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DOUBLE LAYER POTENTIAL REPRESENTATION OF THE SOLUTION OF THE DIRICHLET PROBLEM

(Preliminary communication)

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Let M be an open set in R^m, the Euclidean m-space of dimension $m > 2$, and suppose that its boundary B is compact and non-void. Recall that a point $x \in R^m$ is termed a hit of an open segment S on M provided $x \in S$ and each neighborhood of x meets both $S \cap M$ and $S - M$ in a set of positive linear measure. Given $y \in R^m$, $r > 0$ and $\theta \in \Gamma = \{x \in R^m ; |x| = 1\}$ we shall denote by $n_r(\theta, y)$ the total number of all the hits of $\{y + \varphi\theta ; 0 < \varphi < r\}$ on M. For fixed $r > 0$ and $y \in R^m$, $n_r(\theta, y)$ is a Baire function of the variable θ on Γ and one may define

$$v_r(y) = \int_\Gamma n_r(\theta, y)\, dH(\theta)$$

where H stands for the $(m-1)$-dimensional Hausdorff measure in R^m. As usual, for $Q \subset R^m$ we shall denote by $cl\,Q$ and $fr\,Q$ the closure and the boundary of Q, respectively.

For investigations of the generalized Dirichlet problem on M we adopt the following assumptions:

(1) $$fr\,M = fr\,(R^m - cl\,M),$$

(2) $$\lim_{r \to 0+} \sup_{y \in B} v_r(y) < \frac{1}{2} H(\Gamma).$$

It should be noted here that each $x \in B$ is a regular point for the Dirichlet problem on M.

The symbol H_f will denote the solution of the generalized Dirichlet problem on M, provided f is a resolutive function defined on B; given $x \in M$, μ_x stands for the harmonic measure relative to M and x. Now we are in a position to announce:

<u>Proposition</u>. Let $\hat{H}$ denote the restriction of H to B. If M is connected and $x \in M$, then the measures $\hat{H}$ and μ_x are mutually absolutely continuous.

<u>Corollary</u>. A bounded function on B is resolutive if and only if it is H-measurable.

Put $G = R^m - cl\,M$. It follows from (1),(2) and a result of J. Král that G has only a finite number of components and their closures are mutually disjoint. (Note that the same is true for M.) We shall denote by q ($0 \leq q < \infty$) the number of all bounded components of G. These components will be denoted by G_j and we shall fix an arbitrary $x_j \in G_j \quad (j = 1, \dots, q)$.

Let us recall that a unit vector $\theta \in \Gamma$ is called the exterior normal of M at $y \in R^m$ in the sense of H. Federer provided the symmetric difference of M and the half-space

$$\{x \in R^m ;\ (x - y) \cdot \theta < 0\}$$

has m -dimensional density 0 at y . We put $n(y) = \theta$ if θ is the exterior normal of M ; otherwise $n(y)$ denotes the zero vector.

Finally, let $\tilde{N}$ consist of all functions on B which are equivalent (H) to a linear combination of the characteristic functions of $\mathrm{fr}\, G_j$.

Some results of [2],[1], previous investigations of the author (see Czechoslovak Math.J. 22(1972),312-324, 462-489, 554-580) and a modification of the Riesz-Schauder theory given in [3] permit one to obtain the following results:

<u>Theorem</u>. Given an arbitrary bounded H -measurable function g on B there are the bounded H -measurable function f (determined modulo $\tilde{N}$) and uniquely determined constants a_j such that for each $y \in M$

$$(3) \quad H_g(y) = \int_B f(x) \cdot \frac{n(x) \cdot (x-y)}{|x-y|^m} \, dH(x) + \sum_{j=1}^{q} a_j \, |y - x_j|^{2-m} .$$

Moreover, the nontangential limits of H_g equal g at each $x \in B$ except for a set of H -measure zero.

The concepts employed here have their origin in J. Král's investigations [2]. In § 3 of [2] the representation of the form (3) for continuous boundary conditions is given under a slightly more restrictive assumption on the shape of M .

Complete proofs of the above results together with further details are contained in a paper submitted for publication in Časopis pro pěstování matematiky.

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