Jiří Kopáček

On L_p-estimates for the Cauchy problem for hyperbolic systems (Preliminary communication)

Commentationes Mathematicae Universitatis Carolinae, Vol. 10 (1969), No. 2, 237--239

Persistent URL: `http://dml.cz/dmlcz/105230`

Commentationes Mathematicae Universitatis Carolinae
10,2 (1969)

ON L_p - ESTIMATES FOR THE CAUCHY PROBLEM FOR HYPERBOLIC SYSTEMS

Jiří KOPÁČEK, Praha

(Preliminary communication)

In [1] the following theorem was proved:

Theorem 1. Let the system

$$\frac{\partial u}{\partial t} = \sum_{j=1}^{n} A_j \frac{\partial u}{\partial x_j} + Bu \tag{1}$$

where A_j, B are $N \times N$ constant matrices, be hyperbolic in the following sense:

(H) α) The matrix $A(y) = \sum_{j=1}^{n} y_j A_j$

has for all $y \in R_n$ only real eigenvalues and can be diagonalized by a similarity transformation

$T^{-1}(y) A(y) T(y)$ for all $y \in R_n$.

β) There exist positive constants C_1, C_2, C_3, T such that

$$\| \Psi_t(y) \| = \| e^{t(iA(y)+B)} \| \leq C_1 + C_2 |y|^{C_3}$$

for all $y \in R_n$, $t \in \langle 0, T \rangle$.

Let for some $p \in \langle 1, +\infty \rangle$, $p \neq 2$ and for some $C > 0$ the following inequality hold:

$$\| u_\varphi(t,x) \|_{L_p} \leq C \| \varphi \|_{L_p} \tag{2}$$

for all $0 \leq t \leq T$, $\varphi \in \mathcal{S}$, where u_φ is the solution of (1) with the initial condition $u(0,x) = \varphi(x)$.

Then $A_i A_j = A_j A_i$ for $i, j = 1, 2, \dots, m$.

<u>Remark</u>. Using the Hörmander's result about multipliers in L_p (see corollary 1.3 in [2]) one can prove that (H),(β) and (2) imply (H)(α), so that the assumption (H)(α) can be removed. Note that P. Brenner has proved a more general result which will appear in Math.Scandinavica. Now we want to state the following generalization of the theorem 1.

<u>Theorem 2</u>. Let the system (1) satisfy (H)(β) and for some $k \geqslant 1$ and some $p \in (1, +\infty)$, $p \neq 2$ the inequality

(2')
$$\| u_\varphi (t, x) \|_{W_p^k} \leqslant C \| \varphi \|_{W_p^k}$$

holds for $0 \leqslant t \leqslant T$, $\varphi \in \mathcal{S}$, where W_p^k are the Sobolev's spaces. Then the inequality (2) is valid and $A_i A_j = A_j A_i$, $i, j = 1, 2, \dots, m$.

To prove this theorem, one can firstly obtain the inequality

$$\| u_\varphi (t, x) \|_{W_p^{(-k)}} \leqslant C_1 \| \varphi \|_{W_p^{(-k)}}$$

where $W_p^{(-k)}$ is dual to W_q^k, $\frac{1}{q} + \frac{1}{p} = 1$ and finally one can apply the interpolation theorem of Calderón [3].

Finally we generalize a result of Muravej for the wave equation [4].

<u>Theorem 3</u>. Let the system (1) be hyperbolic in the sense of Petrovski and $B \equiv 0$. Then for its solutions u_φ the following estimation holds:

$$\| u_\varphi (t, x) \|_{L_p} \leqslant C \sum_{\nu=0}^{k} t^\nu \sum_{|\alpha| = \nu} \| D^\alpha \varphi \|_{L_p}$$

for all $t \geq 0$ and $\varphi \in \mathcal{S}$, where C depends on p and k is the smallest integer satisfying

$$k \geq \begin{cases} [\frac{n}{2}]+1 & \text{for } p = +\infty , \\ n(\frac{1}{p}-\frac{1}{2}) & \text{for } p \in \langle 1, 2 \rangle , \\ n(\frac{1}{2}-\frac{1}{p}) & \text{for } p \in (2, +\infty) . \end{cases}$$

For the proof the imbedding theorem and finiteness of the domain of dependence for (1) is used.

References

[1] J. KOPÁČEK: The Cauchy problem for linear hyperbolic systems in L_p, Comment.Math.Univ.Carolinae 8(1967),No 3,458-462.

[2] L. HÖRMANDER: Estimates for translation invariant operators in L_p spaces, Acta Math.104(1960),93-140.

[3] A.P. CALDERÓN: Lebesgue spaces of differentiable functions and distributions,Proc.of symp.in pure Math.Vol.IV,Part.dif.equations,Amer.Math.Soc. Providence 1961.

[4] L.A. MURAVEJ: Zadača Koši dlja volnovogo uravnenija v L_p-prostranstvach, Trudy Mat.inst.im.Steklova 103(1968),172-180.

(Received April 17,1969)