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Explicit solutions for boundary value problems related to the operator equations $X^{(2)} - AX = 0$

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EXPLICIT SOLUTIONS FOR BOUNDARY VALUE PROBLEMS RELATED TO THE OPERATOR EQUATIONS $X^{(2)} - AX = 0$.

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Summary. Cauchy problems, boundary value problems with a boundary value condition and Sturm-Liouville problems related to the operator differential equation $X^{(2)} - AX = 0$ are studied for the general case, even when the algebraic equation $X^2 - A = 0$ is unsolvable. Explicit expressions for the solutions in terms of data problem are given and computable expressions of the solutions for the finite-dimensional case are made available .

Keywords: Sturm-Liouville, operator differential equation, operator algebraic equation.

AMS Subject Classification: 34B05, 34B10, 47A62, 47A60.

1. INTRODUCTION

For the finite-dimensional case, second order operator differential equations are important in the theory of damped oscillatory systems and vibrational systems, [6], [12]. Infinite-dimensional equations occur frequently in the theory of stochastic processes, the degradation of polymers, infinite ladder network theory in engineering, [1], [19], denumerable Markov chains and moment problems, [10], [22].

In [8], the author studies Cauchy problems and boundary value problems related to the operator differential equation

$$X^{(2)} + A_1X^{(1)} + A_0X = 0 \tag{1.1}$$

where A_i, for $i = 0, 1$, are bounded linear operators on a complex separable Hilbert space H. Explicit expressions of the solutions of these problems in terms of data problem and the solutions of the algebraic operator equation

$$X^2 + A_1X + A_0 = 0 \tag{1.2}$$

were given in [8]. The resolution problem of the equation (1.2) is related to the problem of existence of a linear factorization for the operator polynomial $L(z) = z^2 + A_1z + A_0$. So, for the finite dimensional case, P is a solution of (1.2) if and only if the matrix polynomial $zI - P$ is a right divisor of $L(z)$, i.e. $L(z) = L_1(z)(zI - P)$ for some matrix polynomial $L_1(z)$ (which is necessarily linear with the leading coefficient I). Furthermore, this occurs if the companion matrix $C_L = \begin{bmatrix} 0 & I \\ -A_0 & -A_1 \end{bmatrix}$ is diagonalizable, [6], [13]. The infinite-dimensional case is treated in [20] in a more general context. Note that for the operator case, even for the finite-dimensional case, the equation (1.2) may be unsolvable. For example if $A_1 = 0$ and $-A_0$ is a unilateral weighted shift operator, the equation (1.2) is unsolvable, [21], p. 63.

Sturm-Liouville operator problems have been studied by several authors and with different techniques, [14]–[18]. Fot the scalar case, the classical Sturm-Liouville theory yields a complete solution of the problem, [4], [7]. In a recent paper [9], we study the Sturm-Liouville operator problem

$$(1.3) \qquad \begin{aligned} &X^{(2)} - \lambda Q X = 0 \\ &E_1 X(0) + E_2 X^{(1)}(0) = 0 \\ &F_1 X(a) + F_2 X^{(1)}(a) = 0 \\ &0 \leqq t \leqq a\,, \quad \lambda \in \mathbb{C} \end{aligned}$$

where E_i, F_i, for $i = 1, 2$, and Q are bounded operators on H, and C denotes the complex plane.

The paper [9] deals with the problem of finding non-trivial explicit solutions of the problem (1.3) for the case when the corresponding algebraic operator equation $X^2 - \lambda Q = 0$ is solvable. In this paper we are interested in the problem of finding conditions under which the problem admits non-trivial solutions, as well as explicit expressions for the solutions in terms of data problem, for the general case, even when $X^2 - \lambda Q = 0$ is unsolvable. We are also interested in finding explicit expressions for the solutions of boundary value problems and Cauchy problems concerning the operator differential equation

$$(1.4) \qquad X^{(2)} - AX = 0$$

when A is an operator without a square root.

In the following we denote by $L(H)$ the algebra of all bounded linear operators on H. If T lies in $L(H)$, its spectrum will be denoted by $\sigma(T)$, and its compression spectrum $\sigma_{\text{comp}}(T)$ is the set of all complex numbers z such that the range $(zI - T)(H)$ is not dense in H, [2], p. 240.

2. BOUNDARY VALUE PROBLEMS

We begin this section with the study of the Cauchy problem for the equation (1.4). Let us consider the $L(H)$ valued analytic functions defined by

$$(2.1) \qquad g_A(z) = \sum_{k \geqq 0} A^k z^{2k}/(2k)!\,, \quad f_A(z) = \sum_{k \geqq 0} A^k z^{2k+1}/(2k+1)!$$

Note that g_A and f_A are entire functions in the complex plane. If A has a square root B such that $A = B^2$, then $g_A(z) = \cosh(Bz)$, where $\cosh(\cdot)$ denotes the image of the hyperbolic cosine by means of the Riesz-Dunford functional calculus, [5]. Furthermore, if A is an invertible operator such that $A = B^2$, the $f_A(z) = B^{-1}\,\mathrm{sh}\,(Bz)$, where sh $(\cdot)$ denotes the image of the hyperbolic sine by means of the Riesz-Dunford functional calculus. It is integresting to note that for the case when A has not a square root, or A is not invertible, then $g_A(z)$ and $f_A(z)$ are not computable by means of the Riesz-Dunford functional calculus, but for the finite-dimensional case, an explicit and computable expression of these matrix functions is available. Let us suppose that $\alpha \in [0, 2\pi]$ is chosen such that $\sigma(A) - \{0\}$ is contained in $D_\alpha = \mathbb{C} - H_\alpha$ with $H_\alpha = \{-r\exp(i\alpha);\ r \geqq 0\}$, then if $w \in \sigma(A) - \{0\}$ and $z^{1/2} = \exp((\log_\alpha(z))/2)$, one gets

$$(2.2)\qquad h(z) = \sum_{k\geqq 0} z^k/(2k)! = \cosh(z^{1/2}), \quad \text{if} \quad z \in D_\alpha$$

although the series which defines h is an entire function in the complex plane. Thus, the computation of the derivatives $h^{(j)}(w)$ for $j \geqq 0$ and $w \in \sigma(A)$ is very easy even for $z = 0$, considering (2.2) for w in D_α, and taking the series expansion that defines $h(z)$, for $z = 0$. If $\sigma(A) = \{\lambda_i;\ 1 \leqq i \leqq s\}$, $(\lambda_i I - A)^D$ denotes the Drazin inverse of $\lambda_i I - A$, and $E(\lambda_i) = I - (\lambda_i I - A)(\lambda_i I - A)^D$, v_i the index of $\lambda_i I - A$, then

$$(2.3)\qquad g_A(z) = h(z^2 A) = \sum_{i=1}^{s} \sum_{k=0}^{v_i - 1} \frac{h^{(k)}(z^2\lambda_i)}{k!} (A - \lambda_i I)^k E(\lambda_i)\, z^{2k},$$

see [3], Chapter 1 for details. In an analogous way, for computing $g_A(z)$ in the finite dimensional case, this matrix may be computed as a polynomial in A.

Lemma 1. *Let us consider the Cauchy problem*

$$(2.4)\qquad X^{(2)} - AX = 0, \quad X(0) = C_0, \quad X^{(1)}(0) = C_1, \quad -\infty < t < \infty$$

where A, C_0 and C_1 are operators in $L(H)$. Then the only solution of (2.4) *is given by the expression*

$$(2.5)\qquad X(t) = g_A(t)\, C_0 + f_A(t)\, C_1$$

where $g_A(t)$ and $f_A(t)$ are defined by (2.1). *Furthermore, if A is an invertible operator with $B^2 = A$, then*

$$(2.6)\qquad X(t) = \cosh(Bt)\, C_0 + B^{-1}\,\mathrm{sh}\,(Bt)\, C_1 .$$

Proof. Taking $Y_1 = X$, $Y_2 = X^{(1)}$, the Cauchy problem (2.4) is equivalent to the first order extended problem

$$(2.7)\qquad \mathrm{d}/\mathrm{d}t \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}; \quad \begin{bmatrix} Y_1(0) \\ Y_2(0) \end{bmatrix} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}, \quad -\infty < t < \infty .$$

According to [11], the only solution of (2.7) is given by

$$(2.8)\qquad \begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} = \exp\left(\begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix} t\right)\begin{pmatrix} C_0 \\ C_1 \end{pmatrix}.$$

For $k \leqq 0$, one gets

$$(2.9)\qquad \begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix}^{2k} = \begin{bmatrix} A^k & 0 \\ 0 & A^k \end{bmatrix}; \quad \begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix}^{2k+1} = \begin{bmatrix} 0 & A^k \\ A^{k+1} & 0 \end{bmatrix}.$$

From (2.8) and (2.9), it follows that

$$X(t) = Y_1(t) = \Big(\sum_{k \geqq 0} A^k t^{2k}/(2k)!\Big) C_0 + \Big(\sum_{k \geqq 0} A^k t^{2k+1}/(2k+1)!\Big) C_1 =$$

$$= g_A(t)\, C_0 + f_A(t)\, C_1 .$$

If $B^2 = A$, then from the previous remarks to the statement of the lemma, under the invertibility hypothesis imposed on A, one gets (2.6).

The next result is concerned with the boundary value problem

$$(2.10)\qquad X^{(2)} - AX = 0\,, \quad X(a) - X(0) = E$$

where E and A are operators in $L(H)$.

Theorem 1. *Let us consider the boundary value problem* (2.10), *then the following assertions hold*: (i) *The problem* (2.10) *is solvable if* $z^{1/2} \neq (2p\pi i)/a$ *for all* $z \in \sigma(A)$, *where p is any integer.*

(i) *Under the hypothesis* (i) *a solution of* (2.10) *is given by*

$$(2.11)\qquad X(t) = ((g_A(t)\,(g_A(a) - I)^{-1} E\,,$$

where g_A *is defined by* (2.1).

(iii) *If there exists B in* $L(H)$ *such that* $B^2 = A$, *then a solution of* (2.10) *is given by*

$$(2.12)\qquad X(t) = \cosh(Bt)\,(\cosh(Ba) - I)^{-1}\, E$$

Proof. Let us consider the Cauchy problem (2.1) with $C_1 = 0$ and C_0 an arbitrary fixed operator in $L(H)$. By Lemma 1, a solution of this problem is given by

$$(2.13)\qquad X(t) = \Big(\sum_{k=0} A^k t^{2k}/(2k)!\Big)\, C_0 \,.$$

In order to satisfy the boundary value condition of (2.10), the operator C_0 must verify

$$(2.14)\qquad E = X(a) - X(0) = \Big(\sum_{k \geqq 0} A^k a^{2k}/(2k)! - I\Big)\, C_0 =$$

$$= \Big(\sum_{k \geqq 1} A^k a^{2k}/(2k)!\Big)\, C_0 = (g_A(a) - I)\, C_0 \,.$$

From the spectral mapping theorem, [5], one gets

$$(2.15) \qquad \sigma(g_A(a) - I) = \sigma(h(a^2A) - I) = \{h(a^2z) - 1;\, z \in \sigma(A)\} = \\ = \{\cosh(z^{1/2}a) - 1\,;\quad z \in \sigma(A)\}$$

and, as $\cosh(w) = 1$ if and only if $w = 2p\pi i$ with p integer, the hypothesis of (i) implies that the operator $g_A(a) - I$ is invertible in $L(H)$. From (2.14) it follows that

$$(2.16) \qquad C_0 = (g_A(a) - I)^{-1}\, E\,.$$

Hence (i) is proved. Taking this expression of C_0 in (2.13) one gets (ii). If B is an operator with $B^2 = A$, then the expression (2.13) with C_0 given by (2.16) yields

$$X(t) = \Big(\sum_{k \geq 0} (Bt)^{2k}/(2k)!\Big)\, C_0 = \cosh(Bt)\,(\cosh(Ba) - I)^{-1}\, E\,.$$

In accordance with the notation introduced above in (2.1), we denote by $g_A^{(1)}(z)$ the $L(H)$-valued operator series obtained by differentiation in $g_A(z)$, that is

$$(2.17) \qquad g_A^{(1)}(z) = \sum_{k \geq 1} A^k z^{2k-1}/(2k - 1)!\,.$$

Let us consider $f_A(z) = \sum_{k \geq 0} A^k z^{2k+1}/(2k + 1)!$, and note that $g_A^{(1)}(z) = f_A(z)\, A$. The following result is concerned with the Sturm-Liouville problem (1.3) for $\lambda \neq 0$.

Theorem 2. *Let $\lambda \neq 0$, and let A be the operator λQ, then there nontrivial solutions of (1.3) if and only if $0 \in \sigma_{\text{comp}}(S)$, S being the operator matrix*

$$(2.18) \qquad S = \begin{bmatrix} E_1 & E_2 \\ F_1\, g_A(a) + F_2\, g_A^{(1)}(a) & F_1\, f_A(a) + F_2\, g_A(a) \end{bmatrix}.$$

Under the hypothesis $0 \in \sigma_{\text{comp}}(S)$, the solution set for the problem (1.3) is given by

$$X(t) = g_A(t)\, C_0 + f_A(t)\, C_1$$

where C_0 and C_1 are operators in $L(H)$ satisfying

$$(2.19) \qquad S \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = 0\,.$$

If N is a closed subspace of $H \oplus H$ which is orthogonal to the subspace $S(H \oplus H)$, and N_1 and N_2 are the subspaces of $H \oplus H$ defined by $N_1 = N \cap (H \oplus \{0\})$, $N_2 = N \cap (\{0\} \oplus H)$, then the operators C_0 and C_1 may be chosen as the projections on H with ranges N_1 and N_2, respectively.

Proof. By Lemma 1, the general solution of the operator differential equation arising in (1.3) is given by (2.5), where $A = \lambda Q$. If we assume that $X(t)$ given by (2.5) satisfies the boundary value conditions of (1.3), it follows that the operators C_0 and C_1 must verify the conditions

$$(2.20)\qquad \begin{aligned} &E_1(g_A(0)\,C_0 + f_A(0)\,C_1) + E_2(g_A^{(1)}(0)\,C_0 + f_A^{(1)}(0)\,C_1) = 0\,,\\ &F_1(g_A(a)\,C_0 + f_A(a)\,C_1) + F_2(g_A^{(1)}(a)\,C_0 + f_A^{(1)}(a)\,C_1) = 0\,. \end{aligned}$$

From (2.1) and (2.17) one gets $g_A(0) = I, f_A(0) = 0, f_A^{(1)}(0) = g_A(0) = I, g_A^{(1)}(0) = 0$, $f_A^{(1)}(a) = g_A(a)$, $g_A^{(1)}(a) = A f_A(a)$. Thus, the system (2.20) is equivalent to

$$\begin{aligned} &E_1 C_0 + E_2 C_1 = 0\,,\\ &F_1(g_A(a)\,C_0 + f_A(a)\,C_1) + F_2(A f_A(a)\,C_0 + g_A(a)\,C_1) = 0\,. \end{aligned}$$

From here and (2.18) the result is concluded.

Remark 1. Note that for the finite-dimensional case, the hypothesis $0 \in \sigma_{\text{comp}}(S)$ is equivalent to the noninvertibility of the matrix S, and in this case, in order to obtain explicit expressions of the solutions of (1.3), it is sufficient to compute the matrices $f_A(a)$, $g_A(a)$, $g_A^{(1)}(a)$, and to solve the algebraic system (2.19). For the general case different solutions for (1.3) may be found depending on the codimension of the subspace $S(H \oplus H)$.

Corollary 1. *Let us consider the problem* (1.3) *where Q is an invertible operator with a square root D, and $\lambda \neq 0$. Then the problem* (1.3) *is solvable if and only if $0 \in \sigma_{\text{comp}}(T)$, where T is the operator matrix*

$$T = \begin{bmatrix} E_1 & E_2 \\ F_1 \cosh(Da\lambda^{1/2}) + F_2 D \operatorname{sh}(Da\lambda^{1/2}) & F_1(D\lambda^{1/2})^{-1} \operatorname{sh}(Da\lambda^{1/2}) + F_2 \cosh(Da\lambda^{1/2}) \end{bmatrix}.$$

In this case, the solution set for the problem (1.3) *is given by*

$$X(t) = \cosh(D\lambda^{1/2}t)\,C_0 + (D\lambda^{1/2})^{-1} \operatorname{sh}(D\lambda^{1/2}t)\,C_1$$

where C_0 and C_1 are operators satisfying $T = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = 0$.

Proof. It is a consequence of Theorem 2 and Lemma 1.

Let us consider an example to which the method of [9] is not applicable because of the unsolvability of the equation $X^2 - \lambda Q = 0$.

Example 1. Let Q be the matrix $Q = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, *it is easy to show tha Q has no* square root and that $Q^2 = 0$. If we consider the boundary value problem (1.3) with $a = 1, \lambda \in \mathbb{C}, E_1 = F_1 = F_2 = I, E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then by Theorem 2, there are nontrivial solutions of (1.3) if the following matrix is singular:

$$S = \begin{bmatrix} I & I \\ g_{\lambda Q}(1) + g_{\lambda Q}^{(1)}(1) & f_{\lambda Q}(1) + g_{\lambda Q}(1) \end{bmatrix}.$$

In our case one gets

$$g_{\lambda Q}^{(1)}(1) = \lambda Q = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix}; \; g_{\lambda Q}(1) = I + \lambda Q/2 = \begin{bmatrix} 1 & \lambda/2 \\ 0 & 1 \end{bmatrix};$$

$$f_{\lambda Q}(1) = I + \lambda Q/3! = \begin{bmatrix} 1 & \lambda/6 \\ 0 & 1 \end{bmatrix}.$$

So the matrix S arising in (2.18) takes the form

$$S = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 3\lambda/2 & 2 & 2\lambda/3 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

An easy computation yields that the determinant of S is $\det(S) = 6 - 14\lambda$. Thus S is singular only for $\lambda = 3/7$. For this value of λ, the matrix S has in its kernel the vector $v = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix}$. If we consider the matrices $C_0 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$, then the condition (2.19) is verified. By Theorem 2, a nontrivial solution of the problem (1.3) is given by $X(t) = g_A(t)\, C_0 + f_A(t)\, C_1$, where $A = 3Q/7$. From the power series expansions of f_A and g_A, given by (2.1), and from $Q^2 = 0$ it follows that

$$X(t) = (I + At^2/2)\, C_0 + (It + At^3/3!)\, C_1 = \begin{bmatrix} 1 & 3t^2/14 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} t & t^3/14 \\ 0 & t \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 - t + 3t^2/7 - t^3/14 & 0 \\ 2 - t & 0 \end{bmatrix}.$$

It is a straightforward matter to verify that X satisfies the problem (1.3).

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Souhrn

EXPLICITNÍ ŘEŠENÍ OKRAJOVÝCH ÚLOH PŘÍBUZNÝCH OPERÁTOROVÉ ROVNICI $X^{(2)} - AX = 0$

LUCAS JÓDAR, ENRIQUE NAVARRO

V článku jsou vyšetřeny Cauchyovy okrajové a Sturm-Liouvillovy úlohy příbuzné operátorové rovnici $X^{(2)} - AX = 0$ v obecném případě, i když rovnice $X^2 - A = 0$ je neřešitelná. Jsou podány explicitní výrazy pro řešení v konečně dimensionálním případě, které lze použít k výpočtu řešení.

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