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VON KÁRMÁN EQUATIONS

III. SOLVABILITY OF THE VON KÁRMÁN EQUATIONS WITH CONDITIONS FOR GEOMETRY OF THE BOUNDARY OF THE DOMAIN

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Summary. Solvability of the general boundary value problem for von Kármán system of nonlinear equations is studied. The problem is reduced to an operator equation. It is shown that the corresponding functional of energy is coercive and weakly lower semicontinuous. Then the functional of energy attains absolute minimum which is a variational solution of the problem.

Keywords. Variational solution, Sobolev space, linear continuous functional, operator, curvature, property of coerciveness, weakly lower semicontinuous functional, absolute minimum.

AMS Classification. 35D05, 35G30, 73C50, 73K10.

1. INTRODUCTION

This paper is a free continuation of [1]. We deal with the following problem:

$$\Delta^2 w = [\Phi, w] + q \quad \text{in} \quad \Omega, \tag{1.1}$$

$$\Delta^2 \Phi = -[w, w] \quad \text{in} \quad \Omega, \tag{1.2}$$

$$w = w_n = 0 \quad \text{on} \quad \Gamma_1, \tag{1.3}$$

$$w = 0, \quad Mw + k_2 w_n = m_2 \quad \text{on} \quad \Gamma_2, \tag{1.4}$$

$$w_n = 0, \quad Tw + (w_x \Phi_{y\tau} - w_y \Phi_{x\tau}) + k_3 w = r_3 \quad \text{on} \quad \Gamma_3, \tag{1.5}$$

$$\Phi = \Phi_0, \quad \Phi_n = \Phi_1 \quad \text{on} \quad \delta\Omega, \tag{1.6}$$

where:

Ω denotes a bounded simple connected domain in E_2,

$$\Omega \in C^\infty,$$

$\delta\Omega$ denotes the boundary of Ω such that

$$\delta\Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 , \quad \Gamma_i \cap \Gamma_j = \emptyset \quad \text{for} \quad i, j = 1, 2, 3 , \quad i \neq j ,$$

$$u_x = \frac{\delta u}{\delta x} , \quad u_{xy} = \frac{\delta^2 u}{\delta x \, \delta y} , \quad \text{etc.} ,$$

$$\Delta^2 u = \Delta(\Delta u) = u_{xxxx} + 2u_{xxyy} + u_{yyyy} ,$$

$$[u, v] = u_{xx}v_{yy} + u_{yy}v_{xx} - 2u_{xy}v_{xy} ,$$

$n = (n_x, n_y)$ is the unit vector of the outer normal to $\delta\Omega$,

$\tau = (-n_y, n_x)$ is the unit tangent vector to $\delta\Omega$,

σ is the Poisson constant, $0 < \sigma < 1/2$,

$$Mu = \sigma\Delta u + (1 - \sigma) [u_{xx}n_x^2 + 2u_{xy}n_xn_y + u_{yy}n_y^2] ,$$

$$Tu = -(\Delta u)_n + (1 - \sigma) [u_{xx}n_xn_y - u_{xy}(n_x^2 - n_y^2) - u_{yy}n_xn_y]_\tau .$$

In [1] we have established the solvability of the von Kármán equations under the following conditions:

$$K\Phi_0 \geqq 0 \qquad \text{on} \quad \Gamma_2 ,$$

$$K\Phi_0 \geqq 0 , \quad \Phi_1 \geqq 0 \quad \text{on} \quad \Gamma_3 ,$$

where K is the curvature of the curve $\delta\Omega$. In this paper we will show sufficient conditions of solvability which depend on the geometry of the boundary $\delta\Omega$ and do not depend on the values of Φ_0 and Φ_1 on $\delta\Omega$.

Analogously to [1] we shall define a variational solution of the problem (1.1)–(1.6). We shall show that the corresponding functional of energy is coercive and weakly lower semicontinuous. We shall prove the coerciveness of the functional using the idea of Ciarlet-Rabier [4] with the assumption $\Gamma_2 = \Gamma_3 = \emptyset$. The existence of the solution for similar boundary conditions is proved in [6] using the idea of Knightly [7] but for other sufficient conditions. Furhter, in [6] the assumption $\Gamma_3 = \emptyset$ is introduced, but also a free edge with the condition $\Phi_0 = \Phi_1 = 0$ is searched.

2. VARIATIONAL SOLUTION OF THE PROBLEM

We assume (as in [1]):

(2.1) $\qquad k_2, m_2 \in L_p(\Gamma_2) ; \quad k_3, r_3 \in L_1(\Gamma_3) ; \quad q \in L_p(\Omega)$ for $p > 1$,

(2.2) $\qquad k_2 \geqq 0$ a.e. on Γ_2 , $\quad k_3 \geqq 0$ a.e. on Γ_3 ,

(2.3) $\qquad \Phi_0 \in W^{3/2,2}(\delta\Omega) , \quad \Phi_1 \in W^{1/2,2}(\delta\Omega)$.

We denote:

$$A(u, v) = \int_\Omega [u_{xx}(v_{xx} + \sigma v_{yy}) +$$
$$+ 2(1 - \sigma)\, u_{xy}v_{xy} + u_{yy}(v_{yy} + \sigma v_{xx})]\, dx\, dy\,,$$
$$a(u, v) = \int_{\Gamma_2} k_2 u_n v_n\, ds + \int_{\Gamma_3} k_3 uv\, ds\,,$$
$$p(u) = \int_{\Gamma_2} m_2 u_n\, ds + \int_{\Gamma_3} r_3 u\, ds\,,$$
$$(u, v)_{W_0^{2,2}} = \int_\Omega (u_{xx}v_{xx} + 2u_{xy}v_{xy} + u_{yy}v_{yy})\, dx\, dy\,,$$
$$B(u; v, z) = \int_\Omega [(u_{xy}v_y - u_{yy}v_x)\, z_x + (u_{xy}v_x - u_{xx}v_y)\, z_y]\, dx\, dy\,,$$
$$\mathscr{V} = \{u \in C^\infty(\bar{\Omega}) : u = u_n = 0 \text{ on } \Gamma_1,\ u = 0 \text{ on } \Gamma_2,\ u_n = 0 \text{ on } \Gamma_3\}\,.$$
$$V = \bar{\mathscr{V}}\,;$$

here $\bar{\mathscr{V}}$ is the closure in the topology of $W^{2,2}(\Omega)$.

From (2.3) and from $\Omega \in C^\infty$ we obtain the existence of a function $F \in W^{2,2}(\Omega)$ such that

$$F = \Phi_0\,, \quad F_n = \Phi_1 \quad \text{in the sense of traces on } \delta\Omega,$$
$$(F, \psi)_{W_0^{2,2}} = 0 \quad \text{for arbitrary} \quad \psi \in W_0^{2,2}(\Omega).$$

Let

$$f = \Phi - F\,.$$

Definition 2.1. *The variational solution of the problem* (1.1)–(1.6) *is a pair of functions* $(w, f) \in V \times W_0^{2,2}(\Omega)$ *with the following properties*:

$$A(w, \varphi) + a(w, \varphi) = B(F; w, \varphi) + B(f; w, \varphi) + p(\varphi) + \int_\Omega q\varphi\, dx\, dy\,, \tag{2.5}$$
$$(f, \psi)_{W_0^{2,2}} = -B(w; w, \psi) \tag{2.6}$$

for arbitrary $\varphi \in V$, $\psi \in W_0^{2,2}(\Omega)$.

On V we define an inner product

$$(u, v)_V = A(u, v) + a(u, v)\,. \tag{2.7}$$

Under some assumptions (see [5]) we can show that (2.7) is an inner product and the corresponding norm is equivalent to the norm of the Sobolev space $W^{2,2}(\Omega)$. For example, it is sufficient to assume

1° $\operatorname{mes}(\Gamma_1) > 0$ or

2° $\operatorname{mes}(\Gamma_2) > 0$ and Γ_2 is not a part of any straight line.

Using the Riesz representation theorem for linear continuous functionals on Hilbert spaces we get (see [1] or [5]) the existence of operators $L: V \to V$, $C_1: W_0^{2,2}(\Omega) \times V \to V$, $C_2: V \times V \to W_0^{2,2}(\Omega)$ and $q^* \in V$ such that

$$B(F; w, \varphi) = (Lw, \varphi)_V\,, \tag{2.8}$$
$$B(f; w, \varphi) = (C_1(f, w), \varphi)_V\,, \tag{2.9}$$
$$B(w; \bar{w}, \psi) = (C_2(w, \bar{w}), \psi)_{W_0^{2,2}}\,, \tag{2.10}$$
$$p(\varphi) + \int_\Omega q\varphi\, dx\, dy = (q^*, \varphi)_V \tag{2.11}$$

for arbitrary $w, \bar{w}, \varphi \in V$ and $f, \psi \in W_0^{2,2}(\Omega)$. Using (2.8)–(2.11) we get the equivalnce between equations (2.5), (2.6) and equations

$$(2.12) \qquad w = Lw + C_1(f, w) + q^* \quad \text{in} \quad V,$$

$$(2.13) \qquad f = -C_2(w, w) \quad \text{in} \quad W_0^{2,2}(\Omega).$$

If we put

$$Cw = C_1(C_2(w, w), w)$$

then we obtain from (2.13) and (2.12)

$$(2.14) \qquad w - Lw + Cw - q^* = 0 \quad \text{in} \quad V.$$

3. PROPERTIES OF OPERATORS OF THE PROBLEM

In [1] we have shown some properties of the operators L, C_1, C_2 and $[,]$.

P.1. *For an arbitrary sequence* $\{w^n\} \subset V$ *such that*

$$w^n \to w \quad \textit{weakly in} \quad V$$

we have

$$Lw^n \to Lw \quad \text{strongly in} \quad V,$$

$$C_2(w^n, w^n) \to C_2(w, w) \quad \text{strongly in} \quad W_0^{2,2}(\Omega).$$

(See Lemma 4.3 in [1].)

P.2. *The following implication takes place*:

$$C_2(w, w) = 0 \quad \text{in} \quad W_0^{2,2}(\Omega) \Rightarrow [w, w] = 0 \quad \text{in} \quad W^{-2,2}(\Omega).$$

(See Lemma 4.5 in [1].)

P.3. *For arbitrary* $w \in V$, $F \in W^{2,2}(\Omega)$ *the following formula takes place*

$$(3.1) \qquad \int_\Omega [(F_{xy}w_y - F_{yy}w_x)\, w_x + (F_{xy}w_x - F_{xx}w_y)\, w_y]\, dx\, dy =$$
$$= \int_\Omega [w, w]\, F\, dx\, dy - \int_{\Gamma_2} K_2\, F(w_n)^2\, ds - \int_{\Gamma_3} K_3\, F(w_\tau)^2\, ds -$$
$$- \int_{\Gamma_3} F_n(w_\tau)^2\, ds\,,$$

where K_i *denotes the curvature of the curve* Γ_i, $i = 1, 2, 3$.
(See (4.5) in [1] and the boundary condition for w.)

P.4. *The solutions of* (2.14) *are critical points of the fuctional* $J\colon V \to E_1$, *which is defined by the relation*

$$(3.2) \qquad J(w) = \tfrac{1}{2}\|w\|_V^2 - \tfrac{1}{2}(Lw, w)_V + \tfrac{1}{4}\|C_2(w, w)\|^2_{W_0^{2,2}} - (q^*, w)_V\,.$$

(See Lemma 5.1 in [1].)

Let K_i be the curvature of the curve Γ_i $(i = 1, 2, 3)$. In the next part of the paper we shall assume that least one of the following assumptions C.1 – C.6 is fulfilled.

C.1 (*Fig.* 1)

(3.3) $$K_2(x, y) \geqq 0 \quad \text{and} \quad K_3(x, y) > 0 \quad \text{on} \quad \delta\Omega .$$

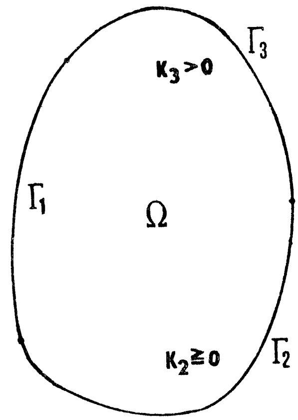

Fig. 1

C.2 (*Fig.* 2)

(3.4) $$K_2(x, y) \leqq 0 \quad \text{and} \quad K_3(x, y) < 0 \quad \text{on} \quad \delta\Omega .$$

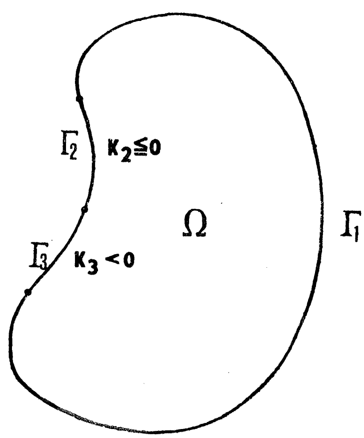

Fig. 2

C.3 (*Fig.* 3) *There exists a line with the equation* $ax + by + c = 0$ *such that*

(3.5) $$K_2(x, y) \,.\, (ax + by + c) \geqq 0 \quad \text{on} \quad \Gamma_2 ,$$

(3.6) $$K_3(x, y) \,.\, (ax + by + c) \geqq 0 \quad \text{on} \quad \Gamma_3 ,$$

(3.7) $$an_x + bn_y > 0 \quad \text{on} \quad \Gamma_3 ,$$

where $\bar{n} = (n_x, n_y)$ *is the unit vector of the normal to* $\delta\Omega$.

Remark. Let α be the angle of vectors $\bar{m} = (a, b)$ and $\bar{n} = (n_x, n_y)$. Then (3.7) has the form

$$(\bar{m}, \bar{n})_{E_2} = |\bar{m}| \cos \alpha > 0 ,$$

and we get

$$\alpha \in (-\pi/2, \pi/2) .$$

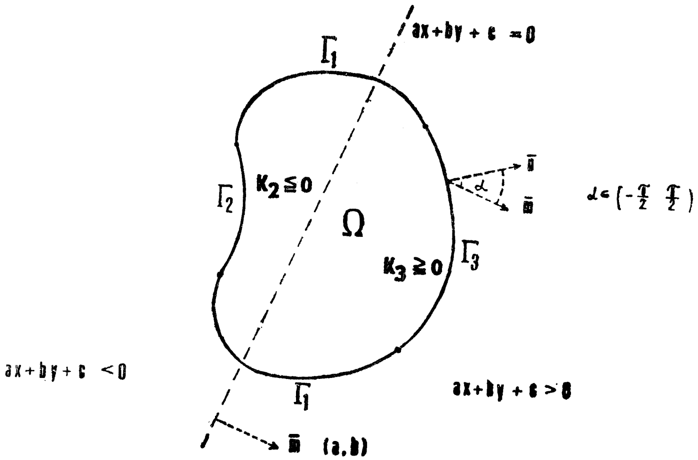

Fig. 3

C.4 (Fig. 4) $\Gamma_3 = \emptyset$ and there exist parallel lines with the equations $ax + by + c_1 = 0$, $ax + by + c_2 = 0$ such that

$$K_2(x, y) . (ax + by + c_1)(ax + by + c_2) \leqq 0 \quad \text{on} \quad \Gamma_2 . \tag{3.8}$$

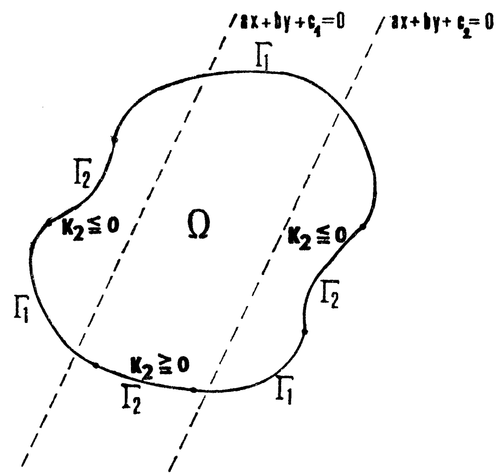

Fig. 4

C.5 (*Fig.* 5) $\Gamma_3 = \emptyset$ *and there exists an ellipse with the equation*

$$\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1$$

such that

$$K_2(x, y) \, . \left[\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} - 1 \right] \leqq 0 \quad \text{on} \quad \Gamma_2 \, . \tag{3.9}$$

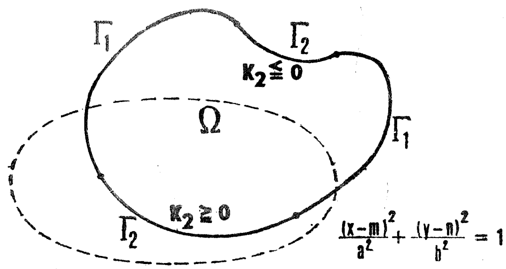

Fig. 5

C.6 (*Fig.* 6) $\Gamma_3 = \emptyset$ *and there exists a parabola with the equation*

$$(x - m)^2 = 2p(y - n)$$

such that

$$K_2(x, y) \, . \, [(x - m)^2 - 2p(y - n)] \leqq 0 \quad \text{on} \quad \Gamma_2 \, . \tag{3.10}$$

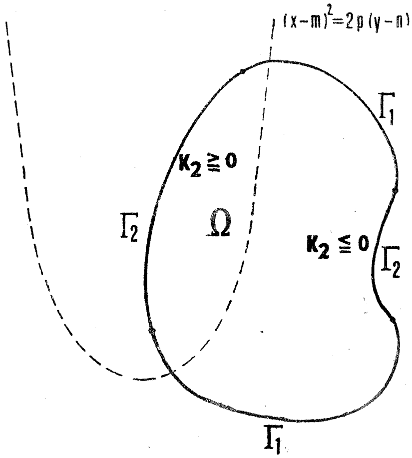

Fig. 6

Lemma 3.1. *Let at least one of the assumption* C.1 – C.6 *take place. Then for an arbitrary* $w \in V$:

$$[w, w] = 0 \Rightarrow w_x = w_y = 0 \quad \text{a.e. in} \quad \Omega . \tag{3.11}$$

Proof. Let C.1 or C.2 take place. In (3.1) we put

$$F = 1 \quad \text{in} \quad \bar{\Omega} ,$$

and we get

$$\int_{\Gamma_2} K_2(w_n)^2 \, ds + \int_{\Gamma_3} K_3(w_\tau)^2 \, ds = 0 .$$

Using the assumptions C.1 or C.2 we obtain

$$K_2(w_n)^2 = 0 \quad \text{on} \quad \Gamma_2 , \tag{3.12}$$

$$w_\tau = 0 \quad \text{on} \quad \Gamma_3 . \tag{3.13}$$

Now we put in (3.1)

$$F = \tfrac{1}{2}(x^2 + y^2) \quad \text{in} \quad \bar{\Omega} .$$

Using (3.12) and (3.13) we get

$$\int_\Omega [(w_x)^2 + (w_y)^2] \, dx \, dy = 0$$

and then

$$w_x = w_y = 0 \quad \text{in} \quad \Omega .$$

Let the assumption C.3 take place and let

$$F = ax + by + c \quad \text{in} \quad \bar{\Omega} .$$

Then (3.1) can be written in the form

$$\int_{\Gamma_2} K_2(ax + by + c)(w_n)^2 \, ds +$$
$$+ \int_{\Gamma_3} K_3(ax + by + c)(w_\tau)^2 \, ds + \int_{\Gamma_3} (an_x + bn_y)(w_\tau)^2 \, ds = 0 .$$

Now using the assumption C.3 we obtain

$$K_2(ax + by + c)(w_n)^2 = 0 \quad \text{on} \quad \Gamma_2 , \tag{3.14}$$

$$w_\tau = 0 \quad \text{on} \quad \Gamma_3 . \tag{3.15}$$

For arbitrary γ_2 such that $\gamma_2 \subset \Gamma_2$ and $\text{mes}(\gamma_2) > 0$ we have

$$ax + by + c = 0 \quad \text{a.e. on} \quad \gamma_2 \Rightarrow K_2 = 0 \quad \text{a.e. on} \quad \gamma_2 .$$

Then (3.14) yields

$$K_2(w_n)^2 = 0 \quad \text{on} \quad \Gamma_2 . \tag{3.16}$$

The rest of the proof is the same as in the case of assumptions C.1 or C.2. We put $F = \frac{1}{2}(x^2 + y^2)$ and use (3.15), (3.16).

Let the assumption C.4 take place and let

$$F = \tfrac{1}{2}(ax + by + c_1)(ax + by + c_2) \quad \text{in} \quad \bar{\Omega} .$$

Then we obtain from (3.1):

$$(3.17) \qquad \int_\Omega [b^2(w_x)^2 - 2abw_xw_y + a^2(w_y)^2] \, dx \, dy = \\ = \int_{\Gamma_2} K_2(ax + by + c_1)(ax + by + c_2)(w_n)^2 \, ds .$$

The left hand side of (3.17) is nonnegative. According to (3.8) the right hand side of (3.17) is nonpositive. Hence

$$(3.18) \qquad K_2(ax + by + c_1)(ax + by + c_2)(w_n)^2 = 0 \quad \text{on} \quad \Gamma_2 .$$

Similarly to the previous part of the proof from (3.18) we have

$$(3.19) \qquad K_2(w_n)^2 = 0 \quad \text{on} \quad \Gamma_2 .$$

If we put $F = \tfrac{1}{2}(x^2 + y^2)$ in (3.1) and use (3.19) we can complete the proof of this part in the same way as in the previous parts.

Let the assumption C.5 take place and let

$$F = \frac{(x - m)^2}{a^2} + \frac{(y - n)^2}{b^2} - 1 \quad \text{in} \quad \bar{\Omega} .$$

Then (3.1) has the form

$$(3.20) \qquad \int_\Omega \left[\frac{2}{b^2}(w_x)^2 + \frac{2}{a^2}(w_y)^2\right] dx \, dy = \\ = \int_{\Gamma_2} K_2 \left[\frac{(x - m)^2}{a^2} + \frac{(y - n)^2}{b^2} - 1\right] (w_n)^2 \, ds .$$

The left hand side of (3.20) is nonnegative. According to (3.9) the right hand side of (3.20) is nonpositive. Hence

$$\frac{2}{b^2}(w_x)^2 + \frac{2}{a^2}(w_y)^2 = 0 \quad \text{in} \quad \Omega$$

and

$$w_x = w_y = 0 \quad \text{in} \quad \Omega .$$

In the end let the assumption C.6 take place. Let

$$F = (x - m)^2 - 2p(y - n) \quad \text{in} \quad \bar{\Omega} .$$

Then we get from (3.1):

$$(3.21) \qquad 2 \int_\Omega (w_x)^2 \, dx \, dy = \int_{\Gamma_2} K_2[(x - m)^2 - 2p(y - n)] (w_n)^2 \, ds .$$

According to (3.10) we obtain from (3.21):

$$w_x = 0 \quad \text{in} \quad \Omega\,,$$

and this implies

$$w(x, y) = C(y) \quad \text{in} \quad \bar{\Omega}\,^{1)}\,.$$

Now we shall prove that $w(x, y) = 0$ in $\bar{\Omega}$. If it were not true, there would exist $(x_0, y_0) \in \Omega$ such that

(3.22) $$w(x_0, y_0) = C(y_0) \neq 0\,.$$

Ω is a bounded and simple connected domain in E_2, therefore there exists $(x_1, y_0) \in \in \delta\Omega$. But on the boundary $\delta\Omega$ we have $w = 0$ and hence

$$w(x_1, y_0) = C(y_0) = 0\,,$$

which is a contradiction with (3.22).

Remark. It is possible to formulate sufficient conditions C.5 and C.6 also for an ellipse and parabola with rotated axes.

4. EXISTENCE OF THE SOLUTION

In what follows we prove coerciveness of the functional J.

Lemma 4.1. *Let at least one of the assumptions* C.1 – C.6 *take place. Then*

(4.1) $$\lim_{R \to \infty} J(w) = \infty\,,$$

where $R = \|w\|_V$.

Proof. (The basic idea is in [1] and [4].) If the functional J were not coercive, there would exist a sequence $\{w^n\}$ in the space V and a constant $c > 0$ such that

(4.2) $$\lim_{n \to \infty} \|w^n\|_V = \infty$$

and

(4.3) $$\tfrac{1}{2}\|w^n\|_V^2 - \tfrac{1}{2}(Lw^n, w^n)_V + \tfrac{1}{4}\|C_2(w^n, w^n)\|_{W_0^{2,2}}^2 - (q^*, w^n)_V \leqq c$$

for arbitrary $n \in N$. According to (4.2) we can suppose $w^n \neq 0$. For all n we set

$$v^n = \frac{w^n}{\|w^n\|_V}$$

---

1) $W^{2,2}(\Omega) \subset C(\bar{\Omega})$ and therefore $w(x, y) \in C(\bar{\Omega})$.

for arbitrary $n \in N$. From (4.3) after dividing by $\|w^n\|_V^2$ and using homogeneity of the operators L and C_2 we obtain

$$(4.4) \qquad \tfrac{1}{2} - \tfrac{1}{2}(Lv^n, v^n)_V + \tfrac{1}{4}\|w^n\|_V^2 \, \|C_2(v^n, v^n)\|_{W_0^{2,2}}^2 \leqq \frac{c}{\|w^n\|_V^2} + \frac{1}{\|w^n\|_V} (q^*, v^n)_V \, .$$

Because $\|v^n\|_V = 1$, the sequence $\{v^n\}$ is bounded in the Hilbert space V. Then there exists a weakly convergent subsequence, which we denote again by $\{v^n\}$ for simplicity, and so

$$v^n \to v \quad \text{weakly in} \quad V \, .$$

With the help of property P.1 and the properties of the inner product we obtain

$$(4.5) \qquad (Lv^n, v^n)_V \to (Lv, v)_V \, ,$$

$$(4.6) \qquad \|C_2(v^n, v^n)\|_{W_0^{2,2}}^2 \to \|C_2(v, v)\|_{W_0^{2,2}}^2 \, ,$$

$$(4.7) \qquad (q^*, v^n)_V \to (q^*, v)_V \, .$$

(i) Let $\|C_2(v, v)\|_{W_0^{2,2}} > 0$. From (4.2) and (4.5)–(4.7) we have that the right hand side of the inequality (4.4) converges to zero while the left hand side diverges to ∞. But this is a contradiction.

(ii) Let $\|C_2(v, v)\|_{W_0^{2,2}} = 0$, i.e. $C_2(v, v) = 0$. According to the property P.2 we have $[v, v] = 0$ and then from Lemma 3.1 we obtain $v_x = v_y = 0$ a.e. in Ω. Now from (2.4) and (2.8) we get

$$(4.8) \qquad (Lv, v)_V = B(F; v, v) = 0 \, .$$

From (4.4) we have

$$(4.9) \qquad \tfrac{1}{2} - \tfrac{1}{2}(Lv^n, v^n)_V \leqq \frac{c}{\|w^n\|_V^2} + \frac{1}{\|w^n\|_V} (q^*, v^n)_V \, .$$

Using (4.2), (4.5), (4.7) and (4.8) we obtain that the left hand side of (4.9) converges to 1/2 while the right hand side converges to 0. And this is a contradiction.

We have just proved that the functional J defined by (3.2) is coercive in V. The functional J is also weakly lower semicontinuous in V (see Definition 5.1 and Lemma 5.2 in [1]). Now, using theorem from [3], we have the existence of at least one point of minima of the functional J. This point is a critical point of J. So we proved the following.

Theorem. *Let the assumptions* (2.1)–(2.3) *and at least one of the assumptions* C.1–C.6 *take place. Then there exists at least one solution of* (2.14), i.e., *there exists at least one variational solution of the problem* (1.1)–(1.6).

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Súhrn

ROVNICE VON KÁRMÁNA
III. RIEŠITEĽNOSŤ PRI PODMIENKACH NA GEOMETRIU HRANICE OBLASTI

JÚLIUS CIBULA

V článku sa skúma riešiteľnosť všeobecnej okrajovej úlohy pre von Kármánovu sústavu nelineárnych rovníc. Variačná formulácia úlohy je ekvivalentná istej operátorovej rovnici. Prislúchajúci funkcionál energie je koercívny a slabo zdola polospjitý. Na základe toho funkcionál nadobúda absolútne minimum aspoň v jednom bode, ktoré je variačným riešením úlohy.

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