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UNIVERSAL CATEGORY WITH LIMITS OF FINITE DIAGRAMS

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The main result of the present note is that there exists a category $\mathfrak{U}$ with limits of finite diagrams and such that every category with limits of finite diagrams may be fully embedded in $\mathfrak{U}$; the embedding preserves these limits.

I. Preliminaries. The present note is written in the Bernays-Gödel set-theory with the axiom of choice for classes, [1]. The same notation and conventions as in [4] are used; knowledge of [4] is assumed.

II. Let V be a normal property x) of categories, W a normal property x) of embeddings. The following metadefinitions are analogous to those given in [4] for properties of $\mathcal{b}$-categories and $\mathcal{b}$-embeddings xx):

Metadefinitions: We shall say that W is categorial if
a) every isofunctor onto has W;
b) the composition of two functors with W has W;
We shall say that W is monotonically additive if every union of monotone systems of embeddings with W has W.

Let $\bar{\mathfrak{k}}$ be a small category. We shall say that V is $\bar{\mathfrak{k}}$ -

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x) i.e. given by a normal formula in the sense of [1]

xx) The metatheorem for $\mathcal{b}$-categories given in [4] must be corrected: W has to be monotonically additive.

invariant if a) every category with V contains $\bar{\mathfrak{k}}$ as a full subcategory, b) if a category $\mathfrak{k}$ has V, φ is an isofunctor of $\mathfrak{k}$ onto $\mathfrak{h}$ which is identical on $\bar{\mathfrak{k}}$, then $\mathfrak{h}$ has V.

We shall say that V is amalgamic with respect to W if every amalgam $\langle \ell, \mathcal{K} \rangle$ with V such that the inclusion functor $\iota_{\mathfrak{k}} : \ell \longrightarrow \mathfrak{k}$ has W for every $\mathfrak{k} \in \mathcal{K}$ has a filling K with V such that for every $\mathfrak{k} \in \mathcal{K}$ the inclusion functor $\iota^*_{\mathfrak{k}} : \mathfrak{k} \longrightarrow K$ has W.

We shall say that V has small W-character if a category K has V if and only if K is the union of a monotone system $\{\mathfrak{k}_\alpha ; \alpha \in T\}$ of small categories with V such that for any $\alpha < \alpha'$ the inclusion functor $\iota_\alpha^{\alpha'} : \mathfrak{k}_\alpha \rightarrow \mathfrak{k}_{\alpha'}$ has W.

III. Metatheorem: Let W be a categorial monotonically additive property of embeddings. Let $\bar{\mathfrak{k}}$ be a small category. Let V be a $\bar{\mathfrak{k}}$-invariant property amalgamic with respect to W and of small W-character. Then there exists a category $\mathcal{U}$ with V such that every category with V may be fully embedded in $\mathcal{U}$. The embedding has W.

Proof if quite analogous to that given in [4] and therefore it is omitted.

IV. Definition. Let S_d (or S_i) be a class of small non-empty categories. We shall say that a category K is $\overrightarrow{S_d}$-complete (or $\overleftarrow{S_i}$-complete) if every diagram in K with schema from S_d (or S_i) has a direct limit (or an inverse limit, respectively) in K. A category which is $\overrightarrow{S_d}$-complete and $\overleftarrow{S_i}$-complete will be called $(\overrightarrow{S_d}, \overleftarrow{S_i})$-complete.

A functor which preserves direct (or universe) limits of all diagrams with schema from S_d (or S_i) will be called $\overrightarrow{S_d}$-preserving (or $\overleftarrow{S_i}$-preserving, respectively). A functor which is $\overrightarrow{S_d}$-preserving and $\overleftarrow{S_i}$-preserving will be called $(\overrightarrow{S_d}, \overleftarrow{S_i})$-preserving.

Definition. Let S be a class of finite non-empty categories containing all h for which $card\ H_h(a,b) \leq 1$ for all $a, b \in h^{\sigma}$. Then S will be called suitable.

V. Let S_d, S_i be suitable classes; we shall consider the following properties:

V_1 is the property of being $\overleftarrow{S_i}$-complete;

W_1 is the property of being $\overleftarrow{S_i}$-preserving;

V_2 is the property of being $(\overrightarrow{S_d}, \overleftarrow{S_i})$-complete;

W_2 is the property of being $(\overrightarrow{S_d}, \overleftarrow{S_i})$-preserving.

We shal verify that the properties V_1, W_1 and V_2, W_2 satisfy the assumptions of the metatheorem, and thus obtain the following results:

There exists an $\overleftarrow{S_i}$-complete category in which every $\overleftarrow{S_i}$-complete category may be fully embedded; the embedding is $\overleftarrow{S_i}$-preserving.

There exists a $(\overrightarrow{S_d}, \overleftarrow{S_i})$-complete category in which every $(\overrightarrow{S_d}, \overleftarrow{S_i})$-complete category may be fully embedded; the embedding is $(\overrightarrow{S_d}, \overleftarrow{S_i})$-preserving.

VI. It isevident that W_1 and W_2 are categorial and monotonically additive. It is also evident that V_1 and V_2 are $\cdot\emptyset$-invariant, where $\emptyset$ denotes the empty category.

VII. Now we prove that V_n has small W_n-character ($n = 1, 2$):

a) Let $\{\mathscr{k}_\alpha ; \alpha \in T\}$ be a monotone system of small categories (T is On-ordered by $\prec$) with V_n such that if $\alpha \prec \alpha'$ then the inclusion functor $\iota_\alpha^{\alpha'} : \mathscr{k}_\alpha \to \mathscr{k}_{\alpha'}$ has W_n. Then evidently $\bigcup_{\alpha \in T} \mathscr{k}_\alpha$ has V_n.

b) Let K be a category with V_n. For every small full subcategory $\mathscr{h}$ of K choose some full small subcategory $\tilde{\mathscr{h}}$ of K with V_n such that $\mathscr{h}$ is a full subcategory of $\tilde{\mathscr{h}}$ and the inclusion functor $\iota_{\tilde{\mathscr{h}}} : \tilde{\mathscr{h}} \to K$ has W_n (this is indeed possible). Let $\prec$ be an On-order for the class K^σ; for every $a \in K^\sigma$ denote by $\mathscr{h}_a$ the full subcategory of K such that $\mathscr{h}_a^\sigma = \{ b \in K^\sigma ; b \prec a \}$. Let ℓ_a be a full subcategory of K such that $\ell_a^\sigma = \mathscr{h}_a^\sigma \cup \bigcup_{b \prec a} \mathscr{k}_b^\sigma$, $\mathscr{k}_a = \tilde{\ell}_a$. Then evidently the monotone system $\{\mathscr{k}_\alpha ; a \in K^\sigma\}$ has the required properties.

VIII. Now we prove that V_1 is amalgamic with respect to W_1.

<u>Convention</u>: Let $\langle \ell , \mathscr{K} \rangle$ be an amalgam. Then there exists its filling K with the following property: if H is a category, $\varphi_{\mathscr{k}} : \mathscr{k} \to H$ functors ($\mathscr{k} \in \mathscr{K}$) such that $\varphi_{\mathscr{k}}/_\ell = \varphi_{\mathscr{k}'}/_\ell$ for every $\mathscr{k}, \mathscr{k}' \in \mathscr{K}$, then there exists exactly one functor $\varphi : K \to H$ such that $\varphi_{\mathscr{k}} = \iota_{\mathscr{k}}^* \cdot \varphi$, where $\iota_{\mathscr{k}}^* : \mathscr{k} \to K$ is the inclusion functor. The filling K will be called the sum of the amalgam $\langle \ell , \mathscr{K} \rangle$.

<u>Lemma 1</u>: Let $\langle \ell , \mathscr{K} \rangle$ be an amalgam, K its sum. Let $\mathscr{k}, \mathscr{k}' \in \mathscr{K}$, $\mathscr{k} \neq \mathscr{k}'$, $\gamma, \gamma' \in \mathscr{k}^m$, $\sigma, \sigma' \in \mathscr{k}'^m$, $\sigma \cdot \gamma = \sigma' \cdot \gamma'$ in K. Then there exist objects $x_1, \dots, x_n$ of ℓ, morphisms $\gamma_1, \dots, \gamma_n$ of $\mathscr{k}$, morphisms $\sigma_1, \dots, \sigma_n$ of $\mathscr{k}'$ and morphisms $\rho_1, \dots, \rho_{n-1}$ of ℓ such that:

1) n is odd, $n \geq 3$, $\varphi_1 \in H_{\ell}(x_1, x_2)$, $\varphi_2 \in H_{\ell}(x_3, x_2), \ldots$
$\ldots, \varphi_{n-1} \in H_{\ell}(x_n, x_{n-1})$;

2) $\delta_1 = \delta$, $\gamma_1 = \gamma$, $\delta_n = \delta'$, $\gamma_n = \gamma'$;

3) $\delta_s \in H_{k'}(\overleftarrow{\sigma}, x_s)$, $\gamma_s \in H_{k}(x_s, \overrightarrow{\gamma})$, $s = 1, 2, \ldots, n$;

4) $\delta_1 \cdot \varphi_1 = \delta_2$; $\delta_3 \cdot \varphi_2 = \delta_2, \ldots, \delta_n \cdot \varphi_{n-1} = \delta_{n-1}$ in k';
$\gamma_1 = \varphi_1 \cdot \gamma_2$, $\gamma_3 = \varphi_2 \cdot \gamma_2, \ldots, \gamma_{n-1} = \varphi_{n-1} \cdot \gamma_n$ in k.

Proof: This follows immediately from the construction of K ,[2].

Note: In the following lemmas 2 and 3 and their proofs the notation from [3] will be used.

Lemma 2: Let $\langle \ell, \mathcal{K} \rangle$ be an amalgam with V_1, and K its sum. Let the inclusion functor $\iota_k : \ell \to k$ have W_1 for every $k \in \mathcal{K}$. Then for every $k \in \mathcal{K}$ the inclusion functor $\iota_k^* : k \to K$ has W_1.

Proof: Let $\mathcal{F} : \mathcal{J} \to k$ be a diagram in $k \in \mathcal{K}$, $\mathcal{J} \in S_i$, let $\langle p, \{\pi_i \ ; \ i \in \mathcal{J}^{\sigma}\}\rangle$, be its inverse limit. We shall prove that it is also an inverse limit of $\mathcal{F}\iota_k^*$.

A) Let $\langle b ; \{\psi_i \ ; \ i \in \mathcal{J}^{\sigma}\}\rangle$ be an inverse bound of $\mathcal{F}\iota_k^*$. We show that there exists an $f \in H_K(b, p)$ such that $f \cdot \pi_i = \psi_i$ for all $i \in \mathcal{J}^{\sigma}$. If $b \in k^{\sigma}$, then this is evident. Let $k' \in \mathcal{K}$, $k' \neq k$, $b \in k'^{\sigma} - k^{\sigma}$. Then there exists $\beta_i \in k'^m$, $\alpha_i \in k^m$ such that $\psi_i = \beta_i \cdot \alpha_i$. For every $\sigma \in H_{\mathcal{J}}(i, i')$ set $\overline{\sigma} = (\sigma)\mathcal{F}$; there is $\beta_i \cdot (\alpha_i \cdot \overline{\sigma}) = \beta_{i'} \cdot \alpha_{i'}$; consequently we may choose objects $x_1^{\sigma}, \ldots, x_{n_\sigma}^{\sigma}$ of ℓ , morphisms $\gamma_1^{\sigma}, \ldots, \gamma_{n_\sigma}^{\sigma}$ of k , morphisms $\delta_1^{\sigma}, \ldots$
$\ldots, \delta_{n_\sigma}^{\sigma}$ of k' and morphisms $\varphi_1^{\sigma}, \ldots, \varphi_{n_\sigma - 1}^{\sigma}$ of ℓ such that the statements 1) – 4) from lemma 1 are satisfied, where we replace δ by β_i , γ by $\alpha_i \cdot \overline{\sigma}$, δ' by $\beta_{i'}$, γ' by $\alpha_{i'}$ and supply the corresponding symbols with an index σ .

Let $\mathcal{J}$ be the following category: $\mathcal{J}^{\sigma} = A \cup B$, where $A = \{\langle \overleftarrow{\alpha}_i, i \rangle ;\ i \in \mathcal{I}^{\sigma}\}$, $B = \{\langle x^{\sigma}_{s}, \sigma, s\rangle ; \sigma \in \mathcal{I}^{m}, 1 < s < n_{\sigma}\}$ (we will assume $A \cup B = \emptyset$); for $\sigma \in H_{\mathcal{I}}(i, i')$ put
$H_{\mathcal{J}}(\langle \overleftarrow{\alpha}_i, i\rangle, \langle x^{\sigma}_2, \sigma, 2\rangle) = \{\langle\langle \overleftarrow{\alpha}_i, i\rangle, \varphi^{\sigma}_1, \langle x^{\sigma}_2, \sigma, 2\rangle\rangle\}$,
$H_{\mathcal{J}}(\langle x^{\sigma}_3, \sigma, 3\rangle, \langle x^{\sigma}_2, \sigma, 2\rangle) = \{\langle\langle x^{\sigma}_3, \sigma, 3\rangle, \varphi^{\sigma}_2, \langle x^{\sigma}_2, \sigma, 2\rangle\rangle\}, \dots,$
$H_{\mathcal{J}}(\langle \overleftarrow{\alpha}_{i'}, i'\rangle, \langle x^{\sigma}_{n_\sigma - 1}, \sigma, n_{\sigma} - 1\rangle) = \{\langle\langle \overleftarrow{\alpha}_{i'}, i'\rangle, \varphi^{\sigma}_{n_\sigma - 1}, \langle x^{\sigma}_{n_\sigma-1}, \sigma, n_{\sigma} - 1\rangle\rangle\}$
(whenever $n_{\sigma} > 3$; the modification of the description for $n_{\sigma} = 3$ is evident); moreover, for every $j \in \mathcal{J}^{\sigma}$ put $H_{\mathcal{J}}(j, j) = \{e_j\}$, where e_j is the identity; in the remaining cases put $H_{\mathcal{J}}(j, j') = \emptyset$; the composition in $\mathcal{J}$ need not be described because every morphism composes with identity only. Evidently $\mathcal{J} \in S_i$. Let $\mathcal{Y} : \mathcal{J} \to \ell$ be the following diagram: let $j \in \mathcal{J}^{\sigma}$; if $j = \langle \overleftarrow{\alpha}_i, i\rangle$, put $(j)\mathcal{Y} = \overleftarrow{\alpha}_i$; if $j = \langle x^{\sigma}_{s}, \sigma, s\rangle$, put $(j)\mathcal{Y} = x^{\sigma}_{s}$; for $\lambda \in \mathcal{J}^{m}$ either $\lambda = e_j$ and then put $(\lambda)\mathcal{Y} = e_{(j)\mathcal{Y}}$, or λ is a triple and then put $(\lambda)\mathcal{Y} = \varphi$, where φ is the middle member of λ . Let $\langle q ; \{\varkappa_j ; j \in \mathcal{J}^{\sigma}\}\rangle$ be an iverse limit of $\mathcal{Y}$. If $j = \langle \overleftarrow{\alpha}_i, i\rangle \in \mathcal{J}^{\sigma}$, set $\vartheta_j = \beta_i$; if $j = \langle x^{\sigma}_{s}, \sigma, s\rangle \in \mathcal{J}^{\sigma}$, set $\vartheta_j = \sigma^{\sigma}_{s}$. Then evidently $\langle b ; \{\vartheta_j ; j \in \mathcal{J}^{\sigma}\}\rangle$ is an inverse bound of $\mathcal{Y}\iota_{k'}$. Since $\iota_{k'}$ is $\overleftarrow{S}_i$ -preserving, there exists an $\xi \in H_{k'}(b, q)$ such that $\xi \cdot \varkappa_j = \vartheta_j$ for all $j \in \mathcal{J}^{\sigma}$. It is easily proved that $\langle q ; \{\varkappa_{\langle \overleftarrow{\alpha}_i, i\rangle} \cdot \alpha_i ; i \in \mathcal{I}^{\sigma}\}\rangle$ is an inverse bound of $\mathcal{F}$, so that there exists an $\zeta \in H_{k}(q, p)$ with $\zeta \cdot \pi_i = \varkappa_{\langle \overleftarrow{\alpha}, i\rangle} \cdot \alpha_i$. Then, of course, $(\xi \cdot \zeta) \cdot \pi_i = \xi \cdot \varkappa_{\langle \overleftarrow{\alpha}, i\rangle} \cdot \alpha_i = \vartheta_{\langle \overleftarrow{\alpha}, i\rangle} \cdot \alpha_i = \beta_i \cdot \alpha_i$.

B) Let ε and ε' be morphisms of K such that $\varepsilon \cdot \pi_i = \varepsilon' \cdot \pi_i$ for all $i \in \mathcal{I}^{\sigma}$. We must prove that $\varepsilon = \varepsilon'$. If $\overleftarrow{\varepsilon} \in k^{\sigma}$, then this is evident. Let $k' \in \mathcal{K} - k + k', b = \overleftarrow{\varepsilon} \in k'^{\sigma} - k^{\sigma}$.

Then morphisms $\beta, \beta' \in k'^m$, $\alpha, \alpha' \in k^m$ may be chosen such that $\varepsilon = \beta \cdot \alpha$, $\varepsilon' = \beta' \cdot \alpha'$. Since $\beta \cdot \alpha \cdot \pi_i = \beta' \cdot \alpha' \cdot \pi_i$, we may choose objects $x_1^i, \dots, x_{n_i}^i$ of ℓ, morphisms $\gamma_1^i, \dots, \gamma_{n_i}^i$ of k, morphisms $\sigma_1^i, \dots, \sigma_{n_i}^i$ of k' and morphisms $\varphi_1^i, \dots, \varphi_{n_i-1}^i$ of ℓ satisfying the statements 1) - 4) from lemma 1, where we replace α by $\alpha \cdot \pi_i$, α' by $\alpha' \cdot \pi_i$ and supply the corresponding symbols with an index i. Let $\mathcal{J}$ be the following category: $\mathcal{J}^\sigma = A \cup B$, where $A = \{\langle \mathfrak{x}, 1 \rangle, \langle \mathfrak{x}', 2 \rangle\}$, $B = \{\langle x_s^i, i, s \rangle ; i \in \mathcal{J}^\sigma, 1 < s < n_i\}$ (we will assume that $A \cup B = \emptyset$); for every $i \in \mathcal{J}^\sigma$ put $H_{\mathcal{J}}(\langle \mathfrak{x}, 1 \rangle, \langle x_2^i, i, 2 \rangle) = \{\langle\langle \mathfrak{x}, 1 \rangle, \varphi_1^i, \langle x_2^i, i, 2 \rangle\rangle\}, \dots,$ $H_{\mathcal{J}}(\langle \mathfrak{x}', 2 \rangle, \langle x_{n_i-1}^i, i, n_i - 1 \rangle) = \{\langle\langle \mathfrak{x}', 2 \rangle, \varphi_{n_i-1}^i, \langle x_{n_i-1}^i, i, n_i - 1 \rangle\rangle\}$; moreover put $H_{\mathcal{J}}(j, j) = \{e_j\}$ for every $j \in \mathcal{J}^\sigma$; in the remaining cases put $H_{\mathcal{J}}(j, j') = \emptyset$. Let $\mathcal{Y} : \mathcal{J} \to \ell$ be the following diagram:

$(\langle \mathfrak{x}, 1 \rangle)\mathcal{Y} = \mathfrak{x}$, $(\langle \mathfrak{x}', 2 \rangle)\mathcal{Y} = \mathfrak{x}'$, $(\langle x_s^i, i, s \rangle)\mathcal{Y} = x_s^i$, $(e_j)\mathcal{Y} = e_{(j)\mathcal{Y}}$; if $\lambda \in \mathcal{J}^m$ is a triple, put $(\lambda)\mathcal{Y} = \rho$, where ρ is the middle member of λ. Let $\langle \mathfrak{q}; \{\varkappa_j ; j \in \mathcal{J}^\sigma\} \rangle$ be an inverse limit of $\mathcal{Y}$; set $\varkappa_{\langle \mathfrak{x}, 1 \rangle} = \varkappa$, $\varkappa_{\langle \mathfrak{x}', 2 \rangle} = \varkappa'$. If $j = \langle \mathfrak{x}, 1 \rangle$, put $\vartheta_j = \beta$; if $j = \langle \mathfrak{x}', 2 \rangle$, put $\vartheta_j = \beta'$; if $j = \langle x_s^i, i, s \rangle \in \mathcal{J}^\sigma$, put $\vartheta_j = \sigma_s^i$. Then evidently $\langle \mathfrak{b}, \{\vartheta_j ; j \in \mathcal{J}^\sigma\} \rangle$ is an inverse bound of $\mathcal{Y}\iota_k$, and therefore there exists an $\xi \in H_{k'}(\mathfrak{b}, \mathfrak{q})$ such that $\xi \cdot \varkappa_j = \vartheta_j$ for all $j \in \mathcal{J}^\sigma$, namely $\xi \cdot \varkappa = \beta$, $\xi \cdot \varkappa' = \beta'$. Evidently $\varkappa \cdot \gamma_1^i = \varkappa' \cdot \gamma_{n_i}^i$, i.e. $\varkappa \cdot \alpha \cdot \pi_i = \varkappa' \cdot \alpha' \cdot \pi_i$ for all i, consequently $\varkappa \cdot \alpha = \varkappa' \cdot \alpha'$. Then $\beta \cdot \alpha = \xi \cdot \varkappa \cdot \alpha = \xi \cdot \varkappa' \cdot \alpha' = \beta' \cdot \alpha'$.

Lemma 3: The property V_1 is amalgamic with respect to W_1.

Proof: Let $\langle \mathcal{L}, \mathcal{K} \rangle$ be an amalgam with V_1 such that for every $\mathcal{k} \in \mathcal{K}$ the inclusion functor $\iota_{\mathcal{k}} : \mathcal{L} \to \mathcal{k}$ has W_1. Let K be the sum of $\langle \mathcal{L}, \mathcal{K} \rangle$. Then for every $\mathcal{k} \in \mathcal{K}$ the inclusion functor $\iota^*_{\mathcal{k}} : \mathcal{k} \to K$ has W_1. Using Theorem II.3 from [3], K may be fully embedded into a small category $\overline{K}$ with V_1 such that the inclusion functor $\iota : K \to \overline{K}$ has W_1. Then $\overline{K}$ is the filling of $\langle \mathcal{L}, \mathcal{K} \rangle$ with the required properties.

IX. Now prove that V_2 is amalgamic with respect to W_2. Let $\langle \mathcal{L}, \mathcal{K} \rangle$ be an amalgam with V_2 such that, for every $\mathcal{k} \in \mathcal{K}$, the inclusion functor $\iota_{\mathcal{k}} : \mathcal{L} \to \mathcal{k}$ has W_2. Let K be the sum of $\langle \mathcal{L}, \mathcal{K} \rangle$. Using VIII Lemma 2 and its dual, it is easy to see that for every $\mathcal{k} \in \mathcal{K}$ the inclusion functor $\iota^*_{\mathcal{k}} : \mathcal{k} \to K$ has W_2. Then use theorem II.5 from [3].

X. Note: One may combine the properties V_1, W_1 and their duals and V_2, W_2 with other properties, for example with the property of being connected x) or of having a singleton. Thus for example the following results are also true (cf. also the proofs of Theorems II.3 and II.5 from [3]): Let S_d, S_i be suitable classes of categories. There exists a connected $\overleftarrow{S_i}$ -complete category in which every connected $\overleftarrow{S_i}$ -complete category may be fully embedded; the embedding is $\overleftarrow{S_i}$ -preserving xx).

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x) A category $\mathcal{k}$ is called connected if for every $a, b \in \mathcal{k}^{\sigma}$ there exist $c_1, \dots, c_n \in \mathcal{k}^{\sigma}$ such that $c_1 = a$, $c_n = b$, $H_{\mathcal{k}}(c_i, c_{i+1}) \cup H_{\mathcal{k}}(c_{i+1}, c_i) \neq \emptyset$ for $i = 1, 2, \dots, n-1$.

There exists a $\overrightarrow{S_d}$ -complete category with a singleton in which every $\overrightarrow{S_d}$ -complete category with a singleton may be fully embedded; the embedding preserves singletons and is $\overrightarrow{S_d}$ -preserving.

There exists an $(\overrightarrow{S_d}, \overleftarrow{S_i})$ -complete category with a system of null morphisms in which every $(\overrightarrow{S_d}, \overleftarrow{S_i})$ -complete category with a system of null morphisms may be fully embedded; the embedding preserves null morphisms and is $(\overrightarrow{S_d}, \overleftarrow{S_i})$-preserving.

XI. Note: The author was adviced by Z. Hedrlín that some results given in [4] remain true if we replace the word "set" by "finite set" and the word "class" by "countable set" (also in the definition of categories). The results of the present note depend not only on results and ideas of 4 but also on the embedding theorems of [3].

Nevertheless, the following proposition holds:

There exists a countable semi-lattice such that every countable semi-lattice is isomorphic with its sub-semi-lattice.

References

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xx) Evidently every category may be fully embedded in an connected category. Thus the result concerning connected categories may be obtained more simply than in [4].

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