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ON TOTAL TRUNCATION ERROR ESTIMATION FOR THE ONE-STEP METHOD

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Summary. In this paper the author establishes estimation of the total truncation error after s steps in the fifth order Ruge-Kutta-Huťa formula for systems of differential equations. The approach is analogous to that used by Vejvoda for the estimation of the classical formulas of the Runge-Kutta type of the 4-th order.

Keywords: Differential equations-Numerical solution, Runge-Kutta method, fifth order, Error analysis.

AMS Classification: 65L05, 65G99.

1. INTRODUCTION

In this paper we state estimates of the total truncation error after s steps in the Runge-Kutta-Huťa formula of the 5-th order [3] (briefly, the formula RKH5) for systems of differential equations.

Our approach is analogous to that used in [10] for the estimation of the classical formulas of the Runge-Kutta type of the 4-th order (briefly, RK4).

If we want to estimate the total truncation error of the approximate solution for the initial problem, first we need to know a local error estimation after the first step. The local error estimation was discussed in detail in [8], [9]. In Section 2 we only briefly mention some results, which we use in the subsequent text. In Section 3 we give a theoretical estimate of the error after s steps of RKH5.

2. A LOCAL ERROR ESTIMATION FOR THE SYSTEM OF DIFFERENTIAL EQUATIONS

Given the first order system of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y) \tag{2.1}$$

with the initial condition

$$y(x_0) = y_0 , \tag{2.2}$$

where $\boldsymbol{y}$, $\boldsymbol{y}_0$ and $\boldsymbol{f}$ are vectors with n components.

Let the components of the vector $\boldsymbol{f}(x, \boldsymbol{y})$ be continuous functions together with their partial derivatives up to the 6-th order in the region Q, where $Q \equiv \langle x_0, X \rangle * * \langle \boldsymbol{y}_0 - b \rangle$; $b > 0, X > x_0$. Then the functions $f_i(x, \boldsymbol{y})$ fulfil the Lipschitz condition in the region Q, that is,

$$|f_i(x, \bar{\boldsymbol{y}}) - f_i(x, \boldsymbol{y})| \leqq K \sum_{j=1}^{n} |\bar{y}_j - y_j| \quad \text{for} \quad i = 1, 2, \ldots, n . \tag{2.3}$$

In the case that the problem (2.1), (2.2) is solved by a formula of the 5-th order the following approach can be used: The interval $\langle x_0, X \rangle$ is divided into N equal subintervals with the dividing points $x_s = x_0 + sh$ where $s = 1, 2, \ldots, N - 1$ and $h = (X - x_0)/N$, $X = x_N$. For $s = r$ and $s = r + 1$ we have the points x_r, x_{r+1}. As an approximate solution of (2.1), (2.2) we can take a continuous curve $\bar{\boldsymbol{y}}(x)$ defined on the interval $\langle x_r, x_{r+1} \rangle$ by

$$\bar{\boldsymbol{y}}_{r+1}(x) = \bar{\boldsymbol{y}}_r(x_r) + (x - x_r) \sum_{t=1}^{6} p_t {}^t\boldsymbol{k}_{r+1} \tag{2.4}$$

where

$${}^t\boldsymbol{k}_{r+1} = \boldsymbol{f}(x_r + \alpha_t(x - x_r), \bar{\boldsymbol{y}}_r(x_r) + \alpha_t {}^{(t-1)}\boldsymbol{R}_{r+1}(x)) ,$$

$${}^{(t-1)}\boldsymbol{R}_{r+1}(x) = (x - x_r) \sum_{j=1}^{t-1} \beta_{jt} {}^j\boldsymbol{k}_{r+1}, \quad {}^0\boldsymbol{R}_{r+1}(x) = 0. \tag{2.5}$$

In our case we have coefficients p_t, α_t in Table 1, and coefficients β_{jt} in Table 2.

Table 1

t	1	2	3	4	5	6
p_t	$\frac{7}{90}$	0	$\frac{32}{90}$	$\frac{12}{90}$	$\frac{32}{90}$	$\frac{7}{90}$
α_t	0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

Table 2

j	t					
	1	2	3	4	5	6
1		1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{4}{7}$
2			$\frac{3}{4}$	$-\frac{3}{2}$	0	$\frac{3}{7}$
3				$\frac{4}{2}$	0	$\frac{12}{7}$
4					$\frac{3}{4}$	$-\frac{12}{7}$
5						$\frac{8}{7}$

Definition 1. *The local error of the approximate solution of* (2.1) *and* (2.2) *on the interval* $\langle x_r, x_{r+1} \rangle$ *is*

$$\omega_{n,r+1} = \max_{\xi \in \langle x_r, x_{r+1} \rangle} \sum_{i=1}^{n} |\bar{y}_{i,r+1}(\xi) - Y_{i,r+1}(\xi)| \tag{2.6}$$

for $r = 0, 1, \ldots, N - 1$, *where* $\bar{y}_{i,r+1}$ *is the i-th component of* $\bar{y}_{r+1}$ (*see* (2.4)), *and* $Y_{i,r+1}$ *is i-th component of* Y_{r+1}.

The integral Y_{r+1} is a local exact solution on the interval $\langle x_r, x_{r+1} \rangle$. The solution Y_{r+1} satisfies the system (2.1) and passes through the point $(x_r, \bar{y}_r(x_r))$, cf. [8], [10].

The local error (2.6) can be written in the form

$$\omega_{n,r+1} = \max_{\xi \in \langle x_r, x_{r+1} \rangle} \sum_{i=1}^{n} \left| A_i(\xi - x_r)^6 + B_i \frac{(\xi - x_r)^7}{7!} \right| \tag{2.7}$$

where A_i and B_i are the coefficients at $(\xi - x_r)^6$ and

$$\frac{(\xi - x_r)^7}{7!}$$

which are described by the formula (18) in [8]. The expression (2.7) *is a theoretical formula* for the local error. Since (2.7) is not suitable for computing the error, we have to content ourselves with its estimation only.

Let there exist such constants L and M that for the functions $f_i(x, y)$ and their derivatives the inequalities (cf. [5], [10], [9])

$$|f_i| \leqq M, \quad \left| \frac{\partial^{l+j_1+\ldots+j_n} f_i}{\partial x^l \, \partial y_1^{j_1} \ldots \partial y_n^{j_n}} \right| \leqq \frac{L^{l+j_1+j_2+\ldots j_n}}{M^{j_1+j_2+\ldots+j_n-1}} \tag{2.8}$$

hold for each $i = 1, 2, \ldots, n$.

In this place it should be noted that there exist some estimations based on different assumptions, see [9], [10]. However, in the sequel our effort will be concentrated on the assumption (2.8) whith is generally well-known in the literature.

The estimation of (2.7) is very complicated and for this reason, auxiliary expressions $V(n, hL)$ and $H(n, hL)$ are introduced. (In [9], $V(n, hL)$ is $VR0$ and $H(n, hL)$ is HCn.) The computations of the auxiliary expressions were executed in fractional arithmetic by a computer. The correctness of the program was verified by comparing the results with lower order methods well-known from literature.

Theorem 1. *Let the components of the vector* $f(x, y)$ *be continuous functions together with their partial derivatives up to the 6-th order in the region Q and let the assumption* (2.8) *be fulfilled. Then for the formula* (2.7) *we have the estimation*

$$\omega_{n,r+1} \leqq C_n = \frac{(hL)^5 \, hM}{3\,628\,000} \left[P_6(n) + V(n, hL) \right] \tag{2.9}$$

where $P_6(n)$ *is a polynomial of the sixth order and* $V(n, hL)$ *is a higher degree polynomial of two variables n and* hL (*hL is the product of h and L*).

We omit the proof of this theorem here. The proof can be seen from the results of the error estimation theoretical formula (2.7). The detailed description of the derivation of (2.9) is found in [9]. For $n = j = 1, 2, 3, 4$, (2.10) $$\omega_{j,r+1} \leqq C_j = \frac{(hL)^5\, hM}{3\,628\,800}\left[a_j + V_j(hL)\right] = hM\left[H_j(hL)\right],$$

where a_j are constants, $V_j(hL) = \left[V(n, hL)\right]_{n=j}$ are polynomials of the 25-th degree of one variable hL; $H_j(hL) = \left[H(n, hL)\right]_{n=j}$ are polynomials of one variable hL.

Some round-off values of $H_j(hL)$ for some of the values of the variable hL are in Table 3. (For details, see [9].)

Table 3

hL	$H_1(hL)$	$H_2(hL)$	$H_3(hL)$	$H_4(hL)$
0·01	0·238*D*-09	0·191*D*-07	0·283*D*-06	0·201*D*-05
0·02	0·160*D*-07	0·135*D*-05	0·209*D*-04	0·156*D*-03
0·03	0·193*D*-06	0·169*D*-04	0·275*D*-03	0·214*D*-02
0·04	0·114*D*-05	0·105*D*-03	0·178*D*-02	0·145*D*-01
0·05	0·459*D*-05	0·442*D*-03	0·783*D*-02	0·665*D*-01
0·10	0·383*D*-03	0·461*D*-01	0·101*D* 01	0·107*D* 02
0·20	0·416*D*-01	0·769*D* 01	0·257*D* 03	0·402*D* 04

3. THE ESTIMATION OF THE ERROR AFTER S STEPS

Let the problem (2.1), (2.2) be solved on the interval $\langle x_0, x_s\rangle$ by the formulas (2.4) where $s \leqq N$. Let the constants L and M from the assumption (2.8) and the Lipschitz constant K from (2.3) be such that the corresponding inequalities are valid on the whole Q. The reader can find the details in [10].

Let $\boldsymbol{Y}_{r+1}$ be the local exact solution of (2.1) initialized at the point $(x_r, \bar{\boldsymbol{y}}_r(x_r))$ for every interval $\langle x_r, x_{r+1}\rangle$.

The error on the interval $\langle \alpha, \beta\rangle \subset \langle x_0, x_N\rangle$ will be understood to be the value

$$\varphi_n(\alpha, \beta) = \max_{\xi \in \langle \alpha, \beta\rangle} \sum_{i=1}^{n} \left|y_i(\xi) - \bar{y}_i(\xi)\right|.$$

As $y_i(x_0) = Y_{i,1}(x_0)$ $(i = 1, 2, \ldots, n)$, the error on the interval $\langle x_0, x_1\rangle$ satisfies

(3.1a) $$\varphi_n(x_0, x_1) = \max_{\xi \in \langle x_0, x_1\rangle} \sum_{i=1}^{n} \left|\bar{y}_{i,1}(\xi) - Y_{i,1}(\xi)\right|.$$

Analogously, for $r = 1, 2, \ldots, N - 1$ we can define the expression

(3.1b) $$\omega_{n,r+1} = \max_{\xi \in \langle x_r, x_{r+1}\rangle} \sum_{i=1}^{n} \left|\bar{y}_{i,r+1}(\xi) - Y_{i,r+1}(\xi)\right|.$$

The expression (3.1a) is a special case of the local error given in (2.6) for $r = 0$. The relations (3.1b) are special cases of the local error (2.6) for $r = 1, 2, \ldots, N - 1$.

Let y_{r+1} be a vector function which fulfils $y_{r+1}(x) = y(x)$ on $\langle x_r, x_{r+1}\rangle$, where $y(x)$ is the exact solution of (2.1), (2.2).

Let $\zeta_{r+1}(x)$ be a vector defined by

$$\zeta_{r+1}(x) = y_r(x_r) + (x - x_r)\sum_{t=1}^{6} p_t\,{}^t\varkappa_{r+1}\,, \tag{3.2}$$

where

$$ {}^t\varkappa_{r+1} = f(x_r + \alpha_t(x - x_r),\ y_r(x_r) + \alpha_t{}^{(t-1)}\,\varrho_{r+1}(x))\,, \tag{3.3}$$

$$ {}^{(0)}\varrho_{r+1}(x) = 0\,,$$

$$ {}^{(t-1)}\varrho_{r+1}(x) = (x - x_r)\sum_{j=1}^{t-1}\beta_{jt}\,{}^j\varkappa_{r+1}\,; \quad \text{for} \quad t = 2, 3, \ldots, 6$$

and p_t, β_{jt} are given in Tables 1 and 2.

Definition 2. *The total truncation error on the interval $\langle x_0, x_s\rangle$ is given by the relation*

$$\varphi_n(x_0, x_s) = \max_{\xi\in\langle x_0, x_s\rangle}\sum_{i=1}^{n}|y_i(\xi) - \bar{y}_i(\xi)|\,.$$

It can be shown that

$$\varphi_n(x_0, x_s) = \max_{\{r=0,1,\ldots,s-1\}}\varphi_n(x_r, x_{r+1})\,, \tag{3.4}$$

where

$$\varphi_n(x_r, x_{r+1}) = \max_{\xi\in\langle x_r, x_{r+1}\rangle}\sum_{i=1}^{n}|y_{i,r+1}(\xi) - \bar{y}_{i,r+1}(\xi)|\,. \tag{3.5}$$

We shall estimate $\varphi_n(x_0, x_s)$ in terms of the estimates for $\varphi_n(x_r, x_{r+1})$. It is obvious that

$$\sum_{i=1}^{n}|y_{i,r+1}(x) - \bar{y}_{i,r+1}(x)| \leqq \tag{3.6}$$

$$\leqq \sum_{i=1}^{n}|y_{i,r+1}(x) - \zeta_{i,r+1}(x)| + \sum_{i=1}^{n}|\zeta_{i,r+1}(x) - \bar{y}_{i,r+1}(x)|\,.$$

The estimation for one step is done for any point of Q. The first term on the right hand side can be estimated by (2.9):

$$\max_{\xi\in\langle x_r, x_{r+1}\rangle}\sum_{i=1}^{n}|y_{i,r+1}(\xi) + \zeta_{i,r+1}(\xi)| \leqq C_n\,. \tag{3.7}$$

From (2.5), (3.3) and the Lipschitz condition we get

$$|{}^1k_{i,r+1} - {}^1\varkappa_{i,r+1}| \leqq K\sum_{i=1}^{n}|y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,, \tag{3.8}$$

$$|^2k_{i,r+1} - {}^2\varkappa_{i,r+1}| \leqq K(1 + \tfrac{1}{6}nhK) \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,,$$

$$|^3k_{i,r+1} - {}^3\varkappa_{i,r+1}| \leqq K[1 + \tfrac{1}{4}nhK + \tfrac{1}{32}(nhK)^2] \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,,$$

$$|^4k_{i,r+1} - {}^4\varkappa_{i,r+1}| \leqq K[1 + 2nhK + \tfrac{3}{8}(nhK)^2 +$$

$$+ \tfrac{1}{32}(nhK)^3] \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,,$$

$$|^5k_{i,r+1} - {}^5\varkappa_{i,r+1}| \leqq K[1 + \tfrac{3}{4}nhK + \tfrac{9}{8}(nhK)^2 +$$

$$+ \tfrac{27}{128}(nhK)^3 + \tfrac{9}{512}(nhK)^4] \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,,$$

$$|^6k_{i,r+1} - {}^6\varkappa_{i,r+1}| \leqq K[1 + \tfrac{39}{7}nhK + \tfrac{67}{14}(nhK)^2 + \tfrac{111}{56}(nhK)^3 +$$

$$+ \tfrac{33}{112}(nhK)^4 + \tfrac{9}{448}(nhK)^5] \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)|\,,$$

where K is the Lipschitz constant.

The relations (3.2), (2.4) and (3.8) yield

$$\text{(3.9)} \qquad \max_{\xi \in \langle x_r, x_{r+1} \rangle} \sum_{i=1}^{n} |\zeta_{i,r+1}(\xi) - \bar{y}_{i,r+1}(\xi)| \leqq [1 + nhK + \tfrac{19}{18}(nhK)^2 +$$

$$+ \tfrac{5}{6}(nhK)^3 + \tfrac{7}{30}(nhK)^4 + \tfrac{7}{240}(nhK)^5 + \tfrac{1}{640}(nhK)^6] \sum_{i=1}^{n} |y_{i,r}(x_r) - \bar{y}_{i,r}(x_r)| \leqq$$

$$\leqq q(nhK) \max_{\xi \in \langle x_r, x_{r+1} \rangle} \sum_{i=1}^{n} |y_{i,r}(\xi) - \bar{y}_{i,r}(\xi)|\,,$$

where $q(nhK)$ is the expression in brackets on the right hand side of the inequality.

Then (3.5) can be estimated by virtue of (3.6), (3.9) nad (3.7) in the following way:

$$\varphi_n(x_r, x_{r+1}) \leqq q(nhK)\, \varphi_n(x_{r-1}, x_r) + C_n\,.$$

This inequality together with (3.4) implies

$$\text{(3.10)} \qquad \varphi_n(x_0, x_s) \leqq C_n \frac{[q(nhK)]^s - 1}{q(nhK) - 1} = C_n\, \Phi_{n,s}(hK)\,.$$

The coefficient $\Phi_{n,s}(hK)$ expresses how much the error increases on the interval $\langle x_0, x_s \rangle$ as compared with the error on the interval $\langle x_0, x_1 \rangle$.

Another approach for the error estimation on the interval $\langle x_0, x_s \rangle$ involving a theorem from the theory of differential equations was shown in [10]:

$$\text{(3.11)} \qquad \varphi_n(x_0, x_s) \leqq C_n \frac{\mathrm{e}^{snhK} - 1}{\mathrm{e}^{nhK} - 1} = C_n\, \Psi_{n,s}(hK)\,.$$

In Table 4 we give some round-off values $\Phi_{n,s}(hK)$ for $n = 1, 2$. In Table 5 we give some round-off values of $\Psi_{n,s}(hK)$ for $n = 1, 2$.

For an error estimation after s steps we have for each type of our formulas two functions Φ and Ψ, which represent the increase of the error estimation on the interval $\langle x_0, x_s \rangle$ as compared with the error estimation on the interval $\langle x_0, x_1 \rangle$.

In [10] it is shown that in the case of $RK4$ the error estimation by the function Φ is better than that by the function Ψ.

A better estimation for Nyström's [6] formulas RKN5 can be obtained by using the function Ψ [4]. A similar situation occurs also in the case of RKH5 formulas.

Table 4 Values of function Φ for $n = 1$

s	$hK = 0.01$	$hK = 0.02$	$hK = 0.04$	$hK = 0.10$
2	0·201 011*D* 01	0·202 043*D* 01	0·204 174*D* 01	0·211 141*D* 01
3	0·303 042*D* 01	0·306 170*D* 01	0·312 697*D* 01	0·334 665*D* 01
4	0·406 105*D* 01	0·412 425*D* 01	0·425 750*D* 01	0·471 951*D* 01
5	0·510 209*D* 01	0·520 851*D* 01	0·543 522*D* 01	0·624 532*D* 01
10	0·104 673*D* 02	0·109 712*D* 02	0·121 036*D* 02	0·168 362*D* 02
20	0·220 418*D* 02	0·244 014*D* 02	0·303 224*D* 02	0·652 530*D* 02
30	0·348 408*D* 02	0·408 417*D* 02	0·577 460*D* 02	0·204 488*D* 03
40	0·489 937*D* 02	0·609 668*D* 02	0·990 250*D* 02	0·604 896*D* 03
50	0·646 439*D* 02	0·856 025*D* 02	0·161 160*D* 03	0·175 637*D* 04
60	0·819 496*D* 02	0·115 760*D* 03	0·254 687*D* 03	0·506 774*D* 04
70	0·101 086*D* 03	0·152 676*D* 03	0·395 469*D* 03	0·145 905*D* 05
80	0·122 247*D* 03	0·197 867*D* 03	0·607 378*D* 03	0·419 755*D* 05
90	0·145 646*D* 03	0·253 186*D* 03	0·926 352*D* 03	0·120 728*D* 06
100	0·171 521*D* 03	0·320 904*D* 03	0·140 648*D* 04	0·347 203*D* 06

Values of function Φ for $n = 2$

s	$hK = 0.01$	$hK = 0.02$	$hK = 0.04$	$hK = 0.10$
2	0·202 043*D* 01	0·204 174*D* 01	0·208 719*D* 01	0·224 927*D* 01
3	0·306 170*D* 01	0·312 697*D* 01	0·326 918*D* 01	0·380 995*D* 01
4	0·412 425*D* 01	0·425 750*D* 01	0·455 422*D* 01	0·575 966*D* 01
5	0·520 851*D* 01	0·543 522*D* 01	0·595 132*D* 01	0·819 539*D* 01
10	0·109 712*D* 02	0·121 036*D* 02	0·149 908*D* 02	0·331 329*D* 02
20	0·244 014*D* 02	0·303 224*D* 02	0·495 757*D* 02	0·339 914*D* 03
30	0·408 417*D* 02	0·577 460*D* 02	0·129 366*D* 03	0·318 043*D* 04
40	0·609 668*D* 02	0·990 250*D* 02	0·313 448*D* 03	0·294 811*D* 05
50	0·856 025*D* 02	0·161 160*D* 03	0·738 138*D* 03	0·273 002*D* 06
60	0·115 760*D* 03	0·254 687*D* 03	0·171 793*D* 04	0·252 778*D* 07
70	0·152 676*D* 03	0·395 469*D* 03	0·397 839*D* 04	0·234 050*D* 08
80	0·197 867*D* 03	0·607 378*D* 03	0·919 345*D* 04	0·216 710*D* 09
90	0·253 186*D* 03	0·926 352*D* 03	0·212 250*D* 05	0·200 654*D* 10
100	0·320 904*D* 03	0·140 648*D* 04	0·489 826*D* 05	0·185 787*D* 11

A little better error estimation on the interval $\langle x_0, x_s \rangle$ can be reached by using the function Ψ (3.11) instead of the function Φ (3.10).

It is generally well known that even in the case of using the Runge-Kutta methods for one differential equation it is very difficult to obtain suitable results for the estimation of the accumulated error [2], [7]. In spite of the unfavourable forecasts the Runge-Kutta methods are used. It is necessary to have some estimates available for the results obtained.. If for some reasons we require the estimate to be guaranteed then it is usually paid for by a relatively pessimistic estimation.

Table 5 Values of function Ψ for $n = 1$

s	$hK = 0{\cdot}01$	$hK = 0{\cdot}02$	$hK = 0{\cdot}04$	$hK = 0{\cdot}1$
2	0·201 005*D* 01	0·202 020*D* 01	0·204 081*D* 01	0·210 517*D* 01
3	0·303 025*D* 01	0·306 101*D* 01	0·312 410*D* 01	0·332 657*D* 01
4	0·406 071*D* 01	0·412 285*D* 01	0·425 159*D* 01	0·467 643*D* 01
5	0·510 152*D* 01	0·520 614*D* 01	0·542 511*D* 01	0·616 826*D* 01
10	0·104 646*D* 02	0·109 598*D* 02	0·120 513*D* 02	0·163 380*D* 02
20	0·220 298*D* 02	0·243 461*D* 02	0·300 298*D* 02	0.607 493*D* 02
30	0·348 112*D* 02	0·406 963*D* 02	0·568 506*D* 02	0·181 472*D* 03
40	0·489 370*D* 02	0·606 663*D* 02	0·968 625*D* 02	0·509 629*D* 03
50	0·645 483*D* 02	0·850 578*D* 02	0·156 553*D* 03	0·140 165*D* 04
60	0·818 015*D* 02	0·114 850*D* 03	0·245 601*D* 03	0·382 643*D* 04
70	0·100 869*D* 03	0·151 237*D* 03	0·378 445*D* 03	0·104 176*D* 05
80	0·121 942*D* 03	0·195 682*D* 03	0·576 625*D* 03	0·283 344*D* 05
90	0·145 232*D* 03	0·249 966*D* 03	0·872 275*D* 03	0·770 373*D* 05
100	0·170 970*D* 03	0·316 269*D* 03	0·131 333*D* 04	0·209 425*D* 06

Values of function Ψ for $n = 2$

s	$hK = 0{\cdot}01$	$hK = 0{\cdot}02$	$hK = 0{\cdot}04$	$hK = 0{\cdot}1$
2	0·202 020*D* 01	0·204 081*D* 01	0·208 329*D* 01	0·222 140*D* 01
3	0·306 101*D* 01	0·312 410*D* 01	0·325 680*D* 01	0·371 323*D* 01
4	0·412 285*D* 01	0·425 159*D* 01	0·452 805*D* 01	0·553 535*D* 01
5	0·520 614*D* 01	0·542 511*D* 01	0·590 517*D* 01	0·776 089*D* 01
10	0·109 598*D* 02	0·120 513*D* 02	0·147 147*D* 02	0·288 572*D* 02
20	0·243 461*D* 02	0·300 298*D* 02	0·474 627*D* 02	0·242 084*D* 03
30	0·406 963*D* 02	0·568 506*D* 02	0·120 345*D* 03	0·181 763*D* 04
40	0·606 663*D* 02	0·968 625*D* 02	0·282 547*D* 03	0·134 594*D* 05
50	0·850 578*D* 02	0·156 553*D* 03	0·643 535*D* 03	0·994 814*D* 05
60	0·114 850*D* 03	0·245 601*D* 03	0·144 693*D* 04	0·735 103*D* 06
70	0·151 237*D* 03	0·378 445*D* 03	0·323 491*D* 04	0·543 174*D* 07
80	0·195 682*D* 03	0·576 625*D* 03	0·721 415*D* 04	0·401 355*D* 08
90	0·249 966*D* 03	0·872 275*D* 03	0·160 701*D* 05	0·296 563*D* 09
100	0·316 269*D* 03	0·131 333*D* 04	0·357 794*D* 05	0·219 132*D* 10

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Súhrn

ODHAD TOTÁLNEJ CHYBY METÓDY PRE JEDNU JEDNOKROKOVÚ METÓDU

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Článok pojednáva o odhade totálnej chyby metódy Huťových vzorcov 5. rádu (RKH5) Rungeho-Kuttaovho typu. Je tu uvedený odhad lokálnej chyby metódy v prípade riešenia systému diferenciálnych rovníc metódou RKH5 na intervale $\langle x_0, x_1 \rangle$. Ďalej sa v článku nachádza odhad totálnej chyby v prípade riešenia úlohy (2.1), (2.2) pomocou vzorcov RKH5 za predpokladu, že riešime danú úlohu na celom intervale $\langle x_0, X \rangle$.

Odhad totálnej chyby je tu realizovaný dvoma spôsobmi. Východiskom pri odvodzovaní odhadov chyby vzorcov RKH5 bol prístup použitý v [10], kde boli odvodené odhady chyby štandardných vzorcov RK4.

Резюме

ОЦЕНКА АККУМУЛИРОВАННОЙ ОШИБКИ УСЕЧЕНИЯ ДЛЯ ОДНОШАГОВОГО МЕТОДА

ANNA VALKOVÁ

В статье предлагаются формулы для оценки локальной и аккумулированной ошибки усечения для численного решения методом Рунге-Кутта-Хутья пятого порядка. Для аккумулированной ошибки усечения даются два возможных способа оценки (3.10), (3.11). Оказывается, что в случае метода пятого порядка лучшуго оценку дает формула (3.11).

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