Commentationes Mathematicae Universitatis Carolinae

Ján Ninčák

Hamiltonian circuits in cubic graphs

Commentationes Mathematicae Universitatis Carolinae, Vol. 15 (1974), No. 4, 627--630

Persistent URL: http://dml.cz/dmlcz/105587

COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINAE

15,4 (1974)

HAMILTONIAN CIRCUITS IN CUBIC GRAPHS

Ján NINČÁK, Košice

Abstract: This remark presents a construction of a cubic graph (Fig.1) without triangles with 18 vertices and just three Hamiltonian circuits, thus solving Bosák's problem from [1]: whether each cubic graph with just three Hamiltonian circuits and at least three vertices has a triangle.

Key words: Graph, Hamiltonian circuit.

AMS: 05C20 Ref. Ž.: 8.83

Let G be the graph of Fig.1. Since G does not contain any triangle, it will suffice to prove that it has only three Hamiltonian circuits. In the graph G all the edges $x_i z_i$ $(0 \leq i \leq 8)$ are called spokes as in [2]. Let us denote the set of all spokes in G by $\mathcal{N}$. Two spokes, t and w, are called conjugated if there exists an edge from the set $\{x_0 x_8\} \cup \{x_i x_{i+1};\ i = 0, 1, 2, \dots, 7\}$ which is adjacent to both edges t and w.

Let $\mathcal{M}$ be the set of all pairs of conjugated spokes from $\mathcal{N}$. It is evident that a Hamiltonian circuit in G cannot contain a spoke not belonging to a pair from $\mathcal{M}$, nor an odd number of spokes from $\mathcal{N}$. Considering certain symmetries of G, we can suppose without loss of generality that a Hamiltonian circuit, if it exists in G, must contain all the spokes from $\mathcal{A}_j$ for some $j \in \{1, 2, \ldots, 9\}$, where $\mathcal{A}_1 = \{x_0 z_0, x_1 z_1\}$, $\mathcal{A}_2 = \{x_i z_i;\ i = 0, 1, 2, 3\}$, $\mathcal{A}_3 = \{x_i z_i;\ i = 0, 1, 3, 4\}$, $\mathcal{A}_4 = \{x_i z_i;\ i = 0, 1, 4, 5\}$, $\mathcal{A}_5 = \{x_i z_i;\ i = 0, 1, 2, 3, 4, 5\}$, $\mathcal{A}_6 = \{x_i z_i;\ i = 0, 1, 2, 3, 5, 6\}$, $\mathcal{A}_7 = \{x_i z_i;\ i = 0, 1, 3, 4, 6, 7\}$, $\mathcal{A}_8 = \{x_i z_i;\ i = 0, 1, 2, 4, 5, 6\}$ and $\mathcal{A}_9 = \{x_i z_i;\ i = 0, 1, 2, 3, 4, 5, 6, 7\}$.

Now, for each $j \in \{1, 2, \ldots, 9\}$ we shall examine all possibilities for a Hamiltonian circuit passing through all the spokes from $\mathcal{A}_j$. In this way we shall find out that in the graph G of Fig. 1 there exists no Hamiltonian circuit which contains just all the spokes from $\mathcal{A}_j$ for $j = 1,2,3,4,5,6,8,9$ and that it has just one Hamiltonian circuit which passes through all the spokes from $\mathcal{A}_7$. This circuit is drawn in heavy line in Fig.1. By rotating (40° and 80°) this line in G we get only two new Hamiltonian circuits. It follows from this fact that the given graph G in Fig. 1 has just three Hamiltonian circuits. Thus the proof is finished.

Evidently, the graph G of Fig. 1 is not planar. There remains a question if there exists a planar cubic graph without triangles with just three Hamiltonian circuits and at least three vertices.

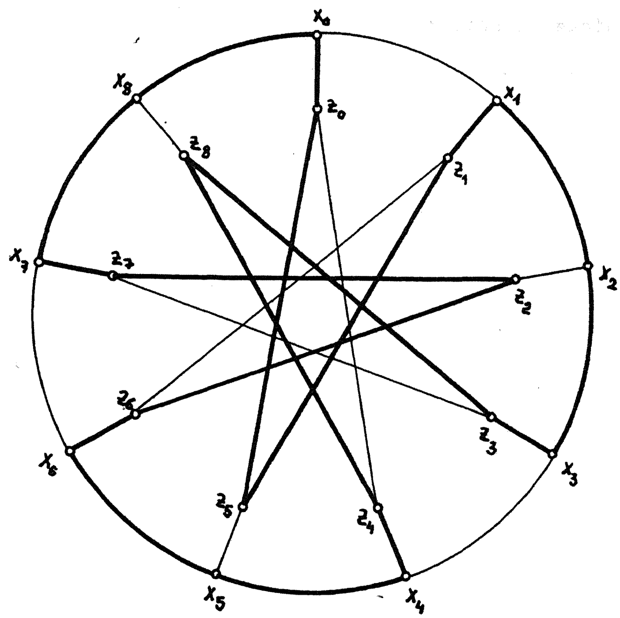

Fig. 1

References

[1] BOSÁK J.; Hamiltonian lines in cubic graphs, Theory of graphs(Proc.Int.Symp.Rome,July 1966,ed. P. Rosenstiehl),Gordon and Breach,New York,1967, 35-46.

[2] CASTANGA F. and PRINS G.: Every generalized Peterson graph has a Tait coloring, Pacif.J.Math.40(1972), 53-58.

Katedra matematickej informatiky
Elektrotechnickej fakulty VŠT
Zbrojnícka 7, Košice
Československo

(Oblatum 5.9.1974)