Commentationes Mathematicae Universitatis Carolinae

Richard C. Brown; Milan Tvrdý

Generalized boundary value problems with abstract side conditions and their adjoints. I.

Commentationes Mathematicae Universitatis Carolinae, Vol. 20 (1979), No. 1, 191

Persistent URL: http://dml.cz/dmlcz/105914

ANNOUNCEMENTS OF NEW RESULTS

GENERALIZED BOUNDARY VALUE PROBLEMS WITH ABSTRACT SIDE CONDITIONS AND THEIR ADJOINTS,I.

R.C. Brown, M. Tvrdý (Českosl. Akad. Věd, Praha 1, Československo), received 30.12. 1978

Notations. C^m is the space of complex m-vectors, $W_m^{1,p}$ ($1 \leq p \leq \infty$) is the Sobolev space of functions $y: [0,1] \to C^m$ which are absolutely continuous on $[0,1]$ and whose derivatives are L^p-integrable (essentially bounded if $p=\infty$) on $[0,1]$ ($y' \in L_m^p$), $W_m^{1,1} = AC_m$. Given a matrix B, B^* is its conjugate transpose.

Assumptions. $1 \leq p < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$ if $p > 1$, $q=\infty$ if $p=1$; A_0, A are $k \times m$-matrix functions, $k \geq m$, $A_0^* = [I, B^*]$ where I is the identity $m \times m$-matrix and B is a $(k-m) \times m$-matrix function essentially bounded on $[0,1]$, A is L^p-integrable on $[0,1]$; F is a loc. conv. top. vector space, F^* its dual and $H: W_m^{1,p} \to F$ is a linear continuous operator defined for every $y \in W_m^{1,p}$.

Results. Let $D = \{y \in W_m^{1,p} : Hy = 0\}$ and $L: y \in D \to A_0 y' + Ay$. Then $Ly \in L_k^p$ for any $y \in W_m^{1,p}$ and L has a closed range in L_k^p. There exist operators $U^*: F^* \to C^m$ and $V^*: F^* \to L_m^q$ such that

$$\varphi(Hy) = y^*(0)(U^*\varphi) + \int_0^1 (y')^* (V^*\varphi)dt \text{ for all } y \in W_m^{1,p} \text{ and } \varphi \in F^*.$$

Let $\mathcal{V}$ be the weak*-closure of the range of V^* in L_m^q. Let us denote

$$\ell_0^+(z,\psi) = A_0^* z + \psi, \quad \ell^+(z,\psi) = -(\ell_0^+(z,\psi))' + A^* z,$$

$$D^+ = \{(z,\psi) \in L_k^q \times \mathcal{V} : \ell_0^+(z,\psi) \in AC_m, \ (\ell_0^+(z,\psi))^* y]_0^1 - \int_0^1 \psi^* y' dt = 0 \text{ for all } y \in D\}.$$

Then the relation

$$L^+ = \{(z, \ell^+(z,\psi)) : (z,\psi) \in D^+\}$$

is the adjoint of L.

The proofs and more details will be published in Časopis pro pěstování matematiky. The case of $p=\infty$ and the n-th-order differential operator will be treated in the second part of the paper.