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Commentationes Mathematicae Universitatis Carolinae, Vol. 14 (1973), No. 1, 135--138

Persistent URL: http://dml.cz/dmlcz/105477

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14,1 (1973)

ON A CONJECTURE BY NASH - WILLIAMS

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Abstract: This note brings forward an example disproving the following conjecture by C.St.I.A. Nash-Williams [1]: Let $\mathcal{D}$ be a directed graph with $n \geq 5$ vertices. If the in-degree as well as the out-degree of every vertex of $\mathcal{D}$ is $\geq \frac{n}{2}$, then in $\mathcal{D}$ at least two edge-disjoint hamiltonian circuits are admitted.

Key words: graph, hamiltonian circuit

AMS, Primary: 05C20 Ref. Ž. 8.83

We show that every two hamiltonian circuits of the graph G in Fig. 1 have a common edge. To do this we associate to G the rooted tree T in Fig. 2 as follows: The root X_1 is the image of x_1. (Because of the symmetry of G the choice is arbitrary.) The neighbours of X_1 are the images X_2, X_3, X_5 of the neighbours x_2, x_3, x_5 of x_1. Analogously it is proceeded with X_2, etc., until the vertex X_i appears in T which is the image of such a vertex of G from which there is an edge directed to x_1. If the length of the path from X_1 to X_i is 5, this path is the image of a hamiltonian line in G. Doing this with all vertices X_2, X_3, X_5 and their neighbours etc. we get all the hamiltonian lines in G starting with x_1, and in that way all the hamiltonian circuits of G. (Of course, if in construing the branch through X_3 or X_5 a vertex appearing already in the second branch through X_2 is met, this vertex need not be considered in this second branch.) In that way from the tree T the graph G is seen to have four hamiltonian circuits no two of which are edge-disjoint.

Problem: Given a positive integer n, determine the maximum number $f(n)$ such that every directed graph on n vertices admits $f(n)$ edge-disjoint hamiltonian circuits, supposing in addition that all in-degrees as well as out-degrees are $\geq \frac{n}{2}$.

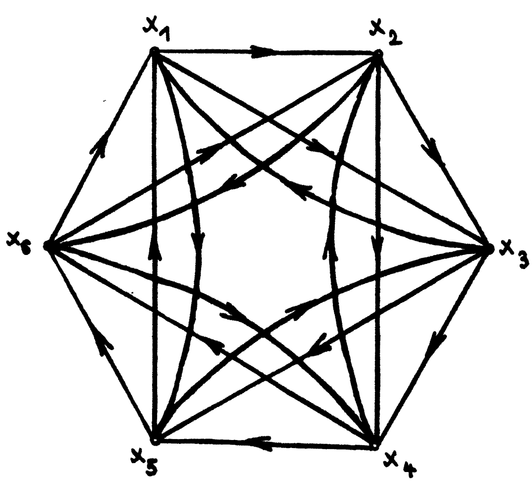
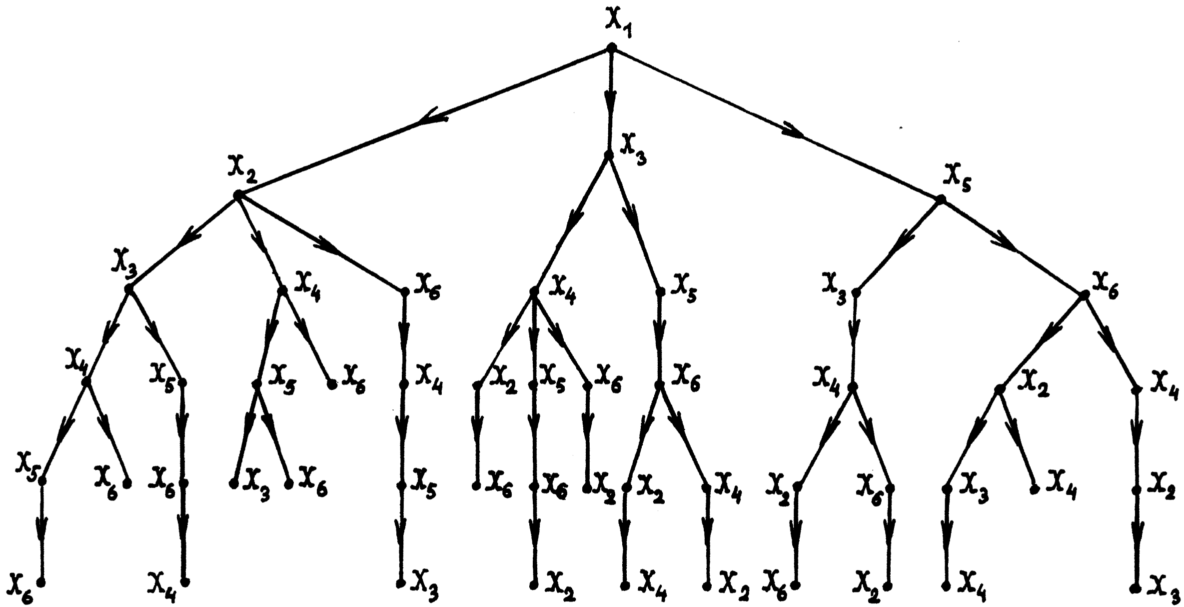


Fig. 1



X_1
X_2 X_3 X_5
X_3 X_4 X_6 X_4 X_5 X_3 X_6
X_4 X_5 X_5 X_6 X_4 X_2 X_5 X_6 X_6 X_4 X_2 X_4
X_5 X_6 X_6 X_3 X_6 X_5 X_6 X_6 X_2 X_2 X_4 X_2 X_6 X_3 X_4 X_2
X_6 X_4 X_3 X_2 X_4 X_2 X_6 X_2 X_4 X_3

Fig. 2

R e f e r e n c e

[1] NASH-WILLIAMS C.St.I.A.: Hamiltonian circuits in graphs and digraphs. The many facets of graph theory (Proceedings of a Conference at Western Michigan University in November 1968, edited by Chartrand, G. and Kapoor, S.F), Springer-Verlag, Berlin, Heidelberg and New York, 1969, pp. 237-243.

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(Oblatum 24.2.1973)