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ON SOME CLOSURE PROPERTIES OF GENERABLE LANGUAGES

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Abstract: In this article it is shown that the intersection of a class of generable languages is a generable language and the necessary and sufficient conditions are given for the union of two generable languages to be a generable language.

Key words: G-machine, output word, word generated by G-machine, language of a G-machine.

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A generable language is the set of all "words" generated by a G-machine [3], which is a certain generalization of machines introduced in [1],[2],[4]. In the above references the term "generable set" instead of "generable language" is used. Since a generable language is a subset of I^{∞} (the set of all nonvoid finite or infinite sequences over a finite set I) the term "generable language" seems to be more suitable. We shall deal with some problems concerning the closure properties of generable languages. It will be seen that generable languages possess closure properties analogous to generable sets studied in [1],[2],[4].

Denotation. Let $T, \overline{T}, T_m, \overline{T}_m$ denote the following sets:

$T = \{1, 2, \dots\}$, $\overline{T} = \{0, 1, 2, \dots\}$, $T_m = \{1, 2, \dots, m\}$, $\overline{T}_m = \{0, 1, 2, \dots, m\}$.

Definition 1. Let I be a finite set (including an empty set). Denote I^∞ the set of all nonvoid finite and infinite sequences of elements of I. These sequences are called words. For $w \in I^\infty$, $m \in T$, $w = (s_0, s_1, \dots, s_{m-1})$ put $\ell(w) = m$. For $w \in I^\infty$, $w = (s_0, s_1, \dots)$ put $\ell(w) = \infty$. The symbol $1(w)$ is called the length of w. Instead of $w = (s_0, s_1, \dots, s_{m-1})$ and $w = (s_0, s_1, \dots)$ we write $w = s_0 s_1 \dots s_{m-1}$ and $w = s_0 s_1 \dots$ respectively or $w = \prod_{i=0}^{m-1} s_i$ and $w = \prod_{i=0}^{\infty} s_i$. Considering an arbitrary word of finite or infinite length we use the denotation $\prod_i s_i$.

By the symbol $(s_0 s_1 \dots s_{m-1})^k$, where $k \in T$ we understand the word $s_0 s_1 \dots s_m s_{m+1} \dots s_{2m} s_{2m+1} \dots s_{km-1}$, where $s_{im+j} = s_j$ for all $i \in \overline{T}_{k-1}$ and all $j \in \overline{T}_{m-1}$. Further, by the symbol $(s_0 s_1 \dots s_{m-1})^\infty$ we understand the word $s_0 s_1 \dots s_m s_{m+1} \dots s_{nm} s_{nm+1} \dots$, where $s_{nm+j} = s_j$ for all $j \in \overline{T}_{m-1}$ and all $n \in \overline{T}$. For $m = 1$ we omit the brackets and write s_0^k, s_0^∞.

Convention. In the relation $C \subseteq I^\infty$ we suppose every element from I is included in at least one sequence from C.

Definition 2. G-machine is a triple $M = (S, I, \sigma)$, where

(i) S is a nonvoid finite set;

(ii) $I \subset S$ $(I \neq S)$;

(iii) σ is a mapping of I into the set of all nonvoid subsets of S, i.e. $\sigma : I$ into 2^S.

In the following, M is to be understood as G-machine $M = (S, I, \sigma)$. Let $m \in T$. A word $\prod_{i=0}^{m-1} s_i$ or $\prod_{i=0}^{\infty} s_i$ respectively is called an output word of the length m or ∞ respectively if $s_0 \in I$, $s_{i+1} \in \sigma(s_i) \cap I$ for all $i \in \overline{T}_{m-2}$ or for all $i \in \overline{T}$. An output word $w = \prod_i s_i$ is called a word generated by M if either $\ell(w) = \infty$ or $\ell(w) = m$ and there exists $v \in \sigma(s_{m-1}) \cap (S - I)$. If it is necessary to specify that a word $\prod_i s_i$ is an output word of G-machine $M = (S, I, \sigma)$, we use the denotation $\prod_i s_i(\sigma)$. The set of all words generated by M is denoted $L(M)$ and called the language of M. An arbitrary set C, $C \subseteq I^{\infty}$ is called a generable language if there exists M such that $C = L(M)$.

<u>Definition 3</u>. Let $M = (S, I, \sigma)$ be a G-machine. A couple (s, v) is called productive if $s \in I$, $v \in \sigma(s) \cap I$ and unproductive if $s \in I$, $v \in \sigma(s) \cap$ $\cap (S - I)$. Denote by P_{σ} the set of all productive couples and N_{σ} the set of all unproductive couples.

<u>Definition 4</u>. Let I be a finite set, $C \subseteq I^{\infty}$ and $I \subset S$ $(I \neq S)$, where S is a nonvoid finite set.

Suppose $I \neq \emptyset$. Denote by c an arbitrary element (sequence) from C and by s_i the $(i+1)$-th element of c, $c = \prod_i s_i$ for all $i \in \overline{T}_{m-1}$ if $\ell(c) = m \in T$ and for all $i \in \overline{T}$ if $\ell(c) = \infty$. For $c \in C$, $c = \prod_{i=0}^{m-1} s_i$ $(m \in T)$ put $P(c) = \bigcup_k (s_k, s_{k+1})$ for all $k \in \overline{T}_{m-2}$ and $N(c) = (s_{m-1}, v)$, where v is an arbitrary element of $(S - I)$. For $c \in C$, $c = \prod_{i=0}^{\infty} s_i$ put $P(c) = \bigcup_k (s_k, s_{k+1})$ for all $k \in \overline{T}$. Denote $P = \bigcup_{c \in C} P(c)$, $N = \bigcup_{c \in C} N(c)$, $\sigma[C] = P \cup N$. If $I \neq \emptyset$, put $\sigma[C] = 0$. Define G-machine $M[C] = (S, I, \sigma[C])$.

Definition 5. Let I be a nonvoid finite set, $C \leq I^{\infty}$. Suppose $c', c'' \in C$, $c' = \prod_{i=0}^{m-1} s'_i$ $(m \in T)$, $c'' = \prod_i s''_i$. Define partial operation $\omega : (C \times C)$ into I^{∞} , where $(C \times C) \subseteq C \times C$ as follows:

$$(1) \quad \omega(c', c'') = s_0 s_1 \dots s_{m-1} s_m \dots ,$$

where $s_j = s'_j$ for all $j \in \overline{T}_{m-1}$, $s_{m-1+k} = s''_k$ for all $k \in T_{l(c'')-1}$ if $l(c'') < \infty$ otherwise for all $k \in T$.

Lemma 1. Let I be a nonvoid finite set, $C \subseteq I^{\infty}$ a generable language, $c', c'' \in C$. Then $\omega(c', c'') \in C$.

Proof. By Definition 2 there exists a G-machine $M = (S, I, \sigma)$ such that $C = L(M)$. Suppose c', c'' satisfy the conditions of Definition 5. First consider $c'' \in C$, $c'' = \prod_{i=0}^{k-1} s''_i$ $(k \in T)$. Using (1) it follows

$\omega(c', c'') = s_0 s_1 \cdots s_{m-1} s_m \cdots s_{m+n+2}$, where $s_j = s'_j$ for all $j \in \overline{T}_{m-1}$, $s_{m-1+k} = s_k$ for all $k \in T_{n-1}$. By Definition 2 $s_{i+1} \in \delta(s_i)$ for all $i \in \overline{T}_{m+n-2}$ and there exists $r \in \delta(s_{m+n-2}) \cap (S - I)$. Hence $\omega(c', c'') \in L(M)$, $\omega(c', c'') \in C$. Now let $c'' \in C$, $c'' = \prod_{i=0}^{\infty} s''_i$. By (1) $\omega(c', c'') = s_0 s_1 \cdots s_{m-1} s_m \cdots$, where $s_j = s'_j$ for all $j \in \overline{T}_{m-1}$ and $s_{m-1+k} = s''_k$ for all $k \in T$. Obviously, $s_{i+1} \in \delta(s_i) \cap I$ for all $i \in \overline{T}$ and by Definition 2 $\omega(c', c'') \in L(M)$, $\omega(c', c'') \in C$.

Corollary 1. Let $C \subseteq I^{\infty}$ be a generable language. Then C is closed under partial operation ω .

Example 1. Decide, whether $C = \{a^n b^n a^n \mid n \in T\}$ is a generable language. Consider $c', c'' \in C$, $c' = aba$, $c'' = aabbaa$. Words c', c'' satisfy the condition of Definition 5 and by (1) $\omega(c', c'') = abraabbaa$. Obviously $\omega(c', c'') \,\overline{\in}\, C$ and by Corollary 1 C is not a generable language.

Lemma 2. Let $C \subseteq I^{\infty}$. Then $C \subseteq L(M[C])$.

Proof. If $I \neq \emptyset$, then $L(M[C]) = \emptyset$ and the statement holds true. Suppose $I \neq \emptyset$. Consider $c \in C$, $c = \prod_{i=0}^{\infty} s_i$. By Definition 4 $(s_i, s_{i+1}) \in P(c)$, $(s_i, s_{i+1}) \in P_{\delta[C]}$ for all $i \in \overline{T}$ and by Definitions 3 and 4 $c \in L(M[C])$. Further, let $c \in C$, $c = \prod_{i=0}^{m-1} s_i$ $(m \in T)$. From Definition 4 it follows $(s_i, s_{i+1}) \in P(c)$ for all $i \in \overline{T}_{m-2}$ and $(s_{m-1}, v) \in N(c)$, which implies $(s_i, s_{i+1}) \in P_{\sigma[C]}$ for all $i \in \overline{T}_{m-2}$ and $(s_{m-1}, v) \in N_{\sigma[C]}$. Hence $c \in L(M[C])$ and $C \subseteq L(M[C])$.

<u>Proposition 1</u>. Let I be a finite set, $C \subseteq I^{\infty}$. C is a generable language iff $C = L(M[C])$.

(See [3] as the consequence of Theorem 6 and Corollary 5.)

<u>Theorem 1</u>. Let $\{C_i \mid i \in K\}$ be a class of generable languages. Then $\bigcap_{i \in K} C_i$ is a generable language.

<u>Proof</u>. From Definition 2 it follows there exist G-machines $M_i = (S_i, I_i, \sigma_i)$ such that $C_i = L(M_i)$ for all $i \in K$. Put $C = \bigcap_{i \in K} C_i$. We shall construct G-machine $M[C] = (S, I, \sigma[C])$ and show $C = L(M[C])$. If $C = \emptyset$, then $L(M[C]) = \emptyset$ and by Proposition 1 C is a generable language. Suppose $C \neq 0$. From Lemma 2 $C \subseteq L(M[C])$. First, suppose $w \in L(M[C])$, $w = \prod_{i=0}^{\infty} a_i$. By Definitions 2 and 3 $(s_k, s_{k+1}) \in P_{\sigma[C]}$ for all $k \in \overline{T}$. Further, by Definition 4 there exists to every (s_k, s_{k+1}) a word $c \in C$ such that $(s_k, s_{k+1}) \in P(c)$, which implies $(s_k, s_{k+1}) \in P_{\sigma_i}$ for all $i \in K$ and all $k \in \overline{T}$. From Definition 2 it follows $w \in L(M_i)$ for all $i \in K$, thus $w \in C$. Second, suppose $w \in L(M[C])$, $w = \prod_{i=0}^{m-1} s_i$ $(m \in T)$. By Definitions 2 and 3 $(s_k, s_{k+1}) \in P_{\sigma[C]}$ for all $k \in \overline{T}_{m-2}$ and $(s_{m-1}, v) \in N_{\sigma[C]}$.

According to Definition 4 there exists to every $(s_k, s_{k+1}) \in P_{\sigma[C]}$ a word $c \in C$ such that $(s_k, s_{k+1}) \in P(c)$. Then $(s_k, s_{k+1}) \in P_{\sigma_i}$ for all $i \in K$ and all $k \in T_{m-2}$. Further, there exists $c \in C$ such that $(s_{m-1}, v) \in N(c)$. Since $c \in \bigcap_{i \in K} C_i$, there exists $v^i \in (S_i - I_i)$ with the property $(s_{m-1}, v^i) \in N_{\sigma_i}$ for every $i \in K$. By Definition 2 $w \in L(M_i)$ for all $i \in K$, hence $w \in C$, $L(M[C]) \subseteq C$ and the proof is completed.

<u>Corollary 2</u>. The class of all generable languages is closed under intersection.

<u>Lemma 3</u>. There exist G-machines $M_1 = (S_1, I_1, \sigma_1)$, $M_2 = (S_2, I_2, \sigma_2)$ for which $C = L(M_1) \cup L(M_2)$ is not a generable language.

<u>Proof</u>. Consider G-machines $M_1 = (S_1, I_1, \sigma_1)$, $M_2 = (S_2, I_2, \sigma_2)$, where $S_1 = S_2 = \{a, b, c, x\}$, $I_1 = I_2 = \{a, b, c\}$, $\sigma_1 : [a \to \{b, x\}, b \to \{b\}, c \to \{c\}]$, $\sigma_2 : [a \to \{b, x\}, b \to \{x\}, c \to \{c\}]$. Then $L(M_1) = \{a, ab^\infty, b^\infty, c^\infty\}$, $L(M_2) = \{a, ab, b, c^\infty\}$, $C = L(M_1) \cup L(M_2) = \{a, ab^\infty, ab, b, b^\infty, c^\infty\}$. By Definition 4 $M[C] = (S, I, \sigma[C])$, where $S = \{a, b, c, x\}$, $I = \{a, b, c\}$, $\sigma[C] : [a \to \{b, x\}, b \to \{b, x\}, c \to \{c\}]$. From here $L(M[C]) = \{ab^\infty, a, ab^k, b^\infty, b^k, c^\infty \mid k \in T\}$. Thus $C \neq L(M[C])$ and according to Proposition 1 C is not a generable language.

Corollary 3. The class of all generable languages is not closed under union.

Lemma 4. Let $M_1 = (S_1, I_1, \sigma_1)$, $M_2 = (S_2, I_2, \sigma_2)$ be G -machines, $C = L(M_1) \cup L(M_2)$. Let for every $i, j \in \in \{1,2\}$, $i \neq j$ and for every $n \in T$

(S) $s_0 s_1 \dots s_{n-1} (\sigma_j)$ and $(s_{n-1}, v) \in P_{\sigma_i}$

implies

$$s_0 s_1 \dots s_{n-1} (\sigma_i) \quad \text{or} \quad (s_{n-1}, v) \in P_{\sigma_i} .$$

Then for every $m \in T$ there exists $k \in \{1, 2\}$ such that $s_0 s_1 \dots s_m (\sigma[C])$ implies $s_0 s_1 \dots s_m (\sigma_k)$.

Proof. For $m = 1$ the statement holds true trivially by Definition 4. Let $m > 1$. We shall prove the statement by induction. Put $m = 2$ and suppose $s_0 s_1 s_2 (\sigma[C])$. By Definition 4 there exists $k \in \{1,2\}$ such that $(s_0, s_1) \in P_{\sigma_k}$. Choose $k_0 \in \{1, 2\}$ such that $(s_0, s_1) \in P_{\sigma_{k_0}}$ and suppose $(s_1, s_2) \in P_{\sigma_{k_1}}$, where $k_1 \in (\{1, 2\} - \{k_0\})$. From Definition 2 and (S) it follows $(s_1, s_2) \in P_{\sigma_{k_0}}$ or $s_0 s_1 (\sigma_{k_1})$. If $(s_1, s_2) \in P_{\sigma_{k_0}}$, then $s_0 s_1 s_2 (\sigma_{k_0})$. In case $s_0 s_1 (\sigma_{k_1})$ by Definition 2 $s_0 s_1 s_2 (\sigma_{k_1})$. Thus, there exists $k \in \{1, 2\}$ such that $s_0 s_1 s_2 (\sigma_k)$. Now suppose that $m = p \in (T - \{1, 2\})$ and $s_0 s_1 \dots s_p (\sigma[C])$ implies there exists $k \in \{1, 2\}$ such that $s_0 s_1 \dots$ $\dots s_p (\sigma_k)$. Choose $k_0 \in \{1, 2\}$ such that $s_0 s_1 \dots s_p (\sigma_{k_0})$

and assume there exists v such that $(s_p, v) \in P_{\sigma_{k_1}}$, $k_1 \in (\{1,2\} - \{k_0\})$. From (S) it follows $(s_p, v) \in P_{\sigma_{k_0}}$ or $s_0 s_1 \dots s_p(\sigma_{k_1})$. If $(s_p, v) \in P_{\sigma_{k_0}}$ then by Definition 2 $s_0 s_1 \dots s_p s_{p+1}(\sigma_{k_0})$, where $s_{p+1} = v$. In case $s_0 s_1 \dots s_p(\sigma_{k_1})$ by Definition 2 $s_0 s_1 \dots s_p s_{p+1}(\sigma_{k_1})$. Hence $s_0 s_1 \dots s_p s_{p+1}(\sigma_k)$ holds true for at least one $k \in \{1,2\}$ and the proof is completed.

<u>Theorem 2</u>. Let $M_1 = (S_1, I_1, \sigma_1)$, $M_2 = (S_2, I_2, \sigma_2)$ be G -machines, $C = L(M_1) \cup L(M_2)$. The following statements (A),(B) are equivalent:

(A) For every $i, j \in \{1,2\}$, $i \neq j$ and every $n \in T$

(α) if $s_0 s_1 \dots s_{n-1}(\sigma_j)$ and $(s_{n-1}, v) \in P_{\sigma_i}$ then

$s_0 s_1 \dots s_{n-1}(\sigma_i)$ or $(s_{n-1}, v) \in P_{\sigma_j}$

and

(β) if $s_0 s_1 \dots s_{n-1}(\sigma_j)$ and $(s_{n-1}, v^i) \in N_{\sigma_i}$ then

$s_0 s_1 \dots s_{n-1}(\sigma_i)$ or there exists $(s_{n-1}, v^j) \in N_{\sigma_j}$.

(B) $C = L(M[C])$.

<u>Proof</u>. If $C \neq \emptyset$ then by Definition 4 $L(M[C]) = 0$. By Proposition 1 C is a generable set and the statement holds true trivially. Suppose $C \neq \emptyset$.

I.(A) $\Longrightarrow$ (B). By Lemma 2 $C \subseteq L(M[C])$. First, assume there exists $c \in L(M[C])$, $c = \prod_{i=0}^{\infty} s_i$. From Definitions

2 and 4 it follows $s_0 s_1 \ldots s_m (\sigma[C])$ for every $m \in T$ and by (A) and Lemma 4 there exists $k \in \{1,2\}$ such that $s_0 s_1 \ldots s_m (\sigma_k)$ for every $m \in T$. Therefore $c \in L(M_k)$ for some $k \in \{1,2\}$, $c \in C$. Second, suppose $c \in L(M[C])$, $c = \prod_{i=0}^{n-1} s_i$ $(n \in T)$. By Definition 2 $s_0 s_1 \ldots s_{n-1} (\sigma[C])$ and there exists x such that $(s_{n-1}, x) \in N_{\sigma[C]}$. By Lemma 4 there exists $k \in \{1,2\}$ such that $s_0 s_1 \ldots s_{n-1} (\sigma_k)$. From Definition 2 it follows there exist $k \in \{1,2\}$ and v^k such that $(s_{n-1}, v^k) \in N_{\sigma_k}$. Choose $k \in \{1,2\}$ with the property $s_0 s_1 \ldots s_{n-1} (\sigma_{k_0})$ and suppose there exists $(s_{n-1}, v^{k_1}) \in N_{\sigma_{k_1}}$, $k_1 \in (\{1,2\} - \{k_0\})$. By (A) $s_0 s_1 \ldots$ $\ldots s_{n-1} (\sigma_{k_0})$ or $(s_{n-1}, v^{k_1}) \in N_{\sigma_{k_1}}$. If $s_0 s_1 \ldots$ $\ldots s_{n-1} (\sigma_{k_0})$ then by Definition 2 $c \in L(M_{k_0})$. In case $(s_{n-1}, v^{k}) \in N_{\sigma_{k_1}}$ by Definition 2 $c \in L(M_{k_0})$. Hence there exists $k \in \{1,2\}$ such that $c \in L(M_k)$, $c \in C$. Thus $C = L(M[C])$ and (B) holds true.

II. (B) $\Longrightarrow$ (A). We shall prove the reverse implication by contraposition. Suppose $C = L(M_1) \cup L(M_2) = L(M[C])$ and (B) does not hold. Let us admit (α) does not hold, i.e. $s_0 s_1 \ldots s_{n-1} (\sigma_j)$, $(s_{n-1}, v) \in P_{\sigma_i}$, $(s_{n-1}, v) \,\overline{\in}\, P_{\sigma_j}$ and $s_0 s_1 \ldots s_{m-1} (\sigma_i)$ does not hold. By Definition 4 there exists $w \in L(M[C])$ beginning with the output word $s_0 s_1 \ldots$ $\ldots s_{m-1} s_m$, where $s_m = v$. Under given assumption $(s_{m-1}, s_m) \,\overline{\in}\, P_{\sigma_j}$, thus $w \,\overline{\in}\, L(M_j)$. If $s_0 s_1 \ldots s_{m-1} (\sigma_i)$ does not hold, then $w \,\overline{\in}\, L(M_i)$, hence $w \,\overline{\in}\, C$. Now suppose (β) does not hold, i.e. $s_0 s_1 \dots s_{n-1}(\sigma_j)$, $(s_{n-1}, v^i) \in$ $\in N_{\sigma_i}$, $(s_{n-1}, v^j) \in N_{\sigma_j}$. for every $v^j \in (S_j - I_j)$ and $s_0 s_1 \dots s_{n-1}(\sigma_i)$ does not hold. By Definition 4 there exists $w \in L(M[C])$, $w = \prod_{i=0}^{n-1} s_i$. Since there does not exist $v^j \in (S_j - I_j)$ such that $(s_{n-1}, v^j) \in N_{\sigma_j}$, then by Definition 2 $w \,\overline{\in}\, L(M_j)$. Further, $s_0 s_1 \dots s_{n-1}(\sigma_i)$ does not hold true, thus $w \,\overline{\in}\, L(M_i)$. Hence $w \,\overline{\in}\, C$ and we reached a contradiction.

<u>Corollary 4</u>. Let $M_1 = (S_1, I_1, \sigma_1)$ and $M_2 = (S_2, I_2, \sigma_2)$ be G-machines. The following statements (A), (B) are equivalent:

(A) For every $i, j \in \{1, 2\}$, $i \neq j$ and for every $n \in T$

(α) if $s_0 s_1 \dots s_{n-1}(\sigma_j)$ and $(s_{n-1}, v) \in P_{\sigma_i}$

then

$s_0 s_1 \dots s_{n-1}(\sigma_i)$ or $(s_{n-1}, v) \in P_{\sigma_j}$ and

(β) if $s_0 s_1 \dots s_{n-1}(\sigma_j)$ and $(s_{n-1}, v^i) \in N_{\sigma_i}$

then

$s_0 s_1 \dots s_{n-1}(\sigma_i)$ or there exists

$(s_{n-1}, v^j) \in N_{\sigma_j}$.

(B) $L(M_1) \cup L(M_2)$ is a generable language.

<u>Corollary 5</u>. Let C_1, C_2 be generable languages. $C = C_1 \cup C_2$ is a generable language iff G-machines

$M[C_1] = (S_1, I_1, \sigma[C_1])$, and $M[C_2] = (S_2, I_2, \sigma[C_2])$ satisfy the condition (A) of Theorem 2.

Example 2. Let $M_1 = (S_1, I_1, \sigma_1)$, $M_2 = (S_2, I_2, \sigma_2)$ be G -machines given as follows: $S_1 = \{a, b, c, d, x\}$, $I_1 = \{a, b, c, d\}$, $\sigma_1 = [a \to \{a, c\}, b \to \{d\}, c \to \{b\}, d \to \{x\}$, $S_2 = \{b, c, e, f, y\}$, $I_2 = \{b, c, e, f\}$, $\sigma_2 :$ $: [b \to \{e\}, c \to \{b\}, e \to \{f\}, f \to \{y\}]$.
By Definitions 2 and 3 $acb(\sigma_1)$, $(b, e) \in P_{\sigma_2}$, $(b, e) \overline{\in} P_{\sigma_1}$ hold true. Further, $acb(\sigma_2)$ does not hold true. By Corollary 4 $L(M_1) \cup L(M_2)$ is not a generable language.

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