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ONCE MORE ABOUT THE MONOTONICITY OF THE TEMPLE QUOTIENTS

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Dedicated to Professor M. Zlámal on the occasion of the sixtieth anniversary of his birth

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Let $\mathscr{H}$ be a real or complex Hilbert space with the inner product $(\cdot, \cdot)$ and the norm $\|\cdot\| = (\cdot, \cdot)^{1/2}$. Let T be a normal bounded linear operator mapping $\mathscr{H}$ into $\mathscr{H}$, of the following structure:

(1) $$T = \lambda_1 P_1 + \lambda_2 P_2 + S \,,$$

where P_1, P_2 are symmetric, S is normal and

(2) $$P_1 P_2 = P_2 P_1 = 0 = P_j S = S P_j \,, \quad j = 1, 2 \,,$$

(3) $$P_1^2 = P_1 \,, \quad P_2^2 = P_2 \,,$$

(4) $$\lambda_1 > |\lambda_2| > r(S) = \sup \{|\lambda| : \lambda \in \sigma(S)\} \,,$$

where $\sigma(S)$ denotes the spectrum of S.

We set $P_1 = P$ and $P_2 = Q$ and take $x_0 \in \mathscr{H}$ such that

(5) $$P x_0 \neq 0 \quad Q x_0 \neq 0 \,.$$

Define sequences

(6) $$x_{k+1} = T x_k$$

and

$$\varkappa_k = \frac{(x_k, x_0)}{(x_{k-1}, x_0)} \,,$$

$$\varrho_k = \frac{(x_k, x_k)^{1/2}}{(x_{k-1}, x_{k-1})^{1/2}} \,,$$

$$\tau_k(L) = \tau_k = \frac{\varkappa_k - L}{1 - \dfrac{L}{\varrho_{k-1}}},$$

$$\sigma_k(L) = \sigma_k = \frac{\varrho_k - L}{1 - \dfrac{L}{\varrho_{k-1}}}, \quad k = 1, 2, \ldots,$$

where L is a real parameter.

The quantities $\varkappa_k$ and τ_k are called the Schwarz and Temple quotients, respectively, while ϱ_k are the Kellogg quotients and σ_k the corresponding Kellogg-Temple quotients.

Under appropriate hypotheses the sequences $\{\varkappa_k\}$, $\{\varrho_k\}$, $\{\tau_k\}$ and $\{\sigma_k\}$ are convergent (see [3], [2]); moreover ([1], [2]),

$$(7) \qquad \varkappa_1 < \varkappa_2 < \ldots < \varkappa_k < \ldots < \lambda_1 < \ldots < \tau_k < \ldots < \tau_2 < \tau_1 ,$$

$$(8) \qquad \lim_{k \to \infty} \varkappa_k = \lim_{k \to \infty} \tau_k = \lambda_1$$

and also [3]

$$(9) \qquad \varrho_1 < \varrho_2 < \ldots < \varrho_k < \ldots < \lambda_1 ,$$

$$(10) \qquad \lim_{k \to \infty} \varrho_k = \lambda_1 .$$

The second part of the monotonicity result (7) has been shown just recently in [1] and re-proved in [2] under identical hypotheses. Although the methods of proof in [1] and [2] are completely different, the results are essentially the same. In our note we offer another approach (naturally different from the previous ones). The idea is very simple and goes back to [3, p. 260–261]. In comparison with the papers mentioned our method has the following features. The monotonicity result is slightly weaker in the sense that we guarantee the monotonicity only asymptotically. On the other hand our approach is much simpler and more transparent; the latter property is used to analysing the interval of admissible L's and it is shown that this interval can be larger than that currently used.

Similarly as in [2] one can generalize our method to much more complex situation. For the sake of simplicity and transparency of the proof we restrict ourselves to the class of operators T having a dominant discrete part of their spectrum. Also the application of our result to the generalized eigenvalue problem of the type $Au = \lambda Bu$ with generally unbounded self-adjoint operators A and B is rather standard.

Lemma 1. *Let T have the form (1) with $\lambda_1 = 1$, and let (2)–(5) be fulfilled. Then there is an integer k_0 such that the following asymptotic expansion holds:*

$$(11) \qquad \varkappa_k = 1 - \frac{(1 - \lambda_2)(Qx_0, x_0)}{(Px_0, x_0)} \lambda_2^{k-1} + o(\lambda_2^k)$$

for $k > k_0$.

Proof. It is well known that [4., p. 346]

(12) $$x_k = T^k x_0 = Px_0 + \lambda_2^k Q x_0 + S^k x_0 , \quad k = 1, 2, \ldots .$$

It follows that

$$\varkappa_k = \frac{(Px_0 + \lambda_2^k Q x_0 + S^k x_0, x_0)}{(Px_0 + \lambda_2^{k-1} Q x_0 + S^{k-1} x_0, x_0)} = 1 + \eta_k + z ,$$

where

$$\eta_k = - \frac{(1 - \lambda_2)(Qx_0, x_0)}{(Px_0, x_0)} \lambda_2^{k-1} ,$$

$$z = \frac{(S^k x_0, x_0) - (S^{k-1} x_0, x_0) - \eta_k(\lambda_2^{k-1} Q x_0 + S^{k-1} x_0, x_0)}{(Px_0 + \lambda_2^{k-1} Q x_0 + S^{k-1} x_0, x_0)} .$$

It is evident that

$$\lim_{k \to \infty} \frac{z}{\lambda_2^k} = 0 .$$

We conclude that

$$\varkappa_k = 1 - \frac{(1 - \lambda_2)(Qx_0, x_0)}{(Px_0, x_0)} \lambda_2^{k-1} + o(\lambda_2^k) = 1 - a \,.\, \lambda_2^{k-1} + o(\lambda_2^k) .$$

This shows the validity of formula (11).

Lemma 2. *Let T have the form (1) with $\lambda_1 = 1$ and $\lambda_2 > 0$, and let (2)–(5) be fulfilled. Then there is a positive integer k_0 such that*

(13) $$\varkappa_k < \varkappa_{k+1} < 1 \quad for \quad k > k_0 .$$

Proof. From (11) we derive that

$$\varkappa_{k+1} - \varkappa_k = \lambda_2(\varkappa_k - \varkappa_{k-1}) + o(\lambda_2^k)$$

and

$$\varkappa_k - \varkappa_{k-1} = a \,.\, (1 - \lambda_2) \,.\, \lambda_2^{k-2} + o(\lambda_2^{k-2}) .$$

This implies that the sequence $\{\varkappa_k\}$ is asymptotically monotonic. Since $\varkappa_k < 1$ for $k > k_0$ (k_0 from Lemma 1), $\varkappa_k \to \lambda_1 = 1$, we deduce that (13) holds and this completes the proof.

Lemma 3. *Let T have the form (1) with $\lambda_1 = 1$ and $\lambda_2 > 0$, and let (2)–(5) be fulfilled. Then there is an integer $k > k_0$ (k_0 from Lemma 2) such that the following expansion holds:*

(14) $$\tau_k = 1 + \frac{(L - \lambda_2)(1 - \lambda_2)(Qx_0, x_0)}{(1 - L)(Px_0, x_0)} \lambda_2^{k-2} + o(\lambda_2^k)$$

for $k > k_1$,
where L is a real parameter, $L \in (\lambda_2, \varkappa_{k_0})$.

Proof. From (12) it follows that

$$\tau_k = \frac{(Px_0 + \lambda_2^k Qx_0 + S^k x_0, x_0) - L(Px_0 + \lambda_2^{k-1} Qx_0 + S^{k-1} x_0, x_0)}{(Px_0 + \lambda_2^{k-1} Qx_0 + S^{k-1} x_0, x_0) - L(Px_0 + \lambda_2^{k-2} Qx_0 + S^{k-2} x_0, x_0)} =$$

$$= 1 + \gamma_k + o(\lambda_2^k),$$

where

$$\gamma_k = \frac{(L - \lambda_2)(1 - \lambda_2)(Qx_0, x_0)}{(1 - L)(Px_0, x_0)} \lambda_2^{k-2}.$$

We deduce that

$$\tau_k = 1 + \frac{(L - \lambda_2)(1 - \lambda_2)(Qx_0, x_0)}{(1 - L)(Px_0, x_0)} \lambda_2^{k-2} + o(\lambda_2^k) = 1 + b\lambda_2^{k-2} + o(\lambda_2^k).$$

This shows the validity of formula (14).

Theorem 1. *Let T have the form (1) with $\lambda_1 = 1$ and $\lambda_2 > 0$, and let (2)–(5) be fulfilled; further, let $L \in (\lambda_2, \varkappa_{k_0})$ (k_0 from Lemma 2). Then there exists a positive integer $k_1 > k_0$ such that*

(15) $$\varkappa_k < \varkappa_{k+1} < 1 = \lambda_1 < \tau_{k+1} < \tau_k$$

holds for $k > k_1$.

Proof. Lemma 2 implies the validity of the left part of (15). It remains to prove that

$$\lambda_1 < \tau_{k+1} < \tau_k \quad \text{for} \quad k > k_1,$$

From (14) we derive that

$$\tau_k - \tau_{k+1} = \lambda_2(\tau_{k+1} - \tau_k) + o(\lambda_2^k)$$

and

$$\tau_k - \tau_{k+1} = b \cdot (1 - \lambda_2) \cdot \lambda_2^{k-2} + o(\lambda_2^k).$$

This implies that the sequence $\{\tau_k\}$ is asymptotically monotonic. Since $\tau_k > 1$ for $k > k_1$ (k_1 from Lemma 3), $\tau_k \to \lambda_1$, we deduce that (15) holds and this completes the proof.

Lemma 4. *Let the hypotheses of Lemma 1 be fulfilled. Then there is a positive integer k_0 such that the following asymptotic expansion holds:*

(16) $$\varrho_k = 1 - \frac{1}{2} \frac{(1 - \lambda_2^2)(Qx_0, Qx_0)}{(Px_0, Px_0)} \lambda_2^{(2k-2)} + o(\lambda_2^{2k-2})$$

for $k > k_0$.

Proof. Using (12) we deduce that

$$\varrho_k^2 = \frac{\|Px_0\|^2 + \lambda_2^{2k}\|Qx_0\|^2 + \|S^{2k}x_0\|^2}{\|Px_0\|^2 + \lambda_2^{2k-2}\|Qx_0\|^2 + \|S^{2k-2}x_0\|^2}$$

and hence,

$$\varrho_k^2 = 1 - \frac{(1-\lambda_2^2)\,\|Qx_0\|^2}{\|Px_0\|^2}\,\lambda_2^{2k-2} + o(\lambda_2^{2k})\,.$$

It follows that

$$\varrho_k = 1 - \frac{1}{2}\frac{(1-\lambda_2^2)\,\|Qx_0\|^2}{\|Px_0\|^2}\,\lambda_2^{2k-2} + o(\lambda_2^{2k}) = 1 - d\lambda_2^{2k-2} + o(\lambda_2^{2k})$$

and this completes the proof.

Before we show the monotonicity of the sequences $\{\varrho_k\}$, $\{\sigma_k\}$ we prove the validity of the following auxiliary assertion (see [3]):

Lemma 5. *There exists a sequence $\{g_k\}$, $g_k \in \mathscr{H}$, such that*

(17) $$x_k = \varrho_0 \cdots \varrho_k g_k\,,$$

where $\varrho_0 = \|x_0\|$ and

(18) $$\|g_k\| = 1\,, \quad k = 1, 2, \ldots.$$

Proof. Let $g_k = x_k/\|x_k\|$, then obviously (18) holds and

$$x_k = \varrho_0 \cdot \varrho_1 \cdots \varrho_k g_k = \|x_k\|\, g_k\,, \quad k = 1, 2, \ldots.$$

Theorem 2. *Let T have the form (1) with $\lambda_1 = 1$ and let (2)–(5) be fulfilled. Then for every $L \in (\lambda_2^2, \varrho_1)$ there exists a positive integer k_0 such that either.*

(19) $$\varrho_1 < \ldots < \varrho_k < \varrho_{k+1} < 1 < \sigma_{k+1} < \sigma_k$$

for $k > k_0$,

or

(20) $$\varrho_1 = \ldots = \varrho_k = 1 = \sigma_k = \ldots = \sigma_1 \quad \textit{for} \quad k = 2, 3, \ldots.$$

Proof. First, we show the monotonicity of the sequence $\{\varrho_k\}$, $k = 1, 2, \ldots$. From (17) and (18) we derive that

$$\varrho_k^2 = \varrho_k \cdot \varrho_{k+1}(g_{k+1}, g_{k-1})\,.$$

Since $\|g_k\| = 1$ this implies that

$$\varrho_k = \varrho_{k+1}(g_{k+1}, g_{k-1}) \leqq \varrho_{k+1} \quad \text{for} \quad k = 1, 2, \ldots.$$

Let us suppose $\varrho_k < \varrho_{k+1}$ for $k = 1, 2, \ldots$, otherwise $\varrho_0 = \varrho_1 = \ldots = \varrho_k = 1$. We compute the difference

$$\sigma = \sigma_k - \sigma_{k+1} = \frac{\|x_k\| - L\|x_{k-1}\|}{\|x_{k-1}\| - L\|x_{k-2}\|} - \frac{\|x_{k+1}\| - L\|x_k\|}{\|x_k\| - L\|x_{k-1}\|}.$$

Using (17) and (18) we deduce that

$$\sigma = \frac{1}{(\|x_k\| - L\|x_{k-1}\|)(\|x_{k-1}\| - L\|x_{k-2}\|)} \cdot \gamma,$$

where

$$\gamma = \|x_{k-1}\| \, \|x_{k-2}\| \, [L(\varrho_k - \varrho_{k-1})(\varrho_k - L) - \varrho_k(\varrho_{k-1} - L)(\varrho_{k+1} - \varrho_k)].$$

Using (16) we obtain

$$\varrho_{k+1} - \varrho_k = \lambda_2^2(\varrho_k - \varrho_{k-1}) + o(\lambda_2^{2k})$$

for $k > k_0$ (k_0 from Lemma 4) and

$$\varrho_{k+1} - \varrho_k = d(1 - \lambda_2^2)\,\lambda_2^{2k-2} + o(\lambda_2^{2k}).$$

For k sufficiently large, we see that

$$\gamma = \|x_{k-1}\| \, \|x_{k-2}\| \, (\varrho_k - \varrho_{k-1})\,[L(\varrho_k - L) - \lambda_2^2\,\varrho_k(\varrho_{k-1} - L)] + o(\lambda_2^{4k}).$$

Therefore, since $\varrho_k > \varrho_{k-1}$ and $\varrho_k > L$ we get

$$\gamma \geqq \|x_{k-1}\| \, \|x_{k-2}\| \, (\varrho_k - \varrho_{k-1})(\varrho_{k-1} - L)(L - \lambda_2^2) + o(\lambda_2^{4k}).$$

It means that $\gamma > 0$ for sufficiently large k and for $L > \lambda_2^2$, and this completes the proof of (19). As concerns formula (20), one can see that

$$\sigma_k = \frac{\varrho_k - L}{1 - \dfrac{L}{\varrho_{k-1}}} = \varrho_k \quad \text{for all} \quad k = 1, 2, \ldots.$$

CONCLUDING REMARKS

1. It is quite evident that the assumption $\lambda_1 = 1$ is in no way a restriction. By using it the analysis is technically simplified.

2. If T is positively semidefinite, then

$$\varkappa_k = 1 - \delta_k, \quad \delta_k \geqq 0, \quad \delta_{k+1} \leqq \delta_k \quad \text{for all} \quad k = 1, 2, \ldots.$$

3. Let

$$\mu_k = \frac{(x_{k+1}, x_k)}{(x_k, x_k)}, \quad k = 0, 1, 2, \ldots.$$

We see that

$$\mu_k = \varkappa_{2k+1}\,, \quad k = 0, 1, \ldots$$

and hence the sequence $\{\mu_k\}$ is increasing and

$$\lim_{k \to \infty} \mu_k = \lambda_1\,.$$

For practical computations this subsequence is preferable, because

$$|\mu_k - \lambda_1| = 0\left(\left|\frac{\lambda_2}{\lambda_1}\right|^{2k}\right)$$

in comparison with

$$|\varkappa_k - \lambda_1| = 0\left(\left|\frac{\lambda_2}{\lambda_1}\right|^{k}\right)$$

while the computational complexity and the computer memory needed are the same for the both suquences.

For the Temple sequence

$$v_k(L) = v_k = \frac{\mu_k - L}{1 - \dfrac{L}{\mu_{k-1}}}$$

corresponding to μ_k we have a statement fully analogous to Theorem 1. From the computational point of view, again, v_k is more advantageous than τ_k.

4. The restriction $L < \varkappa_{k_0}$ or $L < \varrho_1$ is made in order to avoid vanishing of the denominator of the corresponding quotients.

5. If $\lambda_2 < 0$, then $\lambda_2^k = (-1)^k\,|\lambda_2|$ and hence

$$\varkappa_{2k} > \lambda_1 \quad \text{and} \quad \varkappa_{2k+1} < \lambda_1$$

while, assuming $L > \lambda_2$, we have

$$\tau_{2k} > \lambda_1 \quad \text{and} \quad \tau_{2k+1} < \lambda_1$$

for $k > k_1 > k_0$.

6. If we have a more detailed knowledge of T, say, if moreover,

$$S = \lambda_3 P_3 + \ldots + \lambda_N P_N + Z$$

where $\lambda_j \neq 0$ for $j = 3, \ldots, N$, $|\lambda_3| \geqq |\lambda_4| \geqq \ldots \geqq |\lambda_N|$, P_j are symmetric and

$$P_j P_k = P_k P_j = \delta_{jk} P_j\,, \quad j, k = 3, \ldots, N\,,$$

$$P_j Z = Z P_j = 0\,,$$

and if $x_0 \in \mathscr{H}$ is such that $Px_0 \neq 0$, $Qx_0 = 0$, $Wx_0 \neq 0$; where

$$W = P_{j_0}\,, \quad \hat{\lambda} = \lambda_{j_0}$$

with

$$j_0 = \min \{j : P_j x_0 \neq 0, j > 2\},$$

then

$$\varkappa_k = 1 - \frac{(1-\hat{\lambda})(Wx_0, x_0)}{(Px_0, x_0)} \hat{\lambda}^{k-1} + o(\hat{\lambda}^k)$$

and

$$\tau_k = 1 + \frac{(L-\hat{\lambda})(1-\hat{\lambda})(Wx_0, x_0)}{(1-L)(Px_0, x_0)} \hat{\lambda}^{k-2} + o(\hat{\lambda}^k).$$

We see that the sequence $\{\tau_k\}$ is decreasing as soon as $L \in (\hat{\lambda}, \varkappa_{k_0})$, $\hat{\lambda} > 0$ and alternating if $\hat{\lambda} < 0$.

7. Let T have the form

$$T = \lambda_1 P + \lambda_2 Q + \mu_2 R + S, \tag{1'}$$

where P, Q, R are symmetric and S normal,

$$P . Q = Q . P = P . R = R . P = Q . R = R . Q = 0, \tag{2'}$$

$$P . S = S . P = Q . S = S . Q = R . S = S . R = 0,$$

$$P^2 = P, \quad Q^2 = Q, \quad R^2 = R, \tag{3'}$$

$$1 = \lambda_1 > \lambda_2 > r(S), \quad \mu_2 = \alpha . \lambda_2, \quad |\alpha| = 1. \tag{4'}$$

We take $x_0 \in \mathscr{H}$ such that

$$Px_0 \neq 0, \quad Qx_0 + Rx_0 \neq 0, \quad Qx_0 - Rx_0 \neq 0. \tag{5'}$$

For a symmetric T, only two cases are possible: $\alpha = 1$ or $\alpha = -1$. If $\alpha = 1$ then $\mu_2 = \lambda_2$ and

$$T = P + \lambda_2 \bar{Q} + S \quad \text{where} \quad \bar{Q} = Q + R.$$

For this T, (2)–(5) are fulfilled. We have already discussed this case.

Let now $\alpha = -1$. Then $\mu_2 = -\lambda_2$,

$$T = P + \lambda_2(Q - R) + S.$$

In this case

$$x_k = P + \lambda_2^k Q + (-1)^k \lambda_2^k R + S^k, \tag{12'}$$

$$\varkappa_k = 1 - \eta_k + o(\lambda_2^k),$$

where

$$\eta_k = \frac{(1-\lambda_2)(Qx_0, x_0) + (-1)^{k-1}(1+\lambda_2)(Rx_0, x_0)}{(Px_0, x_0)} \lambda_2^{k-1},$$

or else

$$\eta_{2k} = \frac{(1 - \lambda_2)\,\|Qx_0\|^2 - (1 + \lambda_2)\,\|Rx_0\|^2}{\|Px_0\|^2}\,\lambda_2^{2\kappa-1},$$

$$\eta_{2k+1} = \frac{(1 - \lambda_2)\,\|Qx_0\|^2 + (1 + \lambda_2)\,\|Rx_0\|^2}{\|Px_0\|^2}\,\lambda_2^{2k}.$$

One can easily show that

(i) $$\varkappa_{2k} < 1 \Leftrightarrow (1 - \lambda_2)\,\|Qx_0\|^2 - (1 + \lambda_2)\,\|Rx_0\|^2 > 0,$$

$$\varkappa_{2(k+1)} - \varkappa_{2k} = \lambda_2^2(\varkappa_{2k} - \varkappa_{2(k-1)}) + o(\lambda_2^{2k}),$$

and $\varkappa_{2k} \to 1$ for $k \to \infty$.

Thus, either $\|Qx_0\|^2 > [(1 + \lambda_2)/(1 - \lambda_2)]\,\|Rx_0\|^2$ and then for sufficiently large k

$$\varkappa_{2k} < \varkappa_{2(k+1)} < 1 = \lambda_1,$$

or

$$\varkappa_{2k} > \varkappa_{2(k+1)} > 1 = \lambda_1.$$

(ii) For k sufficiently large, $\varkappa_{2k+1} < 1$. Moreover,

$$\varkappa_{2(k+1)+1} - \varkappa_{2k+1} = \lambda_2^2[\varkappa_{2k+1} - \varkappa_{2(k-1)+1}] + o(\lambda_2^{2k})$$

and $\varkappa_{2k+1} \to 1$ for $k \to \infty$.

Thus, $\varkappa_{2(k-1)+1} < \varkappa_{2k+1} < 1 = \lambda_1$ for k sufficiently large. For the sequence $\{\tau_k\}$ (for $\mu_2 = -\lambda_2$), (12′) yields

$$\tau_k = 1 + \chi_k + o(\lambda_2^k),$$

where

$$\chi_k = \frac{(1 - \lambda_2)(L - \lambda_2)(Qx_0, x_0) + (-1)^{k-2}(1 + \lambda_2)(L + \lambda_2)}{(1 - L)(Px_0, x_0)}\,\lambda_2^{k-2} =$$

$$= \frac{\lambda_2^{k-2}}{(1 - L)(Px_0, x_0)}\,\xi_k,$$

with

$$\xi_k = (1 - \lambda_2)(L - \lambda_2)\,\|Qx_0\|^2 + (-1)^{k-2}(1 + \lambda_2)(L + \lambda_2)\,\|Rx_0\|^2$$

or, in more detail,

$$\xi_{2k} = (1 - \lambda_2)(L - \lambda_2)\,\|Qx_0\|^2 + (1 + \lambda_2)(L + \lambda_2)\,\|Rx_0\|^2,$$
$$\xi_{2k+1} = (1 - \lambda_2)(L - \lambda_2)\,\|Qx_0\|^2 - (1 + \lambda_2)(L + \lambda_2)\,\|Rx_0\|^2.$$

One can show that

(i) for k sufficiently large $(L > \lambda_2)$, $\tau_{2k} > 1$. Moreover,

$$\tau_{2(k+1)} - \tau_{2k} = \lambda_2^2(\tau_{2k} - \tau_{2(k-1)}) + o(\lambda_2^{2k})$$

and $\tau_{2k} \to 1$ for $k \to \infty$.

Thus $\tau_{2k} > \tau_{2(k+1)} > 1 = \lambda_1$ for k sufficiently large.

(ii) $$\tau_{2k+1} > 1 \Leftrightarrow \|Qx_0\|^2 > \frac{(1 + \lambda_2)(L + \lambda_2)}{(1 - \lambda_2)(L - \lambda_2)} \|Rx_0\|^2 ,$$

$$\tau_{2(k+1)+1} - \tau_{2k+1} = \lambda_2^2(\tau_{2k+1} - \tau_{2(k-1)+1}) + o(\lambda_2^{2k}) ,$$

$\tau_{2k+1} \to 1$ for $k \to \infty$.

Thus, either

$$\|Qx_0\|^2 > \frac{(1 + \lambda_2)(L + \lambda_2)}{(1 - \lambda_2)(L - \lambda_2)} \|Rx_0\|^2$$

and then (for k sufficiently large)

$$\tau_{2(k+1)+1} > \tau_{2k+1} > 1 = \lambda_1 ,$$

or

$$\tau_{2(k+1)+1} < \tau_{2k+1} < 1 = \lambda_1 .$$

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Souhrn

JEŠTĚ JEDNOU O MONOTONII TEMPLOVÝCH KVOCIENTŮ

DRAHOSLAVA JANOVSKÁ, IVO MAREK

Je podán nový důkaz monotonie Templových kvocientů pro výpočet dominantního vlastního čísla ohraničeného lineárního operátoru. Výklad se provádí pro případ normálního operátoru, výsledky však lze zobecnit na daleko širší třídu lineárních operátorů. Oproti známým důkazům podaným J. Albrechtem a F. Goerischem v [1] a K. Rektorysem v [2] je v této práci předložený důkaz značně jednodušší a metoda vyšetřování obecnější. Navíc je ukázáno, že pro případ Kelloggových-Templeových kvocientů je interval přípustných posuvů obecně větší než ve výše uvedených pracech.

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