma 11. Now lemma 4 shows that φ has no zeroes in $B(x_o,r)$, therefore $f'(z)$ is invertible for all $z \in B(x_o,r)$ by lemma 11.

We claim that f is injective.

Let $x_1, x_2 \in G$, $x_1 \neq x_2$. By lemma 9 there exists a neighborhood U of x_1 in G such that $x_2 \notin U$ and a neighborhood V of $f(x_1)$ and $n_o \in \mathbb{N}$ such that $V \subset \bigcap_{n \geq n_o} f_n(U)$.

Hence $f_n(x_2) \notin V$ for $n \geq n_o$ by the injectivity of f_n. Since $(f_n(x_2)) \to f(x_2)$, we obtain $f(x_1) \neq f(x_2)$.

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