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SOME PROPERTIES OF POTENTIALS OF SIGNED MEASURES

(Preliminary communication)

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An important example of a regular kernel in the Euclidean space R^m $(m > 2)$ is the Newtonian kernel $G(x,y) = |x-y|^{2-m}$. This means that G satisfies the following continuity principle: If μ is an arbitrary nonnegative measure with compact support K such that the restriction to K of $G\mu$ (= the corresponding potential) is finite and continuous, then $G\mu$ is finite and continuous on the whole space. (This is the theorem of Evans-Vasilesco.) Another typical property of G is the following maximum principle of Maria-Frostman:

$$\sup_{x \in R^m} G\mu(x) = \sup_{x \in K} G\mu(x) .$$

In this note we announce two theorems which give an analogy to the principles mentioned above for the case of potentials of signed measure.

<u>Theorem 1</u>. Let μ be a signed measure with compact support $K \subset R^m$ and let $G\mu$ be finite in R^m. If the restriction of $G\mu$ to K is continuous on K, then the potential $G\mu$ is continuous in the whole space.

<u>Theorem 2</u>. If μ is a signed measure with compact support $K \subset R^m$ and $G\mu$ is finite in R^m, then

$$[\inf_{x \in K} G\mu(x)]^- = \inf_{x \in R^m} G\mu(x) \leq \sup_{x \in R^m} G\mu(x) = [\sup_{x \in K} G\mu(x)]^+$$

where $[z]^+$ and $[z]^-$ stand for the positive and the negative parts of z, respectively.

It should be noted that it is possible to construct an example of a signed measure μ such that the potential $G|\mu|$ is bounded, $G\mu$ is continuous in R^m and $G\mu^+$, $G\mu^-$ are discontinuous potentials, so that Theorem 1 is not a consequence of the classical theorem of Evans-Vasilesco.

Theorem 1 answers a question posed by dr. B.-W. Schulze in a discussion on the occasion of the "5. Tagung über Probleme und Methoden der Mathematischen Physik" in Karl-Marx-Stadt (1973). One may prove a more abstract version of the above theorems in the context of the axiomatic theory of harmonic spaces, in which somewhat stronger form of the axiom of domination is satisfied. The proofs make an essential use of fine topology arguments and will be presented in a paper to be published in Czech.Math.Journal.

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