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*Commentationes Mathematicae Universitatis Carolinae*, Vol. 20 (1979), No. 1, 191

Persistent URL: <http://dml.cz/dmlcz/105914>

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## ANNOUNCEMENTS OF NEW RESULTS

GENERALIZED BOUNDARY VALUE PROBLEMS WITH ABSTRACT SIDE CONDITIONS AND THEIR ADJOINTS, I.

R. C. Brown, M. Tvrđý (Českosl. Akad. Věd, Praha 1, Československo), received 30.12. 1978

Notations.  $C^m$  is the space of complex  $m$ -vectors,  $W_m^{1,p}$  ( $1 \leq p \leq \infty$ ) is the Sobolev space of functions  $y: [0,1] \rightarrow C^m$  which are absolutely continuous on  $[0,1]$  and whose derivatives are  $L^p$ -integrable (essentially bounded if  $p = \infty$ ) on  $[0,1]$  ( $y' \in L_m^p$ ),  $W_m^{1,1} = AC_m$ . Given a matrix  $B$ ,  $B^*$  is its conjugate transpose.

Assumptions.  $1 \leq p < \infty$ ,  $\frac{1}{p} + \frac{1}{q} = 1$  if  $p > 1$ ,  $q = \infty$  if  $p = 1$ ;  $A_0, A$  are  $k \times m$ -matrix functions,  $k \geq m$ ,  $A_0^* = [I, B^*]$  where  $I$  is the identity  $m \times m$ -matrix and  $B$  is a  $(k-m) \times m$ -matrix function essentially bounded on  $[0,1]$ ,  $A$  is  $L^p$ -integrable on  $[0,1]$ ;  $F$  is a loc. conv. top. vector space,  $F^*$  its dual and  $H: W_m^{1,p} \rightarrow F$  is a linear continuous operator defined for every  $y \in W_m^{1,p}$ .

Results. Let  $D = \{y \in W_m^{1,p}: Hy = 0\}$  and  $L: y \in D \rightarrow A_0 y' + Ay$ . Then  $Ly \in L_k^p$  for any  $y \in W_m^{1,p}$  and  $L$  has a closed range in  $L_k^p$ . There exist operators  $U^*: F^* \rightarrow C^m$  and  $V^*: F^* \rightarrow L_m^q$  such that

$$\mathcal{G}(Hy) = y^*(0)(U^* \mathcal{G}) + \int_0^1 (y')^* (V^* \mathcal{G}) dt \text{ for all } y \in W_m^{1,p} \text{ and } \mathcal{G} \in F^*.$$

Let  $\mathcal{V}$  be the weak\*-closure of the range of  $V^*$  in  $L_m^q$ . Let us denote

$$\begin{aligned} \ell_0^+(z, \psi) &= A_0^* z + \psi, \quad \ell^+(z, \psi) = -(\ell_0^+(z, \psi))' + A^* z, \\ D^+ &= \{z, \psi \in L_k^q \times \mathcal{V} : \ell_0^+(z, \psi) \in AC_m, (\ell_0^+(z, \psi))^* y|_0^1 - \\ &- \int_0^1 \psi^* y' dt = 0 \text{ for all } y \in D\}. \end{aligned}$$

$L^+ = \{z, \ell^+(z, \psi) : (z, \psi) \in D^+\}$

is the adjoint of  $L$ .

The proofs and more details will be published in Časopis pro pěstování matematiky. The case of  $p = \infty$  and the  $n$ -th-order differential operator will be treated in the second part of the paper.