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AN OPTIMAL CONTROL PROBLEM FOR A PSEUDOPARABOLIC VARIATIONAL INEQUALITY

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Summary. We deal with an optimal control problem governed by a pseudoparabolic variational inequality with controls in coefficients and in convex sets of admissible states. The existence theorem for an optimal control parameter will be proved. We apply the theory to the optimal design problem for a deflection of a viscoelastic plate with an obstacle, where the variable thickness of the plate appears as a control variable.

Keywords: Optimal control, pseudoparabolic variational inequality, convex set, penalization, viscoelastic plate, thickness, obstacle.

AMS classification: 49A29, 49A34, 73F15

1. EXISTENCE AND UNIQUENESS THEOREM FOR A PSEUDOPARABOLIC VARIATIONAL INEQUALITY

This chapter is devoted to the solution of an initial value problem for a pseudoparabolic variational inequality. Such problems are solved in the papers [9] and [12], where the operators on the left-hand side do not depend on the time variable. We will use the method of penalization in a similar way as in [13], where a parabolic problem is considered.

1. Basic Assumptions.

We describe some function spaces. More details can be found in the books [2] or [6] (Appendices).

If $T > 0$, X is a reflexive Banach space with a norm $\|\cdot\|_X$, then we denote by $C([0,T];X)$ the space of all continuous functions $f\colon [0,T] \to X$. Further, $C^k([0,T];X)$ denotes the space of all k-times continuously differentiable functions $f\colon [0,T] \to X$. We proceed to some spaces of integrable vector-valued functions.

If $1 \leqslant p \leqslant \infty$, we denote by $L^p(0,T;X)$ the space of all measurable functions $f\colon [0,T] \to X$ such that $\|f(\cdot)\|_X \in L^p(0,T)$. $L^p(0,T;X)$ is a Banach space with the norm

$$\|f\|_{L^p} = \Big(\int_0^T \|f(t)\|_X^p \, dt \Big)^{\frac{1}{p}} \quad \text{if } 1 \leqslant p < \infty$$

and

$$\|f\|_{L^\infty} = \sup_{t\in[0,T]} \operatorname{ess} \|f(t)\|_X .$$

The spaces $L^p(0,T;X)$, $1 < p < \infty$; are reflexive and the dual space $[L^p(0,T;X)]'$ can be identified with the space $L^q(0,T;X')$, $\frac{1}{p} + \frac{1}{q} = 1$. The space $L^\infty(0,T;X')$ can be identified with the dual space $[L^1(0,T;X)]'$, i.e. for every $F \in [L^1(0,T;X)]'$ there exists a unique function $f \in L^\infty(0,T;X')$ satisfying the relations $\|F\|_* = \|f\|_{L^\infty}$ and

$$F(y) = \int_0^T \langle f(t), y(t) \rangle \, dt \quad \text{for every } y \in L^1(0,T;X).$$

If X is a Hilbert space with an inner product $(\cdot,\cdot)$, then $L^2(0,T;X)$ is a Hilbert space with the inner product

$$(f,g)_{L^2} = \int_0^T (f(t), g(t)) \, dt, \quad f,g \in L^2(0,T;X).$$

Further, we introduce the Sobolev spaces of vector-valued functions. We denote by $W^{m,p}(0,T;X)$ the space of all functions $f \in L^p(0,T;X)$, $m \geqslant 1$, $1 \leqslant p \leqslant \infty$ such that there exist functions $g_i \in L^p(0,T;X)$, $i = 1, \ldots, m$ satisfying the relations

$$\int_0^T \frac{d^i\varphi}{dt^i}(t) f(t) \, dt = (-1)^i \int_0^T \varphi(t) g_i(t) \, dt \quad \text{for every } \varphi \in C_0^\infty(0,T).$$

Functions g_i are generalized derivatives of the i-th order and we set $g_i = \dfrac{d^i f}{dt^i}$, $i = 1, \ldots, m$. $W^{m,p}(0,T;X)$ is a Banach space with the norm

$$\|f\|_{W^{m,p}} = \left(\|f\|_{L^p}^p + \left\| \frac{df}{dt} \right\|_{L^p}^p + \ldots + \left\| \frac{d^m f}{dt^m} \right\|_{L^p}^p \right)^{\frac{1}{p}}, \quad 1 \leqslant p < \infty$$

and

$$\|f\|_{W^{m,\infty}} = \|f\|_{L^\infty} + \left\| \frac{df}{dt} \right\|_{L^\infty} + \ldots + \left\| \frac{d^m f}{dt^m} \right\|_{L^\infty} .$$

If X is a Hilbert space then $W^{m,2}(0,T;X)$ is a Hilbert space with the inner product

$$(f,g)_{W^{m,2}} = (f,g)_{L^2} + \left(\frac{df}{dt}, \frac{dg}{dt} \right)_{L^2} + \ldots + \left(\frac{d^m f}{dt^m}, \frac{d^m g}{dt^m} \right)_{L^2} .$$

Due to Proposition A.6 from [6], every function $f \in W^{1,p}(0,T;X)$ is equal a.e. to an absoutely continuous function $\bar{f}$ of the form

$$\bar{f}(t) = \bar{f}(0) + \int_0^t \frac{df}{ds}\, ds \quad \text{for all } t \in [0,T].$$

Conversely, every function f of the form

$$f(t) = F(0) + \int_0^t g(s)\, ds, \quad t \in [0,T],\ g \in L^p(0,T;X);$$

belongs to the space $W^{m,p}(0,T;X)$ and $g = \dfrac{df}{dt}$..

Let V be a Hilbert space with an inner product $(\cdot,\cdot)$ and a norm $\|\cdot\|$, V' its dual space with the duality pairing $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|_*$, $L(V,V')$ the space of all linear bounded operators from V into V' with the norm $\|\cdot\|_L$.

We consider the initial value problem

$$u(t) \in K \quad \text{for a.e. } t \in [0,T], \tag{1.1}$$

and for a.e. $t \in [0,T]$:

$$\langle A(t)u'(t) + B(t)u(t), v - u(t)\rangle \geqslant \langle f(t), v - u(t)\rangle \quad \text{for all } v \in K, \tag{1.2}$$

$$u(0) = u_0 \in K, \tag{1.3}$$

where K is a closed convex subset of V, $f \in W^{1,2}(0,T;V') \cap C([0,T],V')$ and the operator functions $A(\cdot)$, $B(\cdot)\colon [0,T] \to L(V,V')$ satisfy the assumptions

$$A(\cdot), B(\cdot) \in C^1([0,T], L(V,V')), \tag{1.4}$$

$$\|u\|^2 \leqslant c_1 \langle A(t)u, u\rangle, c_1 > 0, \tag{1.5}$$

$$\langle A(t)u, v\rangle = \langle A(t)v, u\rangle, \langle B(t)u, v\rangle = \langle B(t)v, u\rangle, \tag{1.6}$$

$$\langle [2B(t) - A'(t)]u, u\rangle \geqslant 0 \tag{1.7}$$

for all u, $v \in V$ and $t \in [0,T]$.

2. The Penalized Problem.

We deal with a penalized pseudoparabolic initial value problem corresponding to (1.1), (1.2), (1.3):

$$A(t)u_\varepsilon'(t) + B(t)u_\varepsilon(t) + \frac{1}{\varepsilon}\beta(u_\varepsilon(t)) = f(t), \quad \varepsilon > 0 \tag{1.8}$$

$$u_\varepsilon(0) = u_0. \tag{1.9}$$

$\beta\colon V \to V'$ is the penalty operator defined by

$$\beta(u) = J(u - P_K(u)), \quad u \in V,$$

where

$$J : V \to V', \quad \langle Ju, v\rangle = (u, v), \quad u, v \in V$$

is the canonical isomorphism from V onto V' and

$$P_K : V \to K$$

is the projection operator defined by

$$\|u - P_K u\| = \min_{v\in K} \|u - v\|, \quad u \in V.$$

The operator P_K has the following properties arising directly from its definition ([11], I.2):

i) $P_K u = u \Leftrightarrow u \in K$,

ii) $(P_K u - u,\, v - P_K u) \geqslant 0$ for all $u \in V$, $v \in K$,

iii) $\|P_K u - P_K v\| \leqslant \|u - v\|$ for all u, $v \in V$.

The operator β then fulfils the conditions

$$\begin{aligned} &\text{i)}\ \beta(v) = 0 \Leftrightarrow v \in K,\\ &\text{ii)}\ \langle \beta(u) - \beta(v), u - v\rangle \geqslant 0,\\ &\text{iii)}\ \|\beta(u) - \beta(v)\|_* \leqslant 2\|u - v\| \end{aligned} \tag{1.10}$$

for all u, $v \in V$.

Hence, the penalty operator β is monotone and Lipschitz continuous.

Theorem 1.1. *Let $T > 0$, $\varepsilon > 0$. Then there exists a unique solution $u_\varepsilon \in C^1([0,T], V)$ of the initial value problem (1.8), (1.9), and it satisfies the estimates*

$$\|u(t) - u_0\|^2 \leqslant c_1 e^{c_1 T} \int_0^T \|f(t) - B(t)u_0\|_*^2\, dt, \tag{1.11}$$

$$\begin{aligned} \|u'(t)\|^2 \leqslant c_1 e^{c_1 T}[c_1\|f(0) - B(0)u_0\|_*^2 + 2\int_0^T \|f'(t)\|_*^2\, dt +\\ +4Tc_4^2(\|u_0\|^2 + c_1 e^{c_1 T}\int_0^T \|f(t) - B(t)u_0\|_*^2\, dt)] \end{aligned} \tag{1.12}$$

for all $\varepsilon > 0$ and $t \in [0,T]$,
where c_1 is defined in (1.5) and

$$c_2 = \sup_{t\in[0,T]} \|B(t)\|_L, \tag{1.13}$$

$$c_3 = \sup_{t\in[0,T]} \|A'(t)\|_L, \tag{1.14}$$

$$c_4 = \sup_{t\in[0,T]} \|B'(t)\|_L. \tag{1.15}$$

Proof. The initial velue problem (1.8), (1.9) can be rewritten in the form

$$u_\varepsilon'(t) + C_\varepsilon(t)u_\varepsilon(t) = F(t), \tag{1.16}$$

$$u_\varepsilon(0) = u_0 \tag{1.17}$$

with

$$C_\varepsilon(t)\colon V \to V, \quad C_\varepsilon(t) = A(t)^{-1}[B(t) + \frac{1}{\varepsilon}\beta], \tag{1.18}$$
$$F \in C([0,T], V), \quad F(t) = A(t)^{-1}f(t).$$

The assumption (1.5) implies the existence of the inverse operator $A(t)^{-1}$. Moreover, $A(\cdot)^{-1} \in C^1([0,T], L(V', V))$ and $[A(t)^{-1}]' = -A(t)^{-1}A'(t)A(t)^{-1}$. The operators $C_\varepsilon(t)$ are then uniformly Lipschitz continuous and due to [10] (Ch. V. Th. 1.1) the initial velue problem (1.17), (1.18) has a unique solution which is also a unique solution of the problem (1.8), (1.9).

It remains to verify the estimates (1.11), (1.12). Let us denote

$$w_\varepsilon = u_\varepsilon - u_o. \tag{1.19}$$

The function $w_\varepsilon \in C^1([0,T], V)$ is a solution of the initial value problem

$$A(t)w_\varepsilon'(t) + B(t)w_\varepsilon(t) + \frac{1}{\varepsilon}\beta(u_0 + w_\varepsilon(t)) = f(t) - B(t)u_0, \tag{1.20}$$

$$w_\varepsilon(0) = 0. \tag{1.21}$$

After duality pairing, using the differentiability and the symmetry of $A(t)$ we obtain the relation

$$\frac{d}{dt}\langle A(t)w_\varepsilon(t), w_\varepsilon(t)\rangle + \langle [2B(t) - A'(t)]w_\varepsilon(t), w_\varepsilon(t)\rangle + \tag{1.22}$$
$$+\frac{2}{\varepsilon}\langle \beta(u_0 + w_\varepsilon(t)), w_\varepsilon(t)\rangle = 2\langle f(t) - B(t)u_0, w_\varepsilon(t)\rangle.$$

We have $u_0 \in K$ and hence $\beta(u_0) = 0$. Then we obtain

$$\langle \beta(u_0 + w_\varepsilon(t)), w_\varepsilon(t)\rangle \geqslant 0 \tag{1.23}$$

due to the monotonicity of β.

Let us introduce a real function φ_ε defined by

$$\varphi_\varepsilon(t) = \langle A(t)w_\varepsilon(t), w_\varepsilon(t)\rangle, \quad t \in [0,T]. \tag{1.24}$$

Using the relations (1.5), (1.13), (1.14), (1.22), (1.23) we obtain the inequality

$$\varphi_\varepsilon(t)' \leqslant \|f(t) - B(t)u_0\|_*^2 + c_1\varphi_\varepsilon(t) \quad \text{for all } t \in [0,T]$$

and after integrating, taking into account (1.21), we arrive at

$$\varphi_\varepsilon(t) \leqslant \int_0^T \|f(t) - B(t)u_0\|_*^2 \, dt + c_1 \int_0^t \varphi_\varepsilon(s)\, ds, \quad t \in [0,T]. \tag{1.25}$$

Using the Gronwall-Bellman lemma ([6], Lemma A.4) we obtain the inequality

$$\varphi_\varepsilon(t) \leqslant \Big(\int_0^T \|f(t) - B(t)u_0\|_*^2 \, dt \Big) e^{c_1 t} \quad \text{for all } t \in [0,T]$$

and the estimate (1.11) follows from (1.5), (1.24).

In order to obtain (1.12) we differentiate the equation (1.20) and arrive at

$$\begin{aligned} [A(t)w_\varepsilon'(t)]' + [B(t)w_\varepsilon(t)]' + \frac{1}{\varepsilon}[\beta(u_0 + w_\varepsilon(t))]' = \\ = f'(t) - B'(t)u_0 \quad \text{for a.e. } t \in [0,T]. \end{aligned} \tag{1.26}$$

The function $\beta(u_0 + w(\cdot))\colon [0,T] \to V'$ is Lipschitz continuous due to (1.10 iii). As the space V' is reflexive the function $\beta(u_0 + w(\cdot))$ belongs to the space $W^{1,\infty}(0,T;V')$ ([6], pages 143, 145). Further, the functions $C_\varepsilon(\cdot)u_\varepsilon(\cdot)$, $F(\cdot)$ from the equation (1.16) belong to the spaces $W^{1,\infty}(0,T;V)$ and $W^{1,2}(0,T;V)$, respectively. Then $u_\varepsilon \in W^{2,2}(0,T;V)$ and from (1.19), (1.26) we obtain the equality

$$\begin{aligned} \langle A(t)u_\varepsilon''(t), u_\varepsilon'(t)\rangle + \langle [A'(t) + B(t)]u_\varepsilon'(t), u_\varepsilon'(t)\rangle + \frac{1}{\varepsilon}\langle [\beta u_\varepsilon(t)]', u_\varepsilon'(t)\rangle \\ = \langle f'(t) - B'(t)u_\varepsilon(t), u_\varepsilon'(t)\rangle \quad \text{for a.e. } t \in [0,T] \end{aligned} \tag{1.27}$$

We introduce a real functin ψ_ε defined by

$$\psi_\varepsilon(t) = \langle A(t)u_\varepsilon'(t), u_\varepsilon'(t)\rangle, \quad t \in [0,T]. \tag{1.28}$$

Then the equality (1.27) can be rewritten in form

$$\begin{aligned} \psi_\varepsilon'(t) + \langle [A'(t) + 2B(t)]u_\varepsilon'(t), u_\varepsilon'(t)\rangle + \frac{2}{\varepsilon}\langle [\beta(u_\varepsilon(t)]', u_\varepsilon'(t) \rangle \\ = 2\langle f'(t) - B'(t)u_\varepsilon(t), u_\varepsilon'(t)\rangle. \end{aligned} \tag{1.29}$$

From the monotonicity of β we obtain the inequality

$$\langle [\beta(u_\varepsilon(t)]', u_\varepsilon'(t)\rangle \geqslant 0 \quad \text{for a.e. } t \in [0,T]. \tag{1.30}$$

Using (1.5), (1.11), (1.13)–(1.15), (1.29), (1.30) we arrive at the inequality

$$\Psi_\varepsilon'(t) \leqslant 2\|f'(t)\|_*^2 + 4c_4^2\Big(\|u_0\|^2 + c_1 e^{c_1 T}\int_0^T \|f(t) - B(t)u_0\|_*^2\, dt\Big) + c_1\psi_\varepsilon(t)$$

for a.e. $t \in [0,T]$, and integrating we conclude

$$\begin{aligned}\psi_\varepsilon(t) \leqslant \langle A(0)u'_\varepsilon(0), u'_\varepsilon(0)\rangle + 2\int_0^T \|f'(t)\|_*^2\,dt+\\ + 4Tc_4^2\Big(\|u_0\|^2 + c_1 e^{c_1 T}\int_0^T \|f(t) - B(t)u_0\|_*^2\,dt\Big)+\\ + c_1\int_0^t \psi_\varepsilon(s)\,ds \quad \text{for all } t \in [0,T].\end{aligned} \tag{1.31}$$

From (1.5), (1.8), (1,9), (1.10i) we obtain the relations

$$\langle A(0)u'_\varepsilon(0), u'_\varepsilon(0)\rangle = \langle f(0) - B(0)u_0, u'_\varepsilon(0)\rangle \leqslant c_1\|f(0) - B(0)u_0\|_*^2. \tag{1.32}$$

The Gronwall-Bellman lemma implies the inequality

$$\begin{aligned}\psi_\varepsilon(t) \leqslant e^{c_1 T}\Big(c_0\|f(0) - B(0)u_0\|_*^2 + 2\int_0^T \|f'(t)\|_*^2\,dt\\ + 4Tc_4^2\Big(\|u_0\|^2 + c_0 e^{c_1 T}\int_0^T \|f(t) - B(t)u_0\|_*^2\,dt\Big)\Big)\end{aligned} \tag{1.33}$$

for all $t \in [0,T]$, and the estimate (1.12) follows directly from (1.5), (1.28), (1.33). □

3. Solution of a Pseudoparabolic Variational Inequality.

Using Theorem 1.1 we obtain existence, uniqueness and estimates for a solution of the unilateral problem (1.1), (1.2), (1.3).

Theorem 1.2. *There exists a unique solution $u \in W^{1,\infty}(0,T;V) \cap C([0,T],V)$ of the initial value problem (1.1), (1.2), (1.3) fulfilling the estimates*

$$\|u(t) - u_0\|^2 \leqslant c_1 e^{c_1 T}\int_0^T \|f(t) - B(t)u_0\|_*^2\,dt \quad \textit{for all } t \in [0,T], \tag{1.34}$$

$$\begin{aligned}\|u'(t)\|^2 \leqslant c_1 e^{c_1 T}\Big(c_0\|f(0) - B(0)u_0\|_*^2 + 2\int_0^T \|f'(t)\|_*^2\,dt\\ + 4Tc_4^2\Big(\|u_0\|^2 + c_1 e^{c_1 T}\int_0^R \|f(t) - B(t)u_0\|_*^2\,dt\Big)\Big)\end{aligned} \tag{1.35}$$

for a.e. $t \in [0,T]$, where $c_1, \ldots, c_4$ are the constants from (1.5), (1.13), (1.14), (1.15).

Proof. a) Existence. Let $\varepsilon \to 0$, $\varepsilon > 0$. Due to (1.11), (1.12) the system $\{u_\varepsilon\}$, from Theorem 1.1 is bounded in all spaces $W^{1,p}(0,T;V)$, $1 \leqslant p < \infty$. Hence there exist a sequence $\{\varepsilon_n\}$, $\varepsilon_n > 0$; and a function $\bar{u} \in W^{1,2}(0,T;V)$ such that

$$\lim_{n\to\infty} \varepsilon_n = 0, \tag{1.36}$$

$$u_{\varepsilon_n} \rightharpoonup \bar{u} \quad \text{weakly in } W^{1,2}(0,T;V). \tag{1.37}$$

Further, due to (1.9) we have the relation

$$(u_{\varepsilon_n}(t), v) = \Big(\int_0^{r} u'_{\varepsilon_n}(s)\,ds, v\Big) + (u_0, v) \quad \text{for each } n \in N \text{ and } v \in V. \tag{1.38}$$

The expression $(\int_0^t w'(s)\,ds, v)$, $w \in W^{1,2}(0,T;V)$ represents for each fixed $t \in [0,T]$ and $v \in V$ a linear continuous functional over $W^{1,2}(0,T;V)$ and therefore the sequence $\{(u_{\varepsilon_n}(t), v\}$ is, due to (1.37), convergent for every $t \in [0,T]$ and $v \in V$. Consequently, there exists a function $u\colon [0,T] \to V$ such that

$$u_{\varepsilon_n}(t) \rightharpoonup u(t) \text{ (weakly) in } V \text{ for each } t \in [0,T]. \tag{1.39}$$

Using the Fatou lemma and the Lebesgue theorem ([6], App. 1) we obtain that

$$u \in L^1(0,T;V) \quad \text{and} \quad u_{\varepsilon_n} \rightharpoonup u \in L^1(0,T;V).$$

Comparing it with (1.37) we conclude that $u(t) = \bar{u}(t)$ for a.e. $t \in [0,T]$ and

$$u_{\varepsilon_n} \rightharpoonup u \text{ (weakly) in } W^{1,2}(0,T;V). \tag{1.40}$$

Moreover, the estimates (1.11), (1.12) imply that the sequences $\{u_{\varepsilon_n}\}$, $\{u'_{\varepsilon_n}\}$ are bounded in the space $L^\infty(0,T;V)$ which is the adjoint space to $L^1(0,T;V)$. Hence with respect to (1.40) and due to a theorem of Banach-Alaoglu-Bourbaki ([7], Th. III.15) we have

$$u_{\varepsilon_n} \overset{*}{\rightharpoonup} u \text{ (weakly star) in } L^\infty(0,T;V), \tag{1.41}$$

$$u'_{\varepsilon_n} \overset{*}{\rightharpoonup} u' \text{ (weakly star) in } L^\infty(0,T;V). \tag{1.42}$$

Using Prposition III.12 from [7] and (1.41), (1.42) we obtain the inequalities

$$\|u - u_0\|_{L^\infty} \leqslant \liminf_{n\to\infty} \|u_{\varepsilon_n} - u_0\|_{L^\infty}, \|u'\|_{L^\infty} \leqslant \lim\inf_{n\to\infty} \|u'_{\varepsilon_n}\|_{L^\infty},$$

which imply the estimates (1.34), (1.35).

It remains to verify the relations (1.1), (1.2), (1.3). From (1.8) we have the equality

$$\beta(u_{\varepsilon_n}(t)) = \varepsilon_n[f(t) - A(t)u'_{\varepsilon_n}(t) - B(t)u_{\varepsilon_n}(t)] \quad \text{for every } t \in [0,T].$$

The sequences $\{u_{\varepsilon_n}(t)\}$, $\{u'_{\varepsilon_n}(t)\}$ are bounded in V for every $t \in [0,T]$ and (1.36) implies

(1.43) $$\lim_{n\to\infty} \beta(u_{\varepsilon_n}(t) = 0 \text{ (strongly) in } V' \quad \text{for every } t \in [0,T].$$

Monotonicity of β and the relation (1.39) then imply

(1.44) $$\langle \beta(v), u(t) - v\rangle \leqslant 0 \quad \text{for every } t \in [0,T], v \in V.$$

Inserting $v = u(t) + sw$, $s > 0$, $w \in V$ into (1.44) we obtain

$$\langle \beta(u)(t) + sw), w\rangle \leqslant 0 \quad \text{for all } w \in V$$

and due to the Lipschitz continuity of β the limiting process $s \to 0$ yields

$$\langle \beta(u(t)), w\rangle \geqslant 0 \quad \text{for all } w \in V$$

and hence

(1.45) $$\beta(u(t)) = 0 \quad \text{for all } t \in [0,T],$$

which due to (1.10 i) implies the relation (1.1).

We shall now verify the initial condition (1.3). After changing n on a set of zero measure we obtain

(1.46) $$u \in W^{1,\infty}(0,T;V) \cap C([0,T],V)$$

and

(1.47) $$u(t) = u(0) + \int_0^t u'(s)\,ds \quad \text{for every } t \in [0,T].$$

Simultaneously we have the relation

(1.48) $$u_{\varepsilon_n}(t) = u_0 + \int_0^t u'_{\varepsilon_n}(s)\,ds \quad \text{for every } t \in [0,T], n \in N.$$

The initial condition (1.3) then follows from the convergences (1.39) and (1.40).

Let $w \in L^1(0,T;V)$ be an arbitrary function such that

(1.49) $$w(t) \in K \quad \text{for a.e. } t \in [0,T].$$

We then have the inequalities

(1.50) $$\langle \beta(u_{\varepsilon_n}(t)), w(t) - u_{\varepsilon_n}(t)\rangle \leqslant 0 \quad \begin{matrix} \text{for a.e. } t \in [0,T] \\ \text{for every } n \in N \end{matrix}$$

Integrating we arrive at the inequalities

$$\begin{aligned}
&\langle A(T)\ u_{\varepsilon_n}(T), u_{\varepsilon_n}(T)\rangle + \int_0^T \langle [2B(t) - A'(t)]u_{\varepsilon_n}(t), u_{\varepsilon_n}(t)\rangle\, dt \leqslant \\
&\leqslant \langle A(0)u_{\varepsilon_n}(0), u_{\varepsilon_n}(0)\rangle + 2\int_0^T \big\langle A(t)u'_{\varepsilon_n}(t) + B(t)u_{\varepsilon_n}(t), w(t)\big\rangle\, dt + \\
&+ 2\int_0^T \langle f(t), u_{\varepsilon_n}(t) - w(t)\rangle\, dt \quad \text{for all } n \in N.
\end{aligned} \tag{1.52}$$

The functionals on the left-hand side of (1.52) are weakly lower semicontinuous on the spaces V and $L^2(0,T;V)$, respectively, which follows from the assumptions (1.5)–(1.7). The relations (1.39), (1.40) and the initial conditions (1.9), (1.3) then imply the inequalities

$$\begin{aligned}
&\langle A(T)\ u(T), u(T)\rangle + \int_0^T \langle [2B(t) - A'(t)]u(t), u(t)\rangle \leqslant \\
&\quad \leqslant \liminf_{n\to\infty} \Big(\langle A(T)u_{\varepsilon_n}(T), u_{\varepsilon_n}(T)\rangle + \int_0^T \langle [2B(t) - A'(t)]u_{\varepsilon_n}(t), u_{\varepsilon_n}(t)\rangle\, dt \Big) \leqslant \\
&\quad \leqslant \langle A(0)u(0), u(0)\rangle + 2\int_0^T \langle A(t)u'(t) + B(t)u(t), w(t)\rangle\, dt \\
&\quad + 2\int_0^T \langle f(t), u(t) - w(t)\rangle\, dt
\end{aligned}$$

which can be rewritten in the form

$$\int_0^T \langle A(t)u'(t) + B(t)u(t) - f(t), w(t) - u(t)\rangle\, dt \geqslant 0 \tag{1.53}$$

for all $w \in L^1(0,T;V)$ such that $w(t) \in K$ for a.e. $t \in [0,T]$.

Using Proposition 3 from [5] (App. I) we obtain for a.e. $t \in [0,T]$ the inequality

$$\langle A(t)u'(t) + B(t)u(t) - f(t), v - u(t)\rangle \geqslant 0 \quad \text{for all } v \in K,$$

which proves the inequality (1.2). Hence u is a solution of the problem (1.1), (1.2), (1.3).

a) Uniqueness. Let u_1 and u_2 be two solutions of the problem (1.1), (1.2), (1.3). Inserting successively $u(t) = u_1(s)$, $v = u_2(s)$; $u(t) = u_2(s)$, $v = u_1(s)$ into (1.2) we obtain after summing and integrating the inequality

$$\int_0^t \langle A(s)(u_1 - u_2)'(s) + B(s)(u_1 - u_2)(s), (u_1 - u_2)(s)\rangle\, ds \leqslant 0 \tag{1.54}$$

for every $t \in [0,T]$.

Let us denote $w = u_1 - u_2$. The function w fulfils the initial condition

$$w(0) = 0. \tag{1.55}$$

The inequality (1.54) then implies

$$\langle A(t)w(t), w(t)\rangle + \int_0^t \langle [2B(s) - A'(s)]w(s), w(s)\rangle \, ds \leqslant 0 \quad \text{for all } t \in [0,T]. \tag{1.56}$$

The assumptions (1.5), (1.7) imply

$$w(t) = u_1(t) - u_2(t) = 0$$

and

$$u_1(t) = u_2(t) \quad \text{for all } t \in [0,T],$$

which proves uniqueness of the solution of the problem (1.1), (1.2), (1.3). □

2. Optimal control problem

1. Formulation of the Problem.

We assume that the data in the prblem (1.1), (1.2), (1.3) depend on a control parameter e. Control problems for pseudoparabolic equations were studied in the papers [3], [16], [17]. We assume that the convex set of admissible states depends also on a control parameter e. Such problem in the elliptic case was investigated in [3].

We consider the following state problem:

$$u(t,e) \in K(e) \quad \text{for a.e. } t \in [0,T] \tag{2.1}$$

and for a.e. $t \in [0,T]$:

$$\langle A(t,e)u'(t,e) + B(t,e)u(t,e), v - u(t,e)\rangle \geqslant \langle f(t,e), v - u(t,e)\rangle \quad \text{for all } v \in K(e), \tag{2.2}$$

$$u(0,e) = u_0(e) \in K(e), \tag{2.3}$$

where $K(e)$ is a closed convex subset of a Hilbert space V. With the problem (2.1)–(2.3) we link a minimum problem

$$j(u(\bar{e}),\bar{e}) = \min_{e\in U_{ad}} j(u(e),e), \tag{2.4}$$

where U_{ad} is a compact subset of a Banach space U and the functional $j\colon W^{1,2}(0,T;V) \times U \to R$ is lower bounded and fulfils the assumption

$$u_n \rightharpoonup u \text{ in } W^{1,2}(0,T;V) \text{ and } e_n \to e \text{ in } U \Rightarrow j(u,e) \leqslant \liminf_{n\to\infty} j(u_n,e_n). \tag{2.5}$$

In order to characterize the dependence $e \to K(e)$ we recall a special type of convergence of set sequences introduced in [14].

D e f i n i t i o n 2.1. A sequence $\{K_n\}$ of subsets of a normed space V converges to a set $K \subset V$ if

i) K contains all weak limits of sequences $\{u_k\}$, $u_k \in K_{n_k}$, where $\{K_{n_k}\}$ is an arbitrary subsequence of $\{K_n\}$;

ii) every element $v \in K$ is the strong limit of a sequence $\{v_n\}$, $v_n \in K_n$, $n \in \mathbf{N}$.

$$\text{Notation: } K = \operatorname*{Lim}_{n\to\infty} K_n.$$

We assume that

$$e_n \to e \text{ in } U \Rightarrow K(e) = \operatorname*{Lim}_{n\to\infty} K(e_n). \tag{2.6}$$

Further, we introduce assumptions expressing the dependence of all data in (1.2), (1.3) on the control parameter e:

$$\|u\|^2 \leqslant c_5 \langle A(t,e)u, u\rangle, \quad c_5 > 0, \tag{2.7}$$

$$\langle A(t,e)u, v\rangle = \langle A(t,e)v, u\rangle, \tag{2.8}$$

$$\langle B(t,e)u, v\rangle = \langle B(t,e)v, u\rangle, \tag{2.9}$$

$$\langle [2B(t,e) - A_t'(t,e)]u, u\rangle \geqslant 0 \tag{2.10}$$

$$\|A_t'(t,e)\|_L \leqslant c_0, \tag{2.11}$$

$$\|B(t,e)\|_L \leqslant c_7, \tag{2.12}$$

$$\|B_t'(t,e)\|_L \leqslant c_8, \tag{2.13}$$

$$\|f(\cdot,e)\|_{W^{1,2}} \leqslant c_9 \tag{2.14}$$

for all $u, v \in V, t \in [0,T], e \in U_{ad}$;

$$e_n \to e \text{ in } U \Rightarrow \begin{cases} i)\ A(\cdot,e_n) \to A(\cdot,e) \text{ in } C^1([0,T], L(V,V')), \\ ii)\ B(\cdot,e_n) \to B(\cdot,e) \text{ in } C([0,T], L(V,V')), \\ iii)\ f(\cdot,e_n) \to f(\cdot,e) \text{ in } C([0,T], V'), \\ iv)\ u_0(e_n) \to u_0(e) \text{ in } V. \end{cases} \tag{2.15}$$

2. Existence Theorem.

We will formulate and verify the existence theorem for the optimal control problem (2.1)–(2.4).

Theorem 2.1. *Let the assumptions (2.5)–(2.15) hold. Then there exists at least one solution $\bar{e} \in U_{ad}$ of the optimal control problem (2.1)–(2.4).*

P r o o f. Due to Theorem 1.2 for every $e \in U_{ad}$ there exists a unique solution $u(e) \in W^{1,\infty}(0,T;V) \cap C([0,T],V)$ of the state initial value problem (2.1), (2.2), (2.3). Hence we can define the functional

$$J : U_{ad} \to R, J(e) = j(u(e), e).$$

Let $\{e_n\} \subset U_{ad}$ be a minimizing sequence for J:

$$\lim_{n\to\infty} J(e_n) = \inf_{e\in U_{ad}} J(e). \tag{2.16}$$

Since the set U_{ad} is compact in U, there exist an element $\bar{e} \in U_{ad}$ and a subsequence of $\{e_n\}$ (denoted again by $\{e_n\}$) such that

$$\lim_{n\to\infty} e_n = \bar{e} \text{ in } U. \tag{2.17}$$

Denoting $u(e_n) = u_n$, we rewrite the state problem (2.1), (2.2), (2.3) in the form

$$u_n(t) \in K(e_n) \quad \text{for a.e. } t \in [0,T], \tag{2.18}$$

and for a.e. $t \in [0,T]$:

$$\langle A(t,e_n)u_n'(t) + B(t,e_n)u_n(t) - f(t,e_n), v - u_n(t)\rangle \geqslant 0 \quad \text{for all } v \in K(e_n), \tag{2.19}$$

$$u_n(0) = u_0(e_n) \in K(e_n). \tag{2.20}$$

Using the estimates (1.34), (1.35) and the assumptions (2.7)–(2.15) we obtain the estimate

$$\|u_n\|_{W^{1,\infty}} \leqslant c_{10} \quad \text{for all } n \in \mathbb{N}, \tag{2.21}$$

where the constant c_{10} involves only $c_5, \ldots, c_9$ from (2,7)–(2.14) and the upper bound for the sequence $u_0(e_n)$. Comparing the estimates (1.11), (1.12) and (1.34), (1.35) we can see that c_{10} does not depend on the sequence $\{K(e_n)\}$.

It results from (2.21) that there esist a function $\bar{u} \in W^{1,\infty}(0,T;V) \cap C([0,T],V)$ and a subsequence of $\{u_n\}$ (denote again by $\{u_n\}$) such that

$$u_n \rightharpoonup \bar{u} \text{ in } W^{1,2}(0,T;V), \tag{2.22}$$

$$u_n(t) \rightharpoonup \bar{u}(t) \text{ in } V \quad \text{for a.e. } t \in [0,T], \tag{2.23}$$

$$u_n \overset{*}{\rightharpoonup} \bar{u} \text{ in } L^\infty(0,T;V), \tag{2.24}$$

$$u_n' \overset{*}{\rightharpoonup} \bar{u}' \text{ in } L^\infty(0,T;V). \tag{2.25}$$

Relations (2.18), (2.23) and the assumption (2.6) imply

$$\bar{u}(t) \in K(\bar{e}) \quad \text{for a.e. } t \in [0,T]. \tag{2.26}$$

From the relations

$$u_n(t) = u_0(e_n) + \int_0^t u_n'(s)\,ds,$$

$$\bar{u}(t) = \bar{u}(0) + \int_0^t \bar{u}'(s)\,ds, \quad t \in [0,T]$$

we obtain due to (2.22), (2.23), (2.15 iv) the initial condition

$$\bar{u}(0) = u_0(\bar{e}) \in K(\bar{e}). \tag{2.27}$$

Let $w \in L^1(0,T;V)$ be an arbitrary function such that

$$w(t) \in K(\bar{e}) \quad \text{for a.e. } t \in [0,T]. \tag{2.28}$$

Since the set $K(\bar{e})$ is closed in the space V, we can use Lemma A.O. from [6] (App.) according to which for every $\varepsilon > 0$ there exists a measurable function $v\colon [0,T] \to K(\bar{e})$ with only a finite number of values and such that

$$\int_0^T \|w(t) - v(t)\|\, dt < \varepsilon.$$

The assumption (2.6) and Definition 2.1 then imply the existence of a subsequence of $\{e_n\}$ (denoted again by $\{e_n\}$) and of a sequence $\{v_n\} \subset L^1(0,T;V)$ such that

$$v_n(t) \in K(e_n) \quad \text{for all } t \in [0,T], n \in \mathbf{N} \tag{2.29}$$

and

$$\lim_{n\to\infty} \|v_n - w\|_{L^1} = \lim_{n\to\infty} \int_0^T \|v_n(t) - w(t)\|\, dt = 0. \tag{2.30}$$

The inequality (2.19) then implies

$$\int_0^T \langle A(t,e_n)u_n'(t) + B(t,e_n)u_n(t) - f(t,e_n), \quad v_n(t) - u_n(t)\rangle\, dt \geqslant 0. \tag{2.31}$$

The last inequality can be rewitten in the form

$$\begin{aligned}
&\langle A(T,e_n)u_n(T), u_n(T)\rangle + \int_0^T \langle [2B(t,e_n) - A'(t,e_n)]u_n(t), u_n(t)\rangle\, dt \leqslant \\
&\quad \leqslant \langle A(0,e_n)u_n(0), u_n(0)\rangle + 2\int_0^T \langle A(t,e_n)u_n'(t) + B(t,e_n)u_n(t), v_n(t)\rangle\, dt + \\
&\quad + 2\int_0^T \langle f(t,e_n), u_n(t) - v_n(t)\rangle\, dt,
\end{aligned}$$

and further,

$$\begin{aligned}
&\langle A(T,\bar{e})u_n(T), u_n(T)\rangle + \int_0^T \langle [2B(t,\bar{e}) - A'(t,\bar{e})]u_n(t), u_n(t)\rangle\, dt \leqslant \\
&\leqslant \big\langle \big[A(T,\bar{e}) - A(T,e_n)\big]u_n(T), u_n(T)\big\rangle \\
&+ \int_0^T \langle 2[B(t,\bar{e}) - B(t,e_n)]u_n(t), u_n(t)\rangle\, dt + \int_0^T \langle [A'(t,e_n) - A'(t,\bar{e})]u_n(t), u_n(t)\rangle\, dt \\
&+ \langle [A(0,e_n) - A(0,\bar{e})]u_0(e_n), u_0(e_n)\rangle + \langle A(0,\bar{e})u_0(e_n), u_0(e_n)\rangle \\
&+ 2\int_0^T \langle A(t,e_n)u_n'(t) + B(t,e_n)u_n(t), v_n(t)\rangle\, dt + \\
&+ \int_0^T \langle f(t,e_n), u_n(t) - v_n(t)\rangle\, dt.
\end{aligned}$$

The functionals on the left-hand side of this inequality are weakly lower semicontinuous on the spaces V and $L^2(0,T;V)$ respectively which follows from the assumptions (2.7)–(2.10). Using the assumption (2.15) and the relations (2.17), (2.22)–(2.25), (2.30) we arrive at the inequality

$$\begin{aligned}\langle A(T,\bar e)\bar u(T),\bar u(T)\rangle + \int_0^T \langle [2B(t,\bar e) - A'[(t,\bar e)]\bar u(t),\bar u(t)\rangle \, dt \leqslant \\ \leqslant \langle A(0,\bar e)u_0(\bar e),u_0(\bar e)\rangle + 2\int_0^T \langle A(t,\bar e)\bar u'(t) + B(t,\bar e)\bar u(t), w(t)\rangle \, dt + \\ + 2\int_0^T \langle f(t,\bar e),\bar u(t) - w(t)\rangle \, dt\end{aligned}$$

for all $w \in L^1(0,T;V)$ such that $w(t) \in K(\bar e)$ for a.e. $t \in [0,T]$. Using the initial condition (2.27), the differentiability and symmetry of the operator function $A(\cdot,\bar e)$, we obtain the inequality

$$\int_0^T \langle A(t,\bar e)\bar u'(t) + B(t,\bar e)\bar u(t) - f(t,\bar e), w(t) - \bar u(t)\rangle \, dt \geqslant 0 \tag{2.32}$$

for all $w \in L^1(0,T;V)$ such that $w(t) \in K(\bar e)$ for a.e. $t \in [0,T]$.

Again, using Proposition 3 from [5] (App. I) we arrive at

$$\langle A(t,\bar e)\bar u'(t) + B(t,\bar e)\bar u(t) - f(t,\bar e), v - \bar u(t)\rangle \geqslant 0 \tag{2.33}$$

for a.e. $t \in [0,T]$, for all $v \in K(\bar e)$.

The last inequality together with (2.22), (2.26), (2.27) and the uniqueness of a solution of (2.1)–(2.3) imply the relations

$$\bar u = u(\bar e), \tag{2.34}$$

$$u(e_n) \rightharpoonup u(\bar e) \text{ in } W^{1,2}(0,T;V). \tag{2.35}$$

Finally, using (2.5), (2.16) wi obtain the inequalities

$$\begin{aligned} j(u(\bar e),\bar e) \leqslant \liminf_{n\to\infty} j(u(e_n),e_n) = \lim_{n\to\infty} J(e_n) = \\ = \inf_{e\in U_{ad}} J(e) = \inf_{e\in U_{ad}} j(u(e),e)\end{aligned}$$

and hence (2.4) follows, which completes the proof of the theorem. □

3. Optimal control of a viscoelastic plate bending with an obstacle

Let us condider a thin viscoelastic plate whose middle surface is identified with an open bounded domain $\Omega \subset R^2$ with a Lipschitz boundary $\partial\Omega$. A function $e\colon \bar{\Omega} \to R$ expresses its varying thickness. We assume that the plate is deflected under the perpendicular load $p\colon \Omega \to R$, concentrated loads p_i at the points X_i, $i = 1, \ldots, m$, and under its own weight. Further, we assume that the plate is clamped on its boundary and that an obstacle is expressed via a function $\Phi\colon \Omega \to R$.

The deflection $u \equiv u(x,t,e)$ is then a solution of the inital value problem for the pseudoparabolic variational inequality:

$$u(t,e) \in K(e) \qquad \text{for a.e. } t \in [0,T], \tag{3.1}$$

$$\iint_\Omega e^3(x)\Big(A^{(0)}_{ijkl}(t)\frac{\partial}{\partial t}u_{'ij}(t,e) + A^{(1)}_{ijkl}(t)u_{'ij}(t,e)\Big)\cdot[v-u(t,e)]_{'kl}\,dx \tag{3.2}$$
$$\geqslant \langle f(t,e)v - u(t,e)\rangle \quad \text{for a.e. } t \in [0,T], \quad \text{for all } v \in K(e),$$

$$u(0,e) = u_0(e) \in K(e), \tag{3.3}$$

where

$$K(e) = \{v \in H^2_0(\Omega)\colon v(x) \geqslant \Phi(x) + \frac{1}{2}e(x) \quad \text{for all } x \in \bar{\Omega}\}, \tag{3.4}$$

$u_{'ij} = \dfrac{\partial^2 u}{\partial x_i \partial x_j}$ and the summation convention the indices $i,j,k,l \in \{1,2\}$ is considered.

We assume the Zener model of the plate ([8], [15]), which means that the functional $f(t,e) \in V'$, $V = H^2_0(\Omega)$ has the form

$$\langle f(t,e),v\rangle = \iint_\Omega [k_0(t)p(t,x) + p'_t(t,x) - \rho(x)e(x)]v(x)dx + \tag{3.5}$$
$$+ \sum_{i=1}^m [k_i(t)p_i(t) + p'_i(t)]v(X_i), \quad v \in V,$$

where $X_i \in \Omega$, the real functions k_i are continuous and positive for $i = 0, 1, \ldots, m$; $p(t,\cdot) \in C(\bar{\Omega})$ for every $t \geqslant 0$, $p(\cdot,x)$ $C^1[0,\infty)$ for every $x \in \bar{\Omega}$, $\rho \in C(\bar{\Omega})$ is the plane density of the plate.

The initial function $u_0(e) \in K(e)$ is a solution of the elliptic variational inequality

$$\begin{aligned}(3.6)\qquad &\iint_\Omega e^3(x)A^{(0)}_{ijkl}(0)u_{0,ij}(e)[v-u_0(e)]_{'kl}dx \geqslant\\ &\geqslant \iint_\Omega [p(0,x)-k_0(0)^{-1}\rho(x)e(x)][v-u_0(x)]_{'kl}dx+\\ &+\sum_{i=1}^m p_i(0)[v-u_0(e)](X_i) \quad \text{for all } v\in K(e).\end{aligned}$$

Further, we impose the conditions on the functions e, Φ, $A^{(r)}_{ijkl}$:

$$\begin{aligned}(3.7)\qquad U_{ad} = \{&e\in H^3(\Omega)\colon 0<e_{\min}\leqslant e(x)\leqslant e_{\max} \quad \text{for all } x\in\Omega,\\ &\|e\|_{H^3}\leqslant C_1, \iint_\Omega e(x)dx = C_2,\ e|_{\partial\Omega}=\varphi_0, \frac{\partial e}{\partial n}|_{\partial\Omega}=\varphi_1\},\end{aligned}$$

where φ_o, φ_1 are sufficiently smooth real functions defined on $\partial\Omega$. The set U_{ad} is compact in $U = H^2(\Omega)$, because it is bounded and weakly closed in $H^3(\Omega)$ and $H^3(\Omega)$ is compactly imbedded in $H^2(\Omega)$.

The functions Φ, $A^{(r)}_{ijkl}$ are supposed to satisfy the following conditions:

$$(3.8)\qquad \Phi\in C(\bar\Omega),\quad \Phi(s) < -\frac{1}{2}\varphi_0(s) \quad \text{for all } s\in\partial\Omega,$$

$$\begin{aligned}(3.9)\qquad &A^{(r)}_{ijkl}\in C^1[0,T],\ A^{(r)}_{ijkl}=A^{(r)}_{jikl}=A^{(r)}_{klij}\\ &A^{(o)}_{ijkl}(t)\varepsilon_{ij}\varepsilon_{kl}\geqslant c_0\varepsilon_{ij}\varepsilon_{ij},\quad c_0>0,\\ (3.10)\qquad &\Big(2A^{(1)}_{ijkl}(t)-\frac{d}{dt}A^{(o)}_{ijkl}(t)\Big)\varepsilon_{ij}\varepsilon_{kl}\geqslant 0,\\ &\text{for all } t\in[0,T],\quad \{\varepsilon_{ij}\}\in R^4,\quad \varepsilon_{ij}=\varepsilon_{ji}\end{aligned}$$

If we define the operator functions $A(t,e)$, $B(t,e)\colon H^2_0(\Omega)\to[H^2_0(\Omega)]'$ by

$$\begin{aligned}&\langle A(t,e)u,v\rangle = \iint_\Omega e^3(x)A^{(o)}_{ijkl}(t)u_{'ij}v_{'kl}dx,\\ (3.11)\qquad &\langle B(t,e)u,v\rangle = \iint_\Omega e^3(x)A^{(1)}_{ijkl}(t)u_{'ij}v_{'kl}dx,\\ &t\in[0,T],\quad e\in U;\quad u,v\in H^2_0(\Omega)\end{aligned}$$

then all assumptions of Chapters 1, 2 are fulfilled ((2.6) is verified in [4]).

A cost functional j can have the form

$$(3.12)\qquad j(u,e)=\|Du-z_d\|^2_X+F(e),\quad u\in W^{1,2}(0,T;V), e\in U,$$

where X is any Hilbert space, $D \in L(W^{1,2}((0,T;V),X)$, $z_d \in X$ and $F: U \to R$ is any lower continuous functional. Then there exists an optimal thickness function $\bar{e}$ fulfilling

$$j(u(\bar{e}),\bar{e}) = \min_{e \in U_{ad}} j(u(e),e),$$

where $u(e)$ is a solution of the initial value problem (3.1)–(3.6).

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Súhrn

ÚLOHA OPTIMÁLNEHO RIADENIA PRE PSEUDOPARABOLICKÚ VARIAČNU NEROVNICU

IGOR BOCK, JÁN LOVÍŠEK

Je riešená úloha optimálneho riadenia pre pseudoparabolickú variačnu nerovnicu s riadiacimi parametrami v koeficientoch a v konvexných množinách prípustných stavov. Je dokázaná existencia optimálneho riadenia. Teória je aplikovaná na úlohu optimálneho návrhu väzkopružnej tenkej dosky s prekážkou, pričom premenná hrúbka dosky vystupuje ako riadiaci parameter.

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