Commentationes Mathematicae Universitatis Carolinae

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Commentationes Mathematicae Universitatis Carolinae, Vol. 9 (1968), No. 4, 571--572

Persistent URL: http://dml.cz/dmlcz/105200

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9,4 (1968)

ON A PROBLEM OF H. HERRLICH

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All spaces are assumed to be Hausdorff. The letter k stands for infinite cardinal numbers.

H. Herrlich has defined and studied k-compact spaces in his paper Fortsetzbarkeit stetiger Abbildungen und Kompaktheitsgrad topologischer Räume which appeared in Math.Z.96(1967),64-72. A uniformizable space P is called k-compact if each its ultrafilter of zero sets with k-intersection property is fixed (a collection of sets is said to have k-intersection property if the intersection of less than k members of the collection is nonvoid). Of course, $\aleph_0$-compact spaces coincide with compact spaces and $\aleph_1$-compact spaces coincide with realcompact spaces. It is known that the classes of compact and of realcompact spaces are simple (i.e.,these spaces consist precisely of closed subsets of powers of a space, namely of I or R respectively). H. Herrlich has posed a problem for what k the class of k-compact spaces is simple. We shall state here without details an answer to this problem and introduce some generalizations of known facts for compact and realcompact spaces depending on I and R (the theorem of Kaplansky and Shirota, connections between compactness and completeness). The details and further considerations in this connection will be published elsewhere.

Theorem 1. For each infinite cardinal number k there is a space P_k such that a space is k-compact if and only if it can be embedded as a closed subset into a power P_k^A of P_k.

Corollary. For each infinite cardinal number k the class of k-compact spaces is nonvoid.

H. Herrlich has proved the foregoing corollary for all limit cardinals and for all numbers of a form $\aleph_{\alpha+2}$.

A proximity space is said to be k-complete if each its Cauchy filter with k-intersection property has a limit (we identify proximities and associated totally bounded uniformities).

Theorem 2. A uniformizable space P is k-compact if and only if P has a k-complete proximity.

Corollary. A uniformizable space P is k-compact if and only if the Čech proximity of P is k-complete.

It is possible to find spaces P_k which have natural order structures. These structures carry over by standard methods to $C(P,P_k)$.

Theorem 3. Let P,P' be k-compact spaces, where k is an isolated cardinal number. Then P and P' are homeomorphic if and only if $C(P,P_k)$ and $C(P',P_k)$ are lattice-isomorphic.

(Received June 5, 1968)