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ON THE OPTIMAL CONTROL PROBLEM GOVERNED BY THE EQUATIONS OF VON KÁRMÁN III. THE CASE OF AN ARBITRARY LARGE PERPENDICULAR LOAD

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Summary. We shall deal with an optimal control problem for the system of von Kármán equations for the deflection of a thin elastic plate. We consider the perpendicular load on the plate as the control variable. In contrast to the papers [1], [2], arbitrarily large loads are admitted. As the unicity of a solution of the state equation is not guaranteed, we consider the cost functional defined on the set of admissible controls and states, and the state equation plays the role of the constraint. The existence of an optimal couple (i.e., control and state) is verified. By using Lagrange multipliers, some necessary optimality conditions are derived.

A control problem with the cost functional involving all possible solutions of the state equation for arbitrary perpendicular load-control is investigated in the last part. The optimal control problem is solved via a sequence of penalized optimal control problems.

Key words: optimal control, Kármán's equations, existence proof, conditions of optimality.
AMS Subject classification: 73K10, 73H05, 49A22, 49B22.

1. FORMULATION OF THE STATE PROBLEM

We consider the same state problem as in the paper [2]. Let Ω be a bounded simply connected region with the boundary $\Gamma = \bigcup_{j=1}^{l} S_j$, where S_j are simple smooth arcs and the angles of the tangents at the corners, if there are any, are positive.

Problem I. *To find functions* y, Φ *such that*

$$\Delta^2 y = [\Phi, y] + v\,, \tag{1.1}$$

$$\Delta^2 \Phi = -[y, y] \quad in\ \Omega\,, \quad where \quad [\varphi, \Phi] = \varphi_{11}\psi_{22} + \varphi_{22}\psi_{11} - 2\varphi_{12}\psi_{12}\,,$$

$$\varphi_{ij} = \frac{\partial^2 \varphi}{\partial x_i\, \partial x_j}\,, \quad i, j = 1, 2\,;$$

$$y = y_n = 0 \quad \text{on} \quad \Gamma_1\,, \tag{1.2}$$

$$y = M(y) + k_2 y_n = 0 \quad \text{on} \quad \Gamma_2\,,$$

$$M(y) + k_{31}y_n = T(y) + k_{32}y_n = 0 \quad \text{on} \quad \Gamma_3 ,$$

(1.3) $$\Phi = \varphi_0 , \quad \Phi_n = \varphi_1 \text{ on } \Gamma ,$$

(1.3') $$\Phi_{22}n_1 - \Phi_{12}n_2 = X , \quad \Phi_{11}n_2 - \Phi_{12}n_1 = Y \text{ on } \Gamma_3 ,$$

where

$$\Gamma = \bar{\Gamma}_1 \cup \bar{\Gamma}_2 \cup \bar{\Gamma}_3 , \quad \Gamma_i \cap \Gamma_j = \emptyset \quad \text{for} \quad i \neq j .$$

The data and operators from (1.2), (1.3), (1.3') are specified in the papers [2] or [3].

We introduce a weak solution of the problem (1.1)–(1.3) in the same way as in [3].

Let $L^2(\Omega)$ be the Hilbert space of all real measurable square integrable functions in the Lebesgue sense on Ω with the scalar product

(1.4) $$(u, v)_0 = \int_\Omega uv \, dx$$

and the norm

(1.5) $$|u|_0 = (u, u)_0^{1/2} .$$

We introduce the Sobolev space

$$H^2(\Omega) = \{u \mid u \in L^2(\Omega), \ D^\alpha u \in L^2(\Omega) \text{ for } |\alpha| \leqq 2\}$$

with derivatives

$$D^\alpha u = \frac{\partial^{|\alpha|} u}{\partial x_1^{\alpha_1} \, \partial x_2^{\alpha_2}} , \quad |\alpha| = \alpha_1 + \alpha_2$$

in the distributive sense. $H^2(\Omega)$ is the Hilbert space with the scalar product

(1.6) $$(u, v)_2 = \int_\Omega (uv + \sum_{|\alpha|=2} D^\alpha u D^\alpha v) \, dx$$

and the norm

(1.7) $$\|u\|_2 = (u, u)_2^{1/2} .$$

Let us set

$$\mathscr{V} = \{u \mid u \in C^\infty(\bar{\Omega}), \ u = u_n = 0 \text{ on } \Gamma_1, \ u = 0 \text{ on } \Gamma_2\}$$

and denote by $V = \bar{\mathscr{V}}$ the closure of $\mathscr{V}$ in the space $H^2(\Omega)$. Further we define two bilinear forms on $V \times V$:

$$A(u, v) = \int_\Omega [u_{11}v_{11} + 2(1 - \mu) \, u_{12}v_{12} + u_{22}v_{22} + \mu(u_{11}v_{22} + u_{22}v_{11})] \, dx ,$$

$$a(u, v) = \int_{\Gamma_2} k_2 u_n v_n \, ds + \int_{\Gamma_3} (k_{31}u_n v_n + k_{32}uv) \, ds ,$$

where $\mu \in \langle 0, \frac{1}{2})$ is the Poisson constant appearing in the boundary operators M, T (see [2]). If the partition of the boundary $\Gamma = \bar{\Gamma}_1 \cup \bar{\Gamma}_2 \cup \bar{\Gamma}_3$ satisfies some conditions ([3], Lemma 3.1), the bilinear form

$$((u, v)) = A(u, v) + a(u, v), \quad u, v \in V,$$

determines a scalar product on V with the associated norm $\|u\| = ((u, u))^{1/2}$, which is equivalent to the original norm $\|u\|_2$. Hence V is a Hilbert space with the scalar product $((u, v))$ and the norm $\|u\|$.

Further we introduce the space

$$H_0^2(\Omega) = \{u \mid u \in H^2(\Omega),\ u = u_n = 0 \text{ on } \Gamma \text{ in the sense of traces}\}.$$

$H_0^2(\Omega)$ is a Hilbert space with the scalar product

$$((u, v))_0 = \int_\Omega \Delta u \, \Delta v \, dx$$

and the norm

$$\|u\|_0 = ((u, u))_0^{1/2}.$$

Next, we define the trilinear form on $[H^2(\Omega)]^3$

$$B(u, v, w) = \int_\Omega [u_{12}(v_2 w_1 + v_1 w_2) - u_{22} v_1 w_1 - u_{11} v_2 w_2] \, dx.$$

Let $F \in H^2(\Omega)$ be a function fulfilling the relations

(1.8) $$((F, \psi))_0 = 0 \quad \text{for all} \quad \psi \in H_0^2(\Omega),$$

(1.9) $$F = \varphi_0, \quad F_n = \varphi_1 \quad \text{on} \quad \Gamma.$$

Setting $\Phi = F + f, f \in H_0^2(\Omega)$, we arrive at the following definition of a weak solution of Problem I.

Definition 1.1. *A couple* $[y, f] \in V \times H_0^2(\Omega)$ *is a reduced weak solution of Problem I, if*

(1.10) $$((y, \varphi)) = B(f, y, \varphi) + B(F, y, \varphi) + (v, \varphi)_0 \quad \textit{for all} \quad \varphi \in V,$$

(1.11) $$((f, \psi))_0 = -B(y, y, \psi) \quad \textit{for all} \quad \psi \in H_0^2(\Omega).$$

The system (1.10), (1.11) can be transformed to an operator equation in the space V. Let us define the following operators:

$M: L^2(\Omega) \to V$:

(1.12) $$((Mv, \varphi)) = (v, \varphi)_0 \quad \text{for all} \quad \varphi \in V,$$

$L: V \to V$:

(1.13) $$((Ly, \varphi)) = B(F, y, \varphi) \quad \text{for all} \quad \varphi \in V,$$

C_1: $H_0^2(\Omega) \times V \to V$:

$$((C_1(u, y), \varphi)) = B(u, y, \varphi) \quad \text{for all} \quad \varphi \in V, \tag{1.14}$$

C_2: $V \times V \to H_0^2(\Omega)$:

$$((C_2(y, w), \varphi))_0 = B(y, w, \varphi) \quad \text{for all} \quad \varphi \in H_0^2(\Omega), \tag{1.15}$$

C: $V \to V$:

$$C(y) = C_1(C_2(y, y), y). \tag{1.16}$$

The following lemma expresses some properties of the operators introduced above.

Lemma 1.1. (i) *The operator M is linear and compact*; (*ii*) *the operator L is linear, selfadjoint and compact*; (iii) *the operator C is compact.*

Proof. We shall verify only the compactness of M. All the other properties were proved in the papers [2], [3].

As the norm in the space V is equivalent to the original norm $\|\cdot\|_2$ in the space $H^2(\Omega)$, the imbeddings

$$V \Subset L^2(\Omega) \Subset V^*$$

hold, where V^* with the norm $|\cdot|_*$ is the dual space to V and the symbol $\Subset$ denotes a compact imbedding. Due to the Riesz theorem we obtain the relation

$$\|Mv\| = |v|_* \quad \text{for all} \quad v \in L^2(\Omega). \tag{1.17}$$

Let $v_n \rightharpoonup v$ (weakly) in $L^2(\Omega)$. The compactness of the imbedding $L^2(\Omega) \Subset V^*$ implies $v_n \to v$ (strongly) in V^* and $Mv_n \to Mv$ (strongly) in V. Consequently, the compactness of the operator M: $L^2(\Omega) \to V$ follows.

The system (1.10), (1.11) can be rewritten in the form of an equation in the space V:

$$y - Ly + C(y) = Mv. \tag{1.18}$$

A couple $[y, f]$ is a reduced weak solution of Problem I if and only if y is a solution of (1.18) and $f = -C_2(y, y)$.

The following theorem yields the existence of a solution of the equation (1.18).

Theorem 1.1. *Let $\gamma < 1$ be such that*

$$((Ly, y)) \leqq \gamma\|y\|^2 \quad \forall y \in V. \tag{1.19}$$

Then for arbitrary $v \in L^2(\Omega)$ there exists a solution $y \in V$ of the equation (1.18). *Moreover, the estimate*

$$\|y\| \leqq (1 - \gamma)^{-1} c_0 |v|_0 \tag{1.20}$$

holds, where c_0 is the constant from the inequality

$$|\varphi|_0 \leqq c_0\|\varphi\| \quad \forall \varphi \in V. \tag{1.21}$$

Proof. The existence of a solution $y \in V$ is verified in the papers [3], [5]. The estimate (1.20) results from the relations

$$((Cy, y)) = ((C_1(C_2(y, y), y), y)) = B(C_2(y, y), y, y) =$$
$$= B(y, y, C_2(y, y)) = \|C_2(y, y)\|^2 \geqq 0 \quad \forall y \in V$$

and the equation

$$\|Mv\| \leqq c_0 |v|_0 \quad \forall v \in V, \tag{1.22}$$

which is a consequence of the relation (1.17).

Remark 1.1. The possibilities of satisfying the condition (1.19) are discussed in the paper [3]. We can assume that the functions φ_0, φ_1 in (1.3) are sufficiently small, or that the form $B(F, y, y)$ is nonpositive for all $y \in V$. The latter case corresponds to some state of tension in the plate, determined by an Airy stress function F.

2. OPTIMAL CONTROL PROBLEM WITH THE STATE EQUATION IN THE FORM OF A CONSTRAINT

Let $U_{ad} \subset L^2(\Omega)$ be an arbitrary convex closed and bounded set of admissible controls v:

$$|v|_0 \leqq K \quad \forall v \in U_{ad}. \tag{2.1}$$

We introduce the cost functional $J: V \times U_{ad} \to \mathbb{R}$ of the form

$$J(y, v) = \mathscr{J}(y) + j(v), \quad y \in V, \quad v \in U_{ad}, \tag{2.2}$$

where $\mathscr{J}: V \to \mathbb{R}$, $j: L^2(\Omega) \to \mathbb{R}$ are some functionals.

We shall investigate the following

Optimal Control Problem P_1: to find a couple $(y_0, u) \in V \times U_{ad}$ such that

$$J(y_0, u) = \min_{(y,v) \in \mathscr{K}} J(y, v), \tag{2.3}$$

where

$$\mathscr{K} = \{(y, v) \mid (y, v) \in V \times U_{ad}, \ y - Ly + C(y) - Mv = 0\}. \tag{2.4}$$

Hence the canonical equation (1.18), equivalent to the original Problem (1.1)–(1.3), appears here as a constraint.

We formulate the existence theorem for Problem P_1.

Theorem 2.1. *If (1.19) holds and the functionals $\mathscr{J}$, j are weakly lower semi-continuous on V and $L^2(\Omega)$, respectively, then there exists a solution $(y_0, u) \in \mathscr{K}$ of Optimal Control Problem* P_1.

Proof. Let $\{(y_n, u_n)\} \subset \mathscr{K}$ be a minimizing sequence for the functional J, i.e.

$$\lim_{n\to\infty} J(y_n, u_n) = \inf_{(y,v)\in\mathscr{K}} J(y, v) . \tag{2.5}$$

The set U_{ad} is weakly compact and weakly closed in $L^2(\Omega)$, being bounded, closed and convex. Hence there exists a subsequence $\{u_m\}$ such that

$$u_m \rightharpoonup u \quad \text{(weakly) in } L^2(\Omega)\,, \quad u \in U_{ad} . \tag{2.6}$$

The corresponding sequence $\{y_m\}$ is bounded in V, due to the estimate (1.20), and there exists a subsequence $\{y_k\}$ such that

$$y_k \rightharpoonup y_0 \quad \text{(weakly) in } V, \quad y_0 \in V . \tag{2.7}$$

We have

$$y_k - Ly_k + C(y_k) = Mu_k . \tag{2.8}$$

The operators L, M, C are compact by virtue of Lemma 1.1. Passing to the limit in (2.8), we arrive at

$$y_0 - Ly_0 + C(y_0) = Mu , \tag{2.9}$$

so that $(y_0, u) \in \mathscr{K}$.

As the functionals $\mathscr{J}, j$ are weakly lower semicontinuous, we obtain

$$J(y_0, u) = \mathscr{J}(y_0) + j(u) \leqq \liminf_{k\to\infty} \mathscr{J}(y_k) + \liminf_{k\to\infty} j(u_k) \leqq$$
$$\leqq \liminf_{k\to\infty} J(y_k, u_k) = \inf_{(y,v)\in\mathscr{K}} J(y, v) .$$

Consequently, the couple (y_0, u) is a solution of Optimal Control Problem P_1.

3. NECESSARY CONDITIONS OF OPTIMALITY

Let us first recall the following theorem from the book [4] (Chapt. 1.1.3).

Theorem 3.1. (The extremal principle in smoothly convex problems.) *Let X, Y be Banach spaces, U an arbitrary set, $F: X \times U \to Y, f_i: X \times U \to \mathbb{R}$, $i = 0, 1, \ldots, n$ and*

$$\mathscr{U} \equiv \{(x, u) \mid (x, u) \in X \times U,\ F(x, u) = 0,\ f_i(x, u) \leqq 0,\ i = 1, \ldots, n\} . \tag{3.1}$$

Let $(x_*, u_*) \in \mathscr{U}$ be a couple satisfying the following conditions:
(i) *there exists a neighbourhood $W \subset X$ of x_*, such that*

$$f_0(x_*, u_*) = \min_{(x,u)\in(W\times U)\cap\mathscr{U}} f_0(x, u) ; \tag{3.2}$$

(ii) *the mappings $x \mapsto F(x, u)$ and the functionals $x \mapsto f_i(x, u)$, $i = 0, 1, \ldots, n$, are continuously Fréchet differentiable at the point x_* for each $u \in U$;*

(iii) *the mappings* $u \mapsto F(x, u)$ *and the functionals* $u \mapsto f_i(x, u)$, $i = 0, 1, \ldots, n$, *fulfil for each* $x \in W$ *the following conditions*:

for arbitrary $u_1, u_2 \in U$ *and* $\alpha \in \langle 0, 1 \rangle$ *there exists* $u \in U$ *such that*

$$F(x, u) = \alpha F(x, u_1) + (1 - \alpha) F(x, u_2), \tag{3.3}$$

$$f_i(x, u) \leqq \alpha f_i(x, u_1) + (1 - \alpha) f_i(x, u_2), \quad i = 0, 1, \ldots, n; \tag{3.4}$$

(iv) *the set* $\{y \mid y \in Y,\ y = F'_x(x_*, y_*)\, x,\ x \in X\}$ *is of finite codimension in* Y.

Then there exist Lagrange multipliers $\lambda_0 \geqq 0, \ldots, \lambda_n \geqq 0$, $y^* \in Y$ *not vanishing simultaneously and such that*

$$\mathscr{L}'_x(x_*, u_*, \lambda_0, \ldots, \lambda_n, y^*) = \sum_{i=0}^{n} \lambda_i f'_{ix}(x_*, u_*) + y_0^* F'_x(x_*, u_*) = 0, \tag{3.5}$$

$$\mathscr{L}(x_*, u_*, \lambda_0, \ldots, \lambda_n, y^*) = \min_{u \in U} \mathscr{L}(x_*, u, \lambda_0, \ldots, \lambda_n, y^*), \tag{3.6}$$

$$\lambda_i f_i(x_*, u_*) = 0, \quad i = 1, \ldots, n, \tag{3.7}$$

where $\mathscr{L}$ *is the Lagrange function of the form*

$$\mathscr{L}(x, u, \lambda_0, \ldots, \lambda_n, y^*) \equiv \sum_{i=0}^{n} \lambda_i f_i(x, u) + \langle y^*, F(x, u) \rangle \tag{3.8}$$

and $\langle y^* \circ F'_x(x_*, u_*), x \rangle = \langle y^*, F'_x(x_*, u_*)\, x \rangle$ *for all* $x \in X$.

If, moreover, the set

$$\{y \mid y \in Y,\ y = F'_x(x_*, u_*)\, x + F(x_*, u),\ (x, u) \in X \times U\} \tag{3.9}$$

contains a neighbourhood of zero in Y *and there exists a point* $(x_0, u_0) \in X \times U$ *such that*

$$F'_x(x_*, u_*)\, x_0 + F(x_*, u_0) = 0, \tag{3.10}$$

$$\langle f'_{ix}(x_*, u_*), x_0 \rangle + f_i(x_*, u_0) < 0 \tag{3.11}$$

for all $i > 0$ *such that* $f_i(x_*, u_*) = 0$, *then* $\lambda_0 \neq 0$ *and we can set* $\lambda_0 = 1$.

Using Theorem 3.1, we obtain

Theorem 3.2. (Necessary conditions of optimality.) *Let the estimate* (1.19) *hold and let the couple* $(y_0, u) \in V \times U_{\text{ad}}$ *be a solution of* ***Optimal Control Problem*** P_1 *with a convex functional* j *and continuously Fréchet differentiable functionals* $\mathscr{J}, j$. *Then there exist a number* $\lambda_0 \geqq 0$ *and an element* $z \in V$ *not vanishing simultaneously and such that*

$$[I - L + C'(y_0)]\, z = -\lambda_0 R\, \mathscr{J}'(y_0), \tag{3.12}$$

$$(\lambda_0 j'(u) - z, v - u)_0 \geqq 0 \quad \forall v \in U_{\text{ad}}, \tag{3.13}$$

$$y_0 - L y_0 + C(y_0) = Mu, \tag{3.14}$$

where
$$C'(y_0)\,z = 2C_1(C_2(y_0, z), y_0) + C_1(C_2(y_0, y_0), z)$$
and R: $V^* \to V$ is the Riesz representative operator.

If, moreover,

$$|u|_0 \leqq \alpha^{1/2}(1-\gamma)^{3/2}\, C_0^{-1}(\|C_1\|\,\|C_2\|)^{-1/2}\,, \tag{3.15}$$

where $\alpha \in (0, 1)$ is arbitrary, then $\lambda_0 \neq 0$ and we can set $\lambda_0 = 1$.

Proof. We shall verify the assumptions of Theorem 3.1.

We have $X = Y = V$ – a Hilbert space, $U \equiv U_{\text{ad}}$, $(x_*, u_*) \equiv (y_0, u)$, $\mathscr{F}(y, v) \equiv \equiv y - Ly + C(y) - Mv$, $(y, v) \in V \times U_{\text{ad}}$, $\mathscr{U} = \{(y, v) \mid (y, v) \in V \times U_{\text{ad}}, \mathscr{F}(y, v) = 0\}$, $f_0 = \mathscr{J}(y) + j(v)$.

The mapping $y \to \mathscr{F}(y, v)$ is continuously Fréchet differentiable at each point $y_* \in V$ and we have

$$\mathscr{F}'_y(y_*, v)\,y \equiv y - Ly + C'(y_*)\,y\,, \quad y \in V, \tag{3.16}$$

where

$$C'(y_*)\,y = 2C_1(C_2(y_*, y), y_*) + C_1(C_2(y_*, y_*), y)\,. \tag{3.17}$$

The differentiability results directly from the definition of the Fréchet derivative and of the expression $C(y) = C_1(C_2(y, y), y)$, $y \in V$ (see (1.16)). The continuity of the mapping $\eta \to C'(\eta) \in \mathscr{L}(V, V)$ is a consequence of the estimates (for details cf. [2])

$$\|C_1(\xi, y)\| \leqq \|C_1\|\,\|\xi\|_0\,\|y\| \quad \forall \xi \in H_0^2(\Omega)\,, \quad y \in V, \tag{3.18}$$

$$\|C_2(y, \eta)\|_0 \leqq \|C_2\|\,\|y\|\,\|\eta\| \quad \forall y, \quad \eta \in V. \tag{3.19}$$

The property (3.3) holds for arbitrary elements $u_1, u_2 \in U_{\text{ad}}$ and $\alpha \in (0, 1)$ if we take $u = \alpha u_1 + (1 - \alpha)\,u_2 \in U_{\text{ad}}$, since the set U_{ad} is convex and the operator M in the mapping $\mathscr{F}$ is linear.

The operator $\mathscr{F}'_y(y_*, v) \in \mathscr{L}(V, V)$ can be expressed in the form

$$\mathscr{F}'_y(y_*, v) = I + A\,, \tag{3.20}$$

where

$$A = -L + C'(y_*) \tag{3.21}$$

is a linear compact operator, because L is linear compact due to Lemma 1.1 and the compactness of $C'(y_*)$ results from the compactness of the operator C (see [7], Th. 4.7). Moreover, A is selfadjoint due to Lemma 1.1 and to the form of the operator $C'(y_*)$ (for details we refer to [2], Lemma 5.1). According to the theory of equations with linear compact operators ([6], Chapt. VI, § 2) we have the identity $R(I + A) = = N(I + A)^{\perp}$, where $R(I + A)$ is the range of the operator $I + A$ and $N(I + A)^{\perp}$

is the orthogonal complement of the set $N(I + A) = \{y \mid y \in V,\ (I + A)\, y = 0\}$. The space $N(I + A)$ is finite-dimensional and hence the space $R(I + A)$ has a finite codimension which is equal to the dimension of the space $N(I + A)$. Hence the assumption (iv) of Theorem 3.1 is satisfied, too.

Hence there exist Lagrange multipliers $\lambda_0 \geqq 0$ and $z^* \in V^*$ such that

$$\lambda_0\, \mathscr{J}'(y_0) + z^* \circ [I - L + C'(y_0)] = 0\,, \tag{3.22}$$

$$\begin{aligned} &\lambda_0[\mathscr{J}(y_0) + j(u)] + \langle z^*, F(y_0, u)\rangle = \\ &= \min_{v \in U_{ad}} \{\lambda_0[\mathscr{J}(y_0) + j(v)] + \langle z^*, F(y_0, v)\rangle\}\,. \end{aligned} \tag{3.23}$$

Let R: $V^* \to V$ be the Riesz operator and $Rz^* = z$. Then we have

$$\langle z^*, \eta\rangle = ((Rz^*, \eta)) = ((z, \eta)) \quad \forall \eta \in V\,. \tag{3.24}$$

Rewriting (3.22), we obtain the relations

$$\begin{aligned} 0 &= \langle \lambda_0\, \mathscr{J}'(y_0), \eta\rangle + \langle z^*, [I - L + C'(y_0)]\, \eta\rangle = \\ &= ((\lambda_0 R\, \mathscr{J}'(y_0), \eta)) + ((z, [I - L + C'(y_0)]\, \eta)) = \\ &= ((\lambda_0 R\, \mathscr{J}'(y_0) + [I - L + C'(y_0)]\, z, \eta)) \quad \forall \eta \in V\,, \end{aligned}$$

which imply (3.12) immediately.

Using the convexity of the set U_{ad}, we obtain from (3.23) the inequality

$$(\lambda_0[\mathscr{J}(y_0) + j(v)]'_{v=u_0} + \langle z^*, F(y_0, v)\rangle'_{v=u_0}, v - u)_0 \geqq 0 \quad \forall v \in U_{ad}$$

and, further,

$$\begin{aligned} &0 \leqq (\lambda_0\, j'(u), v - u)_0 - \langle z^*, M(v - u)\rangle = \\ &= (\lambda_0\, j'(u), v - u)_0 - ((z, M(v - u))) = (\lambda_0\, j'(u) - z, v - u)_0 \quad \forall u \in U_{ad}\,, \end{aligned}$$

which yields the inequality (3.13). The state equation (3.14) completes the necessary conditions of optimality.

If (3.15) holds, then due to (1.20) and Theorem 1.1, we arrive at the estimate

$$\|y_0\|^2 \leqq \alpha(1 - \gamma)\, \|C_1\|^{-1}\, \|C_2\|^{-1}\,. \tag{3.25}$$

For arbitrary $y \in V$ we may write

$$\begin{aligned} &((F'_y(y_0, u)\, y, y)) = ((y - Ly + C'(y_0)\, y, y)) = \\ &= \|y\|^2 - ((Ly, y)) + 2((C_1(C_2(y_0, y), y_0), y)) + \\ &+ ((C_1(C_2(y_0, y_0), y), y)) = \|y\|^2 - ((Ly, y)) + \\ &+ 2\|C_2(y_0, y)\|_0^2 + ((C_1(C_2(y_0, y_0), y), y)) \geqq \\ &\geqq (1 - \gamma - \|C_1\|\, \|C_2\|\, \|y_0\|^2)\, \|y\|^2 \geqq (1 - \alpha)(1 - \gamma)\, \|y\|^2 = m\|y\|^2\,, \\ &m = (1 - \alpha)(1 - \gamma) > 0\,. \end{aligned} \tag{3.26}$$

Let $y \in V$, $v \in U_{ad}$. Then we have

$$\mathscr{F}'_y(y_0, u)\, y + \mathscr{F}(y_0, v) = \mathscr{F}'_y(y_0, u)\, y + y_0 - Ly_0 + C(y_0) - Mv =$$
$$= \mathscr{F}'_y(y_0, u)\, y + Mu - Mv\,.$$

Using again the theory of equations with linear compact operators, we conclude that for every $z \in V$ there exists an element $y \in V$ such that

$$\mathscr{F}'_y(y_0, u)\, y + \mathscr{F}(y_0, v) = z\,,$$

and

$$\mathscr{F}'_y(y_0, u)\, 0 + \mathscr{F}(y_0, u) = 0\,.$$

Hence all the assumptions of Theorem 3.1, ensuring $\lambda_0 \neq 0$, are fulfilled and the proof is complete. Q.E.D.

We can show another application of Theorem 3.1 for a particular set U_{ad}.

Theorem 3.3. *Let the couple* $(y_0, u) \in V \times U_{ad}$ *be a solution of Problem* P_1, *where* $U_{ad} = \{v \mid v \in L^2(\Omega),\ |v|_0 \leqq K\}$, *the functionals* $\mathscr{J}$, j *are continuously differentiable in the sense of Fréchet and the functional* j *is convex. Then there exist numbers* $\lambda_0 \geqq 0$, $\lambda_1 \geqq 0$ *and an element* $z \in V$, *not vanishing simultaneously and such that*

$$[I - L + C'(y_0)]\, z = -\lambda_0 R\, \mathscr{J}'(y_0)\,, \tag{3.27}$$

$$\lambda_0\, j(u) + \lambda_1 u - z = 0\,, \tag{3.28}$$

$$\lambda_1(|u|_0 - K) = 0\,, \tag{3.29}$$

$$y_0 - Ly_0 + C(y_0) = Mu\,. \tag{3.30}$$

If, moreover, (3.15) *holds, then* $\lambda_0 \neq 0$ *and we can put* $\lambda_0 = 1$.

Proof. It suffices to verify the assumptions of Theorem 3.1. We put $X = Y = V$, $U = L^2(\Omega)$, $\mathscr{F}(y, v) = y - Ly + C(y) - Mv$, $f_0(y, v) = \mathscr{J}(y) + j(v)$, $f_1(y, v) = \frac{1}{2}(|v|^2 - K^2)$,

$$\mathscr{U} = \{(y, v) \mid (y, v) \in V \times L^2(\Omega),\ \mathscr{F}(y, v) = 0,\ f_1(y, v) \leqq 0\}\,.$$

The equation (3.28) is a necessary condition for the minimum of the differentiable function $v \to \mathscr{L}(y_0, v, \lambda_0, \lambda_1, z)$,

$$\mathscr{L}(y_0, v, \lambda_0, \lambda_1, z) = \lambda_0(\mathscr{J}(y_0) + j(v)) + \tfrac{1}{2}\lambda_1(|v|_0^2 - K^2) + ((z, F(y_0, v)))\,,$$

at the point u on the whole space $L^2(\Omega)$. The relation (3.29) corresponds to the condition (3.7) with $i = 1$.

4. AN OPTIMAL CONTROL PROBLEM WITH A COST FUNCTIONAL INVOLVING ALL SOLUTIONS OF THE STATE EQUATION

We again consider nonempty bounded closed and convex set $U_{ad} \subset L^2(\Omega)$ of admissible controls v, which fulfil the condition

$$(4.1) \qquad |v|_0 \leqq K \quad \forall v \in U_{ad} .$$

Let $\mathscr{F}: V \times U_{ad} \to V$ be the state operator of the form

$$(4.2) \qquad \mathscr{F}(y, v) \equiv y - Ly + C(y) - Mv ,$$

where the operator $L: V \to V$ satisfies the estimate (1.19). We introduce the cost functional of the form

$$(4.3) \qquad J(v) \equiv \sup_{\substack{y \in V \\ \mathscr{F}(y,v)=0}} [\mathscr{J}(y) + j(v)] , \quad v \in U_{ad} ,$$

where $\mathscr{J}: V \to \mathbb{R}$, $j: L^2(\Omega) \to \mathbb{R}$ are given functionals. The functional $J: U_{ad} \to \mathbb{R}$ is defined correctly, as follows from Theorem 1.1 on the existence of solution.

Next, let us define

Optimal Control Problem P_2: to find $u \in U_{ad}$ such that

$$(4.4) \qquad J(u) = \min_{v \in U_{ad}} J(v) .$$

To solve this problem, we shall use the method of penalizations. If $v \in U_{ad}$, then due to Theorem 1.1 every solution $y \in V$ of the equation $\mathscr{F}(y, v) = 0$ fulfils the estimate

$$(4.5) \qquad \|y\| \leqq r , \quad r = (1 - \gamma)^{-1} C_0 K .$$

The functional J can be expressed in the form

$$(4.6) \qquad J(v) = \sup_{\substack{y \in V_r \\ \mathscr{F}(y,v)=0}} \{\mathscr{J}(y) + j(v)\} , \quad v \in U_{ad} ,$$

where $V_r = \{y \mid y \in V, \ \|y\| \leqq r\}$.

J can be also written in the form

$$(4.7) \qquad J(v) = \sup_{y \in V_r} \{\mathscr{J}(y) + j(v) - \beta(y, v)\} , \quad v \in U_{ad} ,$$

where $\beta: V_r \times U_{ad} \to \mathbb{R} \cup \{+\infty\}$ is defined as follows:

$$(4.8) \qquad \beta(y, v) = \begin{cases} 0 , & \text{if } \mathscr{F}(y, v) = 0 \\ +\infty , & \text{if } \mathscr{F}(y, v) \neq 0 . \end{cases}$$

For arbitrary $\varepsilon > 0$ let us consider the functional $J_\varepsilon: U_{ad} \to \mathbb{R}$ of the form

$$(4.9) \qquad J_\varepsilon(v) = \sup_{y \in V_r} \left[\mathscr{J}(y) + j(v) - \frac{1}{\varepsilon} \|\mathscr{F}(y, v)\|\right] , \quad v \in U_{ad} .$$

First we shall solve some penalized Problem P_ε with the functional J_ε instead of J. We verify the existence of a solution u_ε of Problem P_ε. Further, we show the existence of a sequence $\{u_{\varepsilon_n}\}$ weakly convergent to a solution $u \in U_{ad}$ of some modified Optimal Control Problem P_2'.

Lemma 4.1. *Let $\mathscr{J}: V \to \mathbb{R}$ be a weakly continuous functional. Then for each $v \in U_{ad}$ there exists an element $y_v^\varepsilon \in V_r$ such that*

$$J_\varepsilon(v) = \mathscr{J}(y_v^\varepsilon) + j(v) - \frac{1}{\varepsilon} \|\mathscr{F}(y_v^\varepsilon, v)\| . \tag{4.10}$$

Proof. The functional $\tilde{J}_{\varepsilon,v}: V_r \to \mathbb{R}$ defined by

$$\tilde{J}_{\varepsilon,v}(y) = \mathscr{J}(y) + j(v) - \frac{1}{\varepsilon} \|\mathscr{F}(y, v)\| , \quad y \in V_r \tag{4.11}$$

is upper bounded for every $\varepsilon > 0$, $v \in U_{ad}$. Let $\{y_n^{v,\varepsilon}\}_{n=1}^\infty \subset V_r$ be a maximizing sequence for $\tilde{J}_{\varepsilon,v}$ on the set V_r, i.e.

$$\lim_{n\to\infty} \tilde{J}_{\varepsilon,v}(y_n^{v,\varepsilon}) = \sup_{y\in V_r} \tilde{J}_{\varepsilon,v}(y) . \tag{4.12}$$

The sequence $\{y_n^{v,\varepsilon}\}$ is bounded in the Hilbert space V, so that we can extract a subsequence $\{y_m^{v,\varepsilon}\}$ such that

$$y_m^{v,\varepsilon} \rightharpoonup y_v^\varepsilon \quad \text{(weakly) in} \quad V: y_v^\varepsilon \in V_r . \tag{4.13}$$

We have used the fact that the set V_r is closed and convex and hence weakly closed in V. By virtue of the properties of the operators L, C we have

$$\mathscr{F}(y_m^{v,\varepsilon}, v) \rightharpoonup \mathscr{F}(y_v^\varepsilon, v) \quad \text{(weakly) in} \quad V , \tag{4.14}$$

$$\|\mathscr{F}(y_v^\varepsilon, v)\| \leqq \liminf \|\mathscr{F}(y_m^{v,\varepsilon}, v)\| . \tag{4.15}$$

The relations (4.13), (4.15) and the weak continuity of $\mathscr{J}$ imply

$$\tilde{J}_{\varepsilon,v}(y_v^\varepsilon) = \mathscr{J}(y_v^\varepsilon) + j(v) - \frac{1}{\varepsilon} \|\mathscr{F}(y_v, v)\| \geqq$$

$$\geqq \limsup_{m\to\infty} \left[\mathscr{J}(y_m^{v,\varepsilon}) + j(v) - \frac{1}{\varepsilon} \|\mathscr{F}(y_m^{v,\varepsilon}, v)\| \right] = \lim_{m\to\infty} \tilde{J}_{\varepsilon,v}(y_m^{v,\varepsilon}) = \sup_{y\in V_r} \tilde{J}_{\varepsilon,v}(y) .$$

Consequently, we may write

$$J_\varepsilon(v) = \sup_{y\in V_r} \tilde{J}_{\varepsilon,v}(y) = \tilde{J}_{\varepsilon,v}(y_v^\varepsilon) , \tag{4.16}$$

which is equivalent to (4.10).

Next we introduce

Extremal Problem P_ε: to find $u_\varepsilon \in U_{ad}$ such that

$$J_\varepsilon(u_\varepsilon) = \min_{v\in U_{ad}} J_\varepsilon(v) . \tag{4.17}$$

Lemma 4.2. *Let $\mathscr{J}: V \to \mathbb{R}$ be weakly continuous and let $j: L^2(\Omega) \to \mathbb{R}$ be a weakly lower semicontinuous functional. Then there exists a solution $u_\varepsilon \in U_{ad}$ of Extremal Problem* P_ε.

Proof. Let $\{u_n^\varepsilon\} \subset U_{ad}$ be a minimizing sequence for J_ε, i.e.

$$\lim_{n\to\infty} J_\varepsilon(u_n) = \inf_{v\in U_{ad}} J_\varepsilon(v)\,. \tag{4.18}$$

Since the set U_{ad} is bounded and weakly closed, there exists a subsequence $\{u_m^\varepsilon\} \subset U_{ad}$ such that

$$u_m^\varepsilon \rightharpoonup u_\varepsilon \quad \text{(weakly) in } L^2(\Omega)\,, \quad u_\varepsilon \in U_{ad}\,. \tag{4.19}$$

The operator $v \to \mathscr{F}(y, v)$, $v \in U_{ad}$, is compact due to Lemma 1.1 and hence

$$\lim_{m\to\infty} \mathscr{F}(y, u_m^\varepsilon) = \mathscr{F}(y, u_\varepsilon) \quad \forall y \in V. \tag{4.20}$$

Using Lemma 4.1 and the properties of the functionals $\mathscr{J}, j$ we arrive at the relations

$$\begin{aligned}
J_\varepsilon(u_\varepsilon) &= \sup_{y\in V_r}\left[\mathscr{J}(y) + j(u_\varepsilon) - \frac{1}{\varepsilon}\,\|\mathscr{F}(y, u_\varepsilon)\|\right] = \\
&= \mathscr{J}(y_{u_\varepsilon}^\varepsilon) + j(u_\varepsilon) - \frac{1}{\varepsilon}\,\|\mathscr{F}(y_{u_\varepsilon}^\varepsilon, u_\varepsilon)\| \leqq \\
&\leqq \liminf_{m\to\infty}\left[\mathscr{J}(y_{u_\varepsilon}^\varepsilon) + j(u_m^\varepsilon) - \frac{1}{\varepsilon}\,\|\mathscr{F}(y_{u_\varepsilon}^\varepsilon, u_\varepsilon)\|\right] \leqq \\
&\leqq \liminf_{m\to\infty}\sup_{y\in V_r}\left[\mathscr{J}(y) + j(u_m^\varepsilon) - \frac{1}{\varepsilon}\,\|\mathscr{F}(y, u_m^\varepsilon)\|\right] = \\
&= \liminf_{m\to\infty} J_\varepsilon(u_m^\varepsilon) = \inf_{v\in U_{ad}} J_\varepsilon(v)\,. \qquad \text{Q.E.D.}
\end{aligned}$$

Let us now formulate a modified optimal control problem.

Optimal Control Problem P_2': to find a control $u \in U_{ad}$ with a nonempty set $M_u \subset V$ such that

$$\mathscr{F}(z, u) = 0\,, \tag{4.21}$$

$$\mathscr{J}(z) + j(u) \leqq \sup_{\substack{y\in V\\ \mathscr{F}(y,v)=0}} [\mathscr{J}(y) + j(v)] \tag{4.22}$$

for all $z \in M_u$ and all $v \in U_{ad}$.

The main result of this chapter is represented by the following existence theorem.

Theorem 4.1. *Let (1.19) hold, let the functional $\mathscr{J}: V \to \mathbb{R}$ be lower bounded and weakly continuous and let the functional $j: L^2(\Omega) \to \mathbb{R}$ be lower bounded and weakly lower semicontinuous. Then there exists a solution $u \in U_{ad}$ of Optimal Control Problem P_2'. If $\lim_{n\to\infty} \varepsilon_n = 0$, $\varepsilon_n > 0$, then every sequence $\{u_{\varepsilon_n}\} \subset U_{ad}$ of* *solutions of Problem* P_{ε_n} *contains a subsequence* $\{u_{\varepsilon_m}\}$ *such that*

$$(4.23) \qquad u_{\varepsilon_m} \rightharpoonup \bar{u} \quad \text{(weakly) } in \quad L^2(\Omega)\,,$$

where $\bar{u} \in U_{ad}$ *is a solution of Problem* P'_2.

Proof. Let $\{\varepsilon_n\}$ be a sequence of positive numbers such that

$$(4.24) \qquad \lim_{n \to \infty} \varepsilon_n = 0\,.$$

We denote $u_n = u_{\varepsilon_n}$, $n = 1, 2, \ldots$, where $u_{\varepsilon_n} \in U_{ad}$ is a solution of Extremal Problem P_{ε_n}. There exists a subsequence $\{u_m\} \subset U_{ad}$ such that

$$(4.25) \qquad u_m \rightharpoonup u \quad \text{(weakly) in} \quad L^2(\Omega)\,, \quad u \in U_{ad}\,.$$

Let $y_{u_m} \in V_r$ be any solution of the equation

$$(4.26) \qquad \mathscr{F}(y_{u_m}, u_m) = 0\,, \quad m = 1, 2, \ldots.$$

The existence of y_{u_m} results from Theorem 1.1. As the set V_r is convex, closed and bounded and the operators L, C, M appearing in the expression (4.2) are compact, there exists a subsequence $\{y_{u_k}\}$ such that

$$(4.27) \qquad y_{u_k} \to y_u \quad \text{(strongly) in} \quad V,$$

where

$$(4.28) \qquad \mathscr{F}(y_u, u) = 0\,.$$

Since the functional $\mathscr{J}$ and j are weakly continuous and weakly lower semicontinuous, respectively, considering (4.25), (4.26), (4.27) and the character of the elements $u_k = u_{\varepsilon_k}$, $k = 1, 2, \ldots$, we are led to the inequalities

$$(4.29) \qquad \begin{aligned} \mathscr{J}(y_u) + j(u) &\leqq \liminf_{k\to\infty} \left[\mathscr{J}(y_{u_k}) + j(u_k)\right] \leqq \\ &\leqq \liminf_{k\to\infty} \sup_{y \in V_r} \left[\mathscr{J}(y) + j(u_k) - \frac{1}{\varepsilon_k} \|\mathscr{F}(y, u_k)\|\right] \leqq \\ &\leqq \liminf_{k\to\infty} \sup_{y \in V_r} \left[\mathscr{J}(y) + j(v) - \frac{1}{\varepsilon_k} \|\mathscr{F}(y, v)\|\right] = \\ &= \liminf_{k\to\infty} \left[\mathscr{J}(y_v^k) + j(v) - \frac{1}{\varepsilon_k} \|\mathscr{F}(y_v^k, v)\|\right] \leqq \\ &\leqq \liminf_{k\to\infty} \left[\mathscr{J}(y_v^k) + j(v)\right] \quad \text{for all} \quad v \in U_{ad}\,, \end{aligned}$$

where $y_v^k = y_v^{\varepsilon_k}$ and the element $y_v^{\varepsilon_k}$ is defined via Lemma 4.1.

There exists a subsequence $\{y_v^s\} \subset V_r$ such that

$$(4.30) \qquad y_v^s \rightharpoonup \bar{y}_v \quad \text{(weakly) in} \quad V, \quad \bar{y}_v \in V_r\,.$$

Let $y_v \in V_r$ satisfy the equation $\mathscr{F}(y_v, v) = 0$. As the funtionals $\mathscr{J}$, j are lower bounded, there exists a constant $c \in \mathbb{R}$, such that

$$c \leqq \mathscr{J}(y_v) + j(v) \leqq \mathscr{J}(y_v^s) + j(v) - \frac{1}{\varepsilon_s} \|\mathscr{F}(y_v^s, v)\| , \quad s = 1, 2, \ldots .$$

Consequently,

$$\|\mathscr{F}(y_v^s, v)\| \leqq \varepsilon_s \, c(v) , \quad c(v) \in \mathbb{R} , \tag{4.31}$$

where we have used (4.30) and the weak continuity of $\mathscr{J}$. Since the operators L, C are compact, we have

$$\mathscr{F}(y_v^s, v) \rightharpoonup \mathscr{F}(\bar{y}_v, v) \quad \text{(weakly) in } V$$

and (4.31) implies

$$\mathscr{F}(\bar{y}_v, v) = 0 \tag{4.32}$$

due to the inequality

$$\|\mathscr{F}(\bar{y}_v, v)\| \leqq \liminf_{s \to \infty} \|\mathscr{F}(y_v^s, v)\| = 0 .$$

Weak continuity of the functional $\mathscr{J}$ and the relations (4.29), (4.30), (4.31) imply the relations

$$\begin{aligned} \mathscr{J}(y_u) + j(u) &\leqq \liminf_{s \to \infty} [\mathscr{J}(y_v^s) + j(v)] = \\ &= \mathscr{J}(\bar{y}_v) + j(v) \leqq \sup_{\substack{y \in V_r \\ \mathscr{F}(y,v)=0}} [\mathscr{J}(y) + j(v)] . \end{aligned} \tag{4.33}$$

Since y_u is a solution of the state equation $\mathscr{F}(y, u) = 0$ and v is an arbitrary element from U_{ad} we see that $y_u \in M_u$ and u is a solution of Optimal Control Problem P_2', which completes the proof.

Remark 4.1. The nonempty set M_u contains all *stable* solutions y_u of the equation $\mathscr{F}(y, u) = 0$, i.e. the solutions satisfying the condition:

$u_n \rightharpoonup u$ (weakly) in $L_2(\Omega)$ implies the existence of $y_{u_n} \in V$, $n = 1, 2, \ldots$, $\mathscr{F}(y_{u_n}, u_n) = 0$, $y_{u_n} \to y_u$ in V.

Remark 4.2. Instead of the penalized functional J_ε we can consider in (4.9) the functional

$$J_\varepsilon^g(v) = \sup_{y \in V_r} \left[\mathscr{J}(y) + j(v) - \frac{1}{\varepsilon} g(\mathscr{F}(v, y)) \right] , \tag{4.35}$$

where $g \colon V \to \mathbb{R}$ is weakly lower semicontinuous, continuous on V and satisfies the conditions

$$g(0) = 0 , \quad g(y) > 0 \quad \text{for all} \quad y \in V , \quad y \neq 0 . \tag{4.36}$$

Conditions for weak lower continuity of a functional g are described in the book [8], (Chapt. III, § 8). One of such conditions is, for instance, the differentiability of g and the monotonicity of its Gâteaux derivative. Then we can put

$$g(y) = \|y\|^2 , \quad y \in V . \tag{4.37}$$

Remark 4.3. Due to the compact imbeddings of the Sobolev space $H^2(\Omega)$, the following functionals are weakly continuous:

$$\mathscr{I}_0(y) = |y - z_0|_0^2 , \quad z_0 \in L^2(\Omega) , \tag{4.38}$$

$$\mathscr{I}_1(y) = \|y - z_1\|_{H^1}^2 , \quad z_1 \in H^1(\Omega) , \tag{4.39}$$

$$\mathscr{I}_2(y) = \sup_{x \in \bar{\Omega}} |y(x) - z_2(x)| , \quad z_2 \in C(\bar{\Omega}) , \quad y \in V . \tag{4.40}$$

References

[1] *I. Bock, I. Hlaváček, J. Lovíšek:* On the optimal control problem governed by the equations of von Kárman, I. The homogeneous Dirichlet boundary conditions. Aplikace mat. 29 (1984), 303—314.
[2] *I. Bock, I. Hlaváček, J. Lovíšek:* On the optimal control problem governed by the equations of von Kármán. II. Mixed boundary conditions. Aplikace mat. 30 (1985), 375—392.
[3] *I. Hlaváček, J. Neumann:* Inhomogeneous boundary value problems for the von Kármán equations. I. Aplikace mat. 19 (1974), 253—269.
[4] *A. D. Joffe, V. M. Tichomirov:* The theory of extremal problems (in Russian). Moskva, Nauka 1974.
[5] *O. John, J. Nečas:* On the solvability of von Kármán equations. Aplikace mat. 20 (1975), 48—62.
[6] *L. A. Ljusternik, V. I. Sobolev:* The elements of functional analysis (in Russian). Moskva, Nauka 1965.
[7] *M. M. Vajnberg:* Variational methods of investigating nonlinear operators (in Russian). Moskva, Gostechizdat 1956.
[8] *M. M. Vajnberg:* Variational method and a method of monotone operators (in Russian). Moskva, Nauka 1972.

Souhrn

O PROBLÉMU OPTIMÁLNÍHO ŘEŠENÍ PRO KÁRMÁNOVY ROVNICE III. PŘÍPAD LIBOVOLNĚ VELKÉHO PŘÍČNÉHO ZATÍŽENÍ

IGOR BOCK, IVAN HLAVÁČEK, JÁN LOVÍŠEK

Je studována úloha řízení systému nelineárních Kármánových rovnic pro tenkou desku prostřednictvím pravé strany rovnice rovnováhy. Na okraji desky se uvažují kombinované podmínky. Na rozdíl od částí I a II této práce připouští se zde libovolně velké příčné zatížení, takže není zaručena jednoznačnost řešení. Pro dva typy účelového funkcionálu se dokazuje existence řešení optimalizační úlohy, v prvém případě jsou odvozeny též nutné podmínky optimality.

Резюме

ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ СИСТЕМОЙ УРАВНЕНИЙ КАРМАНА III. СЛУЧАЙ ПРОИЗВОЛЬНО БОЛЬШОЙ ПРАВОЙ ЧАСТИ УРАВНЕНИЯ РАВНОВЕСИЯ

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Рассматривается управление системой нелинейных уравнений Кармана для тонкой упругой плиты посредством правой части уравнения равновесия. На границе предполагаются смешанные краевые условия. В отличие от частей I и II этой работы допускается здесь произвольно большая поперечная нагрузка, так что не следует однозначность решения задачи состояния. Для двух типов целевой функции доказывается существование оптимального решения и в первом случае выведены тоже необходимые условия оптимальности.

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