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CONSTRUCTION OF SPECIAL FUNCTORS AND ITS APPLICATIONS

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Let three categories $\mathcal{K}_1, \mathcal{K}_2, \mathcal{C}$ and two faithful functors $F_i : \mathcal{K}_i \longrightarrow \mathcal{C}$ be given. We want to construct "the best functor" G from $\mathcal{K}_1$ into $\mathcal{K}_2$ commuting with F_i and having given values at some objects $X_i \in obj\, \mathcal{K}_1$. "The best" for G means to have as many as possible good properties (to be one-to-one, full, to preserve products, quotients, etc.). It follows that if each object X of $\mathcal{K}_1$ can be embedded into a product of $\{X_i\}$, then GX must be embedded into the product of $\{GX_i\}$. This fact is the basic idea for the construction of G. We shall prove under certain conditions that G is the only functor preserving products and substructures (Theorem 8) or that G is full whenever there is a full functor from $\mathcal{K}_1$ into $\mathcal{K}_2$ (Theorem 3 and its Corollary). In some cases $G[\mathcal{K}_1]$ is a coreflective subcategory of $\mathcal{K}_2$ (Theorem 5) and G is the only full functor from $\mathcal{K}_1$ into $\mathcal{K}_2$ (Theorem 9).

As applications of our general theory there are some illustrative examples concerning relations between Top, Prox, Unif. We shall obtain also new results in Examples 4 and 5 - characterizations of continuity structures and a generalization of Smirnov theorem on the equivalence between compactifications and proximity spaces.

We shall use the notation from [2] and from [4]. It seems to be suitable to recall concepts being often used in the sequel (we shall deal with projective cases only - the dual ones are inductive). In this paper, the term "functor" means "covariant functor". A non-void indexed class $\{f_i\}$ of morphisms is said to be projective if all the f_i have the same domain. The following definition is a modification of the projective generation from [4] (see [1]):

Let $F : \mathcal{K} \longrightarrow \mathcal{C}$ be a faithful functor, $\mathcal{C}_1$ a subcategory of $\mathcal{C}$, $\{X_i\}$ an indexed class of objects from $\mathcal{K}$ and let $\{\varphi_i\}$ be a projective indexed class in $\mathcal{C}$ such that $\{FX_i\} = \{E\varphi_i\}$. We shall denote by

$$\langle F, \mathcal{C}_1 \rangle - \underleftarrow{lim} \langle \{\varphi_i\}, \{X_i\} \rangle$$

an object X of $\mathcal{K}$ with the following properties:

there exists an $f_i : X \longrightarrow X_i$ for each i such that $Ff_i = \varphi_i$;

if $\{g_i\}$ is a projective indexed class in $\mathcal{K}$, $g_i : Y \longrightarrow X_i$, $Fg_i = \varphi_i \circ \varphi$ for each i, where $\varphi \in \mathcal{C}_1$, then $Fg = \varphi$ for some $g : Y \longrightarrow X$.

In the case that $\{\varphi_i\} = \{Fh_i\}$, $\{Eh_i\} = \{X_i\}$, $\{h_i\}$ is a projective indexed class, we shall denote X by $\langle F, \mathcal{C}_1 \rangle - \underleftarrow{lim} \{h_i\}$. The symbol $\langle F, \mathcal{C}_1 \rangle - proj\ \mathcal{K}'$, where $\mathcal{K}'$ is a subcategory of $\mathcal{K}$, designates the class $E\{\langle F, \mathcal{C}_1 \rangle - \underleftarrow{lim} \{h_i\} \mid \{h_i\}$ is a projective indexed class with ranges in $\mathcal{K}'\}$.

Let us have two faithful functors $F_i : \mathcal{K}_i \longrightarrow \mathcal{C}$. A functor $G : \langle \mathcal{K}_1, F_1 \rangle \longrightarrow \langle \mathcal{K}_2, F_2 \rangle$ (i.e. $F_2 \circ G = F_1$) is said to be projectively F_2 -preserving with respect to $\mathcal{C}_1$ if $G[\langle F_1, \mathcal{C}_1\rangle - \underleftarrow{\lim}\{f_i\}] = \langle F_2, \mathcal{C}_1\rangle - \underleftarrow{\lim}\{Gf_i\}$ whenever the left side exists.

In most cases (see [4]) the generation does not depend on $\mathcal{C}_1$ and therefore $\mathcal{C}_1$ will be omitted in this case.

Now, we shall describe the main construction: Let $F_i : \mathcal{K}_i \longrightarrow \mathcal{C}$ be faithful functors, $\mathcal{K}_1'$ a subcategory of $\mathcal{K}_1$ and let G' be a functor $\langle \mathcal{K}_1', F_{1\mathcal{K}_1'}\rangle \to \langle \mathcal{K}_2, F_2\rangle$. Assume that the object

$GX = \langle F_2, F_1[\mathcal{K}_1]\rangle - \underleftarrow{\lim} \langle \{F_1 f \mid Df = X, Ef \in \mathcal{K}_1'\}, \{G'Ef\}\rangle$

exists for each $X \in \operatorname{obj} \mathcal{K}_1$. Then the mapping G can be extended to a functor $G : \langle \mathcal{K}_1, F_1\rangle \longrightarrow \langle \mathcal{K}_2, F_2\rangle$ in this way: For each $f : X \to Y$, $Y \in \mathcal{K}_1'$ there exists an $f_{G'} : GX \longrightarrow G'Y$ such that $F_2 f_{G'} = F_1 f$. It follows that $GX = \langle F_2, F_1[\mathcal{K}_1]\rangle - \underleftarrow{\lim}\{f_{G'}\}$ and, hence, for each $h \in Hom_{\mathcal{K}_1}\langle X', X\rangle$ there is a morphism $Gh \in Hom_{\mathcal{K}_2}\langle GX', GX\rangle$ such that $F_2 Gh = F_1 h$.

Definition. The functor G is said to be projectively $\langle F_1, F_2\rangle$-generated by the functor G'.

In the sequel, we shall use the notation from the preceding construction and we shall suppose that F_i are amnestic (see [2]) - i.e. the conditions $F_i f = 1$, f is an equivalence imply $f = 1$ (see also [1]).

The following two theorems deal with cases when G is one-to-one (i.e. an embedding).

Theorem 1. If G is one-to-one then each $X \in \operatorname{obj} \mathcal{K}_1$ is a maximal object having the given morphisms into $\mathcal{K}_1'$. Consequently, if the projective generation exists in $\langle \mathcal{K}_1, F_1\rangle$, then $\mathcal{K}_1 = \langle F_1, \operatorname{obj} \mathcal{C}\rangle - proj\ \mathcal{K}_1'$.

Evidently, we must add some further assumptions for the converse statement to be true:

Theorem 2. Suppose that $F_1[Hom_{\mathcal{K}_1}\langle X,Y\rangle] = F_2[Hom_{\mathcal{K}_2}\langle GX, G'Y\rangle]$ for each $X \in \mathcal{K}_1$, $Y \in \mathcal{K}_1'$. Then G is an embedding if $\mathcal{K}_1 = \langle F_1, obj\ \mathcal{C}\rangle - proj\ \mathcal{K}_1'$.

Now, we approach to the more special case of an embedding in our case - to the fullness of G.

Theorem 3. The conditions $F_1[Hom_{\mathcal{K}_1}\langle X,Y\rangle] = F_2[Hom_{\mathcal{K}_2}\langle GX, G'Y\rangle]$ for each $X \in \mathcal{K}_1$, $Y \in \mathcal{K}_1'$ and $\mathcal{K}_1 = \langle F_1, F_2[\widehat{G[\mathcal{K}_1]}]\rangle - proj\ \mathcal{K}_1'$ ($\widehat{G[\mathcal{K}_1]}$ is the full subcategory of $\mathcal{K}_2$ generated by $G[\mathcal{K}_1]$) are sufficient for G to be full. If G extends G' then they are also necessary.

Proof. Let $g \in Hom_{\mathcal{K}_2}\langle GX, GX'\rangle$. By the first condition, there is a mapping $\{f \to f'\}: Hom_{\mathcal{K}_1}\langle X', Y\rangle \to Hom_{\mathcal{K}_1}\langle X, Y\rangle$ for each $Y \in \mathcal{K}_1'$ such that $f_{G'} \circ g = f'_{G'}$. By the second condition, $F_2 g = F_1 h$ for some $h: X \to X'$. Clearly, $Gh = g$. The converse statement is obvious.

Corollary. G is full provided that there exists a full functor $H: \langle \mathcal{K}_1, F_1\rangle \to \langle \mathcal{K}_2, F_2\rangle$ which extends G' and that $\mathcal{K}_1 = \langle F_1, F_2[\mathcal{K}_2]\rangle - proj\ \mathcal{K}_1'$.

Theorem 4. G is full provided that $\mathcal{K}_1 = \langle F_1, obj\ \mathcal{C}\rangle - proj\ \mathcal{K}_1'$ and that there exists a projectively $\langle F_2, F_1[\mathcal{K}_1]\rangle$-preserving functor $H: \langle \mathcal{K}_2, F_2\rangle \to \langle \mathcal{K}_1, F_1\rangle$ with the property $H \circ G' = 1_{\mathcal{K}_1'}$.

Proof. It is sufficient to prove the equality $H \circ G = 1_{\mathcal{K}_1}$. By the definition $GX = \langle F_2, F_1[\mathcal{K}_1]\rangle - \underline{lim}\{f_{G'} \mid f: X \to Y,\ Y \in \mathcal{K}_1'\}$. Thus $HGX = \langle F_1, F_1[\mathcal{K}_1]\rangle - \underline{lim}\{Hf_{G'} \mid f: X \to Y,\ Y \in \mathcal{K}_1'\}$. Because $\mathcal{E}Hf_{G'} = \mathcal{E}f$, $F_1 Hf_{G'} = F_1 f$ we have $HGX = \langle F_1, F_1[\mathcal{K}_1]\rangle - \underline{lim}\{f \mid f: X \to Y,\ Y \in \mathcal{K}_1'\} = X$.

Now, we shall turn our attention to other properties of G - to the preservation of generations.

Theorem 5. Let $F_1[\mathrm{Hom}_{\mathcal{K}_1}\langle X, Y\rangle] = F_2[\mathrm{Hom}_{\mathcal{K}_2}\langle GX, G'Y\rangle]$ for each $X \in \mathcal{K}_1$, $Y \in \mathcal{K}_1'$. Then the functor $G: \mathcal{K}_1 \to \mathcal{K}_2'$, where $\mathcal{K}_2'$ is the full subcategory of $\mathcal{K}_2$ generated by $\langle F_2, \mathcal{C}\rangle$ - $proj\ G'[\mathcal{K}_1']$ is inductively $\langle F_{2\mathcal{K}_2'}, \mathcal{C}\rangle$-preserving.

Proof. Assume that $X = \langle F_1, \mathcal{C}\rangle - \varinjlim \{f_i\}$, $g_i: G\mathcal{D}f_i \to A$, $A \in \mathcal{K}_2'$, $F_2 g_i = \varphi \circ F_1 f_i$. We are to prove the existence of a morphism $g: GX \to A$ such that $F_2 g = \varphi$. First, suppose that $A = G'Y$. Then $g_i = h_{iG'}$ for some $h_i:$ $: \mathcal{D}f_i \to Y$ and, consequently, $\varphi = F_1 h$ for some $h: X \to$ $\to Y$. It follows that the requested g is equal to $h_{G'}$. Let $A = \langle F_2, \mathcal{C}\rangle - \varprojlim \{k_j\}$, where $\mathcal{E}k_j \in G'[\mathcal{K}_1']$. Then, by the preceding proof, there exist morphism $l_j: GX \to$ $\to \mathcal{E}k_j$ such that $F_2 l_j = F_2 k_j \circ \varphi$. It follows from the definition of A that $F_2 g = \varphi$ for some $g: GX \to A$.

Of course, it is possible to state Theorem 5 more generally for $\mathcal{C}_1$ instead of $\mathcal{C}$. But in that case we must add a condition ensuring that $\mathcal{C}_1$ is stable under compositions with morphisms into $G'[\mathcal{K}_1']$.

Theorem 6. Let $\mathcal{K}_1 = \langle F_1, \mathcal{C}\rangle$ - $proj\ \mathcal{K}_1'$. If $X = \langle F_1, \mathcal{C}'\rangle - \varprojlim \{f^i\}$, $\mathcal{E}f^i \in \mathcal{K}_1'$ implies $GX =$ $= \langle F_2, \mathcal{C}'\rangle - \varprojlim \{f^i_{G'}\}$ then G is projectively F_2-preserving with respect to $\mathcal{C}'$. If G extends G' then the convers holds, too.

The last theorems deal with relations between G and other functors $H: \langle \mathcal{K}_1, F_1\rangle \to \langle \mathcal{K}_2, F_2\rangle$. We shall denote

$H < G$ if $HX <_{F_2} GX$ for each $X \in \mathcal{K}_1$ (i.e. if there is an $f : HX \longrightarrow GX$ such that $F_2 f = 1$), e.g. we have always $G_{\mathcal{K}_1'} < G'$.

Theorem 7. Let $\tilde{\mathcal{K}}_1$ be a subcategory of $\mathcal{K}_1$ such that $\mathcal{K}_1' \subset \tilde{\mathcal{K}}_1$ and that $Hom_{\tilde{\mathcal{K}}_1}\langle X, Y\rangle = Hom_{\mathcal{K}_1}\langle X, Y\rangle$ for each $X \in \tilde{\mathcal{K}}_1$, $Y \in \mathcal{K}_1'$. Assume that $H_{\mathcal{K}_1'} < G'$ for a functor $H : \langle \tilde{\mathcal{K}}_1, F_{1\tilde{\mathcal{K}}_1}\rangle \rightarrow \langle \mathcal{K}_2, F_2\rangle$. Then $H < G_{\tilde{\mathcal{K}}_1}$.

Corollary. G extends G' if and only if G' can be extended to the full subcategory of $\mathcal{K}_1$ generated by $\mathcal{K}_1'$ with preservation of the equality $F_2 \circ G' = F_1$.

Theorem 8. Suppose that $\tilde{\mathcal{K}}_1$, H fulfil the conditions of Theorem 7. If moreover $H_{\mathcal{K}_1'} = G'$, H is projectively $\langle F_1, obj\, \mathcal{C}\rangle$ -preserving, $\tilde{\mathcal{K}}_1 \subset \langle F_1, obj\, \mathcal{C}\rangle$ - $proj\, \mathcal{K}_1'$, then $H = G_{\tilde{\mathcal{K}}_1}$.

Theorem 9. Let $\mathcal{K}_2 = \langle F_2, obj\, \mathcal{C}\rangle$ - $proj\, G'[\mathcal{K}_1']$. If H is a full functor from $\langle \mathcal{K}_1, F_1\rangle \longrightarrow \langle \mathcal{K}_2, F_2\rangle$ which extends G' then $H = G$.

Proof. It follows from the fullness of H that if $g : HX \longrightarrow G'Y$ then $F_1 f = F_2 g$ for some $f : X \rightarrow Y$. Consequently, $GX <_{F_2} HX$. Hence, $G = H$.

Now, we shall proceed to the applications of the preceding general theorems to special categories. The first ones are only illustrative. The main applications occuring in examples 4 and 5 are presented here without details (they will appear elsewhere).

Example 1. Let $\mathcal{K}_1$ be the category of semi-uniformizable spaces (i.e. $x \in u(y)$ implies $y \in u(x)$), $\mathcal{K}_1'$ ~~be the~~ category of proximity spaces (in the sense of [2] i.e.

they need not be uniformizable).

The functors F_i are the obvious forgetful functors into $\mathcal{E}ns$.

We choose for $\mathcal{K}_1'$ the least space projectively generating $\mathcal{K}_1$.

It is the three-point space $\langle (a, b, c), u \rangle$, where $u(a) = (a, b)$, $u(c) = (c, b)$, $u(b) = (a, b, c)$. If we want for G to be full we must put $G' \langle (a, b, c), u \rangle = \langle (a, b, c), p \rangle$, where the only non-void non-proximal sets are $(a), (c)$. By Theorem 9, G is the only full embedding of $\langle \mathcal{K}_1, F_1 \rangle$ into $\langle \mathcal{K}_2, F_2 \rangle$ (it is the known embedding onto fine proximities). $G[\mathcal{K}_1]$ is coreflective in $\mathcal{K}_2$ by Theorem 5 and is not reflective.

From the inductivity of $G[\mathcal{K}_1]$ in $\mathcal{K}_2$ the uniqueness of G follows also in another way. We know that $\mathcal{K}_1$ is inductively generated by paracompact T_1-spaces with at most one accumulation point. It follows from Theorem 7 that every full functor from $\langle \mathcal{K}_1, F_1 \rangle$ into $\langle \mathcal{K}_2, F_2 \rangle$ coincides with G on discrete spaces and, consequently, on the paracompact T_1-spaces with at most one accumulation point. If we construct the full embedding from these facts inductively we obtain G (without using Theorem 9).

Example 2. The second method from the preceding example (a comparison of the functors generated projectively or inductively) is useful in the cases when $\mathcal{K}_2 \neq \operatorname{proj} G[\mathcal{K}_1']$. The last inequality holds if we replace the proximity spaces by the semi-uniform spaces in Example 1. We shall prove that there is a class of different full embeddings of $\langle \mathcal{K}_1, F_1 \rangle$ into $\langle \mathcal{K}_2, F_2 \rangle$ in this case. Let m be an infinite

cardinal number, A_m a discrete space of cardinality m, $P_m = A_m \times \langle (a, b, c), u \rangle$, $G'_m P_m$ the fine semi-uniform space inducing P_m, and G_m be the functor projectively $\langle F_1, F_2 \rangle$ -generated by G'_m.
Then $G_m \neq G_n$ for $m \neq n$.
If we use also the inductive generation we shall get that the value HP of an arbitrary full functor $H : \mathcal{K}_1 \to \mathcal{K}_2$ is uniformly coarser than the fine semi-uniformity and is uniformly finer than the fine proximally coarse semi-uniformity of P.
The same results can be proved for uniformizable and uniform spaces (we use the closed unit interval instead of $\langle (a, b, c), u \rangle$).

Example 3. Let $\mathcal{K}_1 = \mathcal{U}nif$, $\mathcal{K}_2$ be the category of uniformizable proximities, F_i the forgetful functors into $\mathcal{E}ns$, $\mathcal{K}'_i$ the full subcategory of $\mathcal{K}_i$ generated by metrizable spaces and G' be the obvious isomorphism of $\mathcal{K}'_1$ onto $\mathcal{K}'_2$. Then the functor G projectively $\langle F_1, F_2 \rangle$ -generated by G' is the forgetful functor (the left adjoint of the embedding $\mathcal{K}_2$ onto totally bounded uniform spaces).
As a consequence of Theorem 5 for the restriction of G to the proximally fine uniformities we shall get the coreflectivity in $\mathcal{K}_2$ of the proximities having proximally fine uniformity (this result (see also [7]) can be more easily proved directly or as a consequence of Theorem 6 in [4]).

Example 4. Now, we describe a method which serves to characterizations of continuity structures. (A similar method was used by H. Kowalsky in [6] to a characterization of the category of topological T_1 -spaces.)

E.G. we want to find conditions for a category $\mathcal{K}$ to be equivalent to Top. First, there must exist a faithful functor F from $\mathcal{K}$ into $\mathcal{E}ns$ and an object D of $\mathcal{K}$ such that FD is a two-point set and $\mathcal{K} = \langle F, \mathcal{E}ns \rangle - proj\ (D)$. Now it is sufficient to construct the functor G projectively $\langle F, F_{Top} \rangle$ -generated by G' (G' assigns to D a connected T_0 -topology on FD) and to find conditions under which G is full and onto a representative subcategory of Top. Using Theorem 3 we shall get the following proposition:

A category $\mathcal{K}$ is equivalent with Top if and only if there exists an object D in $\mathcal{K}$ such that:

1) $Hom_{\mathcal{K}}\langle D, D\rangle = (\alpha, \beta, 1_D)$, where $\alpha \circ \beta = \alpha,\ \beta \circ \alpha = \beta$.

2) If $h_i \in Hom_{\mathcal{K}}\langle X, Y\rangle,\ h_1 \neq h_2$ then there is an $f \in Hom_{\mathcal{K}}\langle D, X\rangle$ with the properties $f \circ \alpha = f \circ \beta = f$, $h_1 \circ f \neq h_2 \circ f$ (let us denote by $c_{\mathcal{K}}\langle D, X\rangle$ the set $\mathcal{E}\{f \mid f \in Hom_{\mathcal{K}}\langle D, X\rangle,\ f \circ \alpha = f \circ \beta = f\}$).

3) Assume we have given mappings $\psi : Hom_{\mathcal{K}}\langle X, D\rangle \to Hom_{\mathcal{K}}\langle Y, D\rangle$, $\varphi : c_{\mathcal{K}}\langle D, Y\rangle \to c_{\mathcal{K}}\langle D, X\rangle$ such that $f \circ \varphi h = \psi f \circ h$ for each h, f. Then $\varphi = \{h \to g \circ h\}$ for some $g : Y \to X$.

4) Let φ be a mapping $c_{\mathcal{K}}\langle D, X\rangle \to (\alpha, \beta)$ which can be described by means of a family $\{g_j^i \mid i \in I, j \in J_i$, J_i are finite$\}$ from $Hom_{\mathcal{K}}\langle X, D\rangle$ in this way: $\varphi g = \alpha$ if and only if $g_j^i \circ g = \alpha$ for some $i \in I$ and each $j \in J_i$. Then $\varphi = \{g \to h \circ g\}$ for some $h : X \to D$.

5) Let S be a set and Φ be a subset of

$Hom_{Ens}\langle S,(\alpha,\beta)\rangle$ satisfying the condition (4) with S and Φ instead of $c_{\mathcal{K}}\langle D, X\rangle$ and $Hom_{\mathcal{K}}\langle X, D\rangle$. Then there is an $X \in obj\,\mathcal{K}$ and a bijective mapping φ : $: S \to c_{\mathcal{K}}\langle D, X\rangle$ such that the mapping $\{f \to \{x \to f\circ\varphi x\}\}:$ $: Hom_{\mathcal{K}}\langle X, D\rangle \to \Phi$ is bijective, too.

One obtains similar propositions for closure and proximity spaces using three-point space. It is also possible to give external characterizations as the greatest categories having certain properties etc. A characterization of the category of topological spaces by means of a two-point space was recently found also by D. Schlomiuk (see [8]), but I do not know any further details of that characterization.

Example 5. Let $\mathcal{K}_1$ be the category of those proximity spaces satisfying the implication $\overline{X}\,\not\!p\,\overline{Y} \Rightarrow X\,\not\!p\,Y$, and F_1 be the forgetful functor of $\mathcal{K}_1$ into Top. Let $\mathcal{K}_2$ be the category of compactifications of topological spaces, i.e. the following category:
objects of $\mathcal{K}_2$ are triples $\langle P, f, Q\rangle$ where P is a topological space and f is a homeomorphism of P onto a dense subspace of the compact topological space Q, morphisms of $\mathcal{K}_2$ are triples $\langle\varphi, \langle P, f, Q\rangle, \langle P', f', Q'\rangle\rangle$, where φ is a continuous mapping P into P' such that there is a continuous mapping $\varphi' : Q \to Q'$ making the obvious diagram commutative ($\varphi'\circ f = f'\circ\varphi$). Denote by F_2 the faithful functor $\{\langle\varphi, \langle P, f, Q\rangle, \langle P', f', Q'\rangle\rangle \to$ $\to \langle\varphi, P, P'\rangle\} : \mathcal{K}_2 \to Top$.

The Smirnov theorem, concerning the equivalence between the uniformizable proximities and the uniformizable compactifications, is of the following form ($\mathcal{K}_1''$ means the

category of uniformizable proximities):
There exists a unique full embedding $G : \langle \mathcal{K}_1^u, F_{1\mathcal{K}_1^u} \rangle \longrightarrow$
$\longrightarrow \langle \mathcal{K}_2, F_2 \rangle$ such that $G[\mathcal{K}_1^u]$ is a representative subcategory of uniformizable compactifications. (The proof follows also from our Theorem if we take the closed unit interval for $\mathcal{K}_1'$.)
It suggests generalizations of Smirnov theorem for other proximity spaces and other categories $\mathcal{K}_1'$. We shall mention here only one such a generalization. The details and other generalizations will appear elsewhere.

Let $\mathcal{K}_1' = \mathcal{E}\{\langle P, p_P \rangle \mid P$ is a set$\}$, where p_P is the only proximity from $\mathcal{K}_1$ inducing the coarsest T_1-topology u_P on P (p_P is the Wallman proximity of u_P) and G' be the functor $\{P \longrightarrow \langle P, 1_P, P \rangle\}$. Then the functor G projectively $\langle F_1, F_2 \rangle$-generated by G' is a full embedding of $\langle \mathcal{K}_1, F_1 \rangle$ into $\langle \mathcal{K}_2, F_2 \rangle$. Each $P \in obj\, \mathcal{K}_1$ is a dense subspace of the third member of GP with the Wallman proximity. $G[\mathcal{K}_1]$ is coreflective in $\langle F_2, Top \rangle$-proj $G[\mathcal{K}_1']$ (this last class is a trivial projective extension of $G[\mathcal{K}_1']$-compact spaces in the sense of [3]) and, hence, GP has obvious extension properties for each P.

The advantage of this method is in the fact that we shall get functor (i.e. continuous extensions of some mappings), which is impossible in some similar embeddings concerning only spaces and not mappings (e.g. in [5]).

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