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PERIOD DOUBLING BIFURCATIONS IN A TWO-BOX MODEL OF THE BRUSSELATOR

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Two theorems about period doubling bifurcations are proved. A special case, where one multiplier of the homogeneous solution is equal to $+1$, is discussed in Appendix.

1. PRELIMINARIES

1.1. A two-box model of the reaction-diffusion system with Brusselator kinetics has been discussed by several authors, see for example [1]–[4], [9]. The system is described by the following set of four differential equations:

(1)
$$\begin{aligned} \dot{x}_1 &= A - (B+1)\,x_1 + x_1^2 y_1 + D_1(x_2 - x_1) \\ \dot{y}_1 &= Bx_1 - x_1^2 y_1 + D_2(y_2 - y_1) \\ \dot{x}_2 &= A - (B+1)\,x_2 + x_2^2 y_2 + D_1(x_1 - x_2) \\ \dot{y}_2 &= Bx_2 - x_2^2 y_2 + D_2(y_1 - y_2) \end{aligned}$$

where A, B, D_1, D_2 are adjusted parameters. Hence, the state of the system is determined by the quadruple $\mathbf{x} = [x_1, y_1, x_2, y_2] \in \mathbb{R}^4$. By Δ we shall denote the diagonal in $\mathbb{R}^4$, that is, the set

$$\Delta = \{[x_1, y_1, x_2, y_2] \in \mathbb{R}^4,\ x_1 = x_2 \wedge y_1 = y_2\}\,.$$

1.2. Let us consider a mapping $g : \mathbb{R}^4 \to \mathbb{R}^4$ defined by the relation

$$g(x_1, y_1, x_2, y_2) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix}$$

i.e. in a short form

(2)
$$g(x_1, y_1, x_2, y_2) = (x_2, y_2, x_1, y_1)\,.$$

It is easy to see that the following statements are valid.

(i) $g^2 = g \circ g = id$.

(ii) g is a linear diffecmorphism of $\mathbb{R}^4$.

(iii) $\mathbf{x} \in \Delta$ iff $g(\mathbf{x}) = \mathbf{x}$, i.e. $Fix(g) = \Delta$.

(iv) The matrix A defining the mapping g has two double eigenvalues $+1$ and -1.

The eigenvectors corresponding to $+1$ are $\mathbf{e}_1 = [1, 0, 1, 0]$, $\mathbf{e}_2 = [0, 1, 0, 1]$ and the eigenvectors corresponding to -1 are $\mathbf{e}_3 = [1, 0, -1, 0]$, $\mathbf{e}_4 = [0, 1, 0, -1]$. These eigenvectors form an orthogonal system.

1.3. The vector field $\mathbf{v}(\mathbf{x})$ on the right hand side of the system (1) is *invariant* under the diffeomorphism g, hence the relation

$$\mathbf{v}(g(\mathbf{x})) = (g_*)_x\,(\mathbf{v}(\mathbf{x})) \tag{3}$$

holds. Since the mapping g is linear, $(g_*)_x = g$ for all $\mathbf{x} \in \mathbb{R}^4$ and the verification of the relation (3) is easy.

1.4. The diagonal Δ forms an *invariant manifold* (integral surface) of the vector field $\mathbf{v}$, as for every $\mathbf{x} \in \Delta$, $\mathbf{v}(\mathbf{x}) \in T_x(\Delta)$.

1.5. From now on we shall consider a more general system of ordinary differential equations than (1), viz. the system

$$\dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}, \mu)\,, \tag{4}$$

where $\mathbf{x} = [x_1, y_1, x_2, y_2] \in \mathbb{R}^4$, $\mu \in \mathbb{R}^1$ and $\mathbf{v}(\mathbf{x}, \mu)$ is a one-parameter system of vector fields on $\mathbb{R}^4$ satisfying the following conditions:

a) $\mathbf{v}(\mathbf{x}, \mu)$ is of class C^∞ in both variables $\mathbf{x}$ and μ.

b) The vector field $\mathbf{v}(\cdot, \mu)$ is *invariant* under the diffeomorphism g for every $\mu \in \mathbb{R}$.

c) For every $\mu \in \mathbb{R}$, the flow T_μ^t, $t \in \mathbb{R}$, of the system (4) exists.

As the vector field $\mathbf{v}(\mathbf{x}, \mu)$ is invariant under the diffeomorphism g, hence if $\mathbf{x}(t)$ is a solution of (4), then $g(\mathbf{x}(t))$ is also a solution of (4), see [5], and every trajectory γ of (4) has a corresponding trajectory $g(\gamma)$.

1.6. The following lemma is well-known, *see* [6], p. 141.

Lemma 1. *Let T_μ^t be the flow of the vector field $\mathbf{v}(\mathbf{x}, \mu)$, which is invariant under the diffeomorphism g for all $\mu \in \mathbb{R}$. Then*

$$g \circ T_\mu^t = T_\mu^t \circ g \tag{5}$$

for all $t \in \mathbb{R}$ and $\mu \in \mathbb{R}$.

1.7. Definition 1. *The periodic solution* $\mathbf{x}_\mu(t)$ *of* (4) *will be called a g-invariant solution iff its trajectory* γ_μ *is an invariant set of the mapping* g, *i.e.* $g(\gamma_\mu) = \gamma_\mu$.

The g-invariant solution $\mathbf{x}_\mu(t)$ *for which* $\gamma_\mu \subset \Delta$ *will be called a homogeneous solution* – (*HS*).

The g-invariant solution $\mathbf{x}_\mu(t)$ *for which* $\gamma_\mu \cap \Delta = \emptyset$ *will be called a* Δ*-symmetric solution.*

The following lemma yields a useful characterization of the Δ-symmetric solution.

Lemma 2. *Let* $\mathbf{x}_\mu(t)$ *be a periodic solution of* (4) *and* γ_μ *its trajectory. Let both the points* $\mathbf{x}$ *and* $g(\mathbf{x}) \neq \mathbf{x}$ *lie on* γ_μ. *Then the point* $g(\mathbf{y}) \neq \mathbf{y}$ *lies on* γ_μ *for every* $\mathbf{y} \in \gamma_\mu$ *and hence* $g(\gamma_\mu) = \gamma_\mu$. *The phase shift of the points* $\mathbf{y} \in \gamma_\mu$ *and* $g(\mathbf{y}) \in \gamma_\mu$ *is one half of the period of the solution* $\mathbf{x}_\mu(t)$.

Proof. [1]) Let ω be the smallest period of the solution $\mathbf{x}(t)$. Under our assumption the points $\mathbf{x}$ and $g(\mathbf{x}) \neq \mathbf{x}$ lie on γ, hence $T^\omega(\mathbf{x}) = \mathbf{x}$ and $T^\omega(g(\mathbf{x})) = g(\mathbf{x})$. Then there exists a number $s \in (0, \omega)$ such that

$$T^s(\mathbf{x}) = g(\mathbf{x})\,. \tag{6}$$

From (5) and with help of $g \circ g = id$ we obtain

$$\mathbf{x} = g^2(\mathbf{x}) = g(g(\mathbf{x})) = g(T^s(\mathbf{x})) = T^s(g(\mathbf{x})) = T^s(T^s(\mathbf{x})) = T^{2s}(\mathbf{x})\,.$$

Hence

$$2s = \omega\,, \quad s = \frac{\omega}{2} \quad \text{and} \quad g(\mathbf{x}) = T^{\frac{1}{2}\omega}(\mathbf{x})\,.$$

Let $\mathbf{y}$ be an arbitrary point of γ. A number $r \in (0, \omega)$ can be found such that $\mathbf{y} = T^r(\mathbf{x})$. Then

$$T^{\frac{1}{2}\omega}(\mathbf{y}) = T^{(\frac{1}{2}\omega + r)}(\mathbf{x}) = T^r(T^{\frac{1}{2}\omega}(\mathbf{x})) = T^r(g(\mathbf{x})) = g(T^r(\mathbf{x})) = g(\mathbf{y})\,, \quad \text{QED}\,.$$

2. THE PERIOD DOUBLING BIFURCATION OF (HS)

2.1. Let $\gamma_{\mu_0} \subset \Delta$ be the trajectory of an (HS) of the system (4) for $\mu = \mu_0$. A *Poincaré map* will be used for the description of the bifurcation phenomena. Let $\mathbf{x}_{\mu_0} \in \gamma_{\mu_0}$. By Σ we shall denote the hyperplane of the codimension 1, which intersects transversally the trajectory γ_{μ_0} at the point $\mathbf{x}_{\mu_0}$. By $\Sigma(\mathbf{x}_{\mu_0})$ we denote $B_\varepsilon(\mathbf{x}_{\mu_0}) \cap \Delta$, where $B_\varepsilon(\mathbf{x}_{\mu_0})$ is an appropriate neighbourhood of the point $\mathbf{x}_{\mu_0}$ in $\mathbb{R}^4$. The hyperplane Σ may be chosen in such a way that[2])

$$g(\Sigma) = \Sigma\,. \tag{7}$$

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[1]) The subscript μ will usually be omitted from now on.

[2]) See Appendix, relation (13).

2.2. Let us denote by $P_{\mu_0} : \Sigma(\mathbf{x}_{\mu_0}) \to \Sigma$ the Poincaré map corresponding to the closed trajectory γ_{μ_0}. We suppose that none of the multipliers of this trajectory equals one. In this case there exists a one-parameter family P_μ of Poincaré maps corresponding to closed trajectories γ_μ, $\mu \in O(\mu_0)$ and $O(\mu_0)$ is an appropriate neighbourhood of μ_0. (P_μ is defined on an appropriately chosen $\Sigma(\mathbf{x}_{\mu_0})$.)

Lemma 3. *For every $\mu \in O(\mu_0)$ we have*

$$g \circ P_\mu = P_\mu \circ g \,, \tag{8}$$

whenever $P_\mu \circ g$ is defined.

Proof. The Poincaré map P_μ can be expressed with help of the flow T^t_μ, *see* [7]. If ω_μ is the period of the corresponding (HS), then

$$P_\mu(\mathbf{x}) = T^{[\omega_\mu + \delta_\mu(x)]}(\mathbf{x}) \tag{9}$$

where $\delta_\mu : \Sigma(\mathbf{x}_{\mu_0}) \to \mathbb{R}$, $\delta_\mu(\mathbf{x}_\mu) = 0$, $\mathbf{x}_\mu \in \Sigma(\mathbf{x}_{\mu_0}) \cap \gamma_\mu$.
Let us denote

$$\omega_\mu(\mathbf{x}) = \omega_\mu + \delta_\mu(\mathbf{x}) \,. \tag{10}$$

For $\mathbf{x} \in \Sigma(\mathbf{x}_{\mu_0})$ we have

$$g(P_\mu(\mathbf{x})) = g(T^{\omega_\mu(\mathbf{x})}(\mathbf{x})) = T^{\omega_\mu(g(\mathbf{x}))}(g(\mathbf{x})) = P_\mu(g(\mathbf{x})) \,.$$

The validity of the relation $\omega_\mu(g(\mathbf{x})) = \omega_\mu(\mathbf{x})$ results from the following consideration: The trajectory γ starting at the point $\mathbf{x} \in \Sigma(\mathbf{x}_{\mu_0})$ intersects Σ for the first time at the same moment as the trajectory $g(\gamma)$ starting at the point $g(\mathbf{x}) \in \Sigma(\mathbf{x}_{\mu_0})$ intersects Σ.

2.3. Theorem 1. *After a period doubling bifurcation of an (HS), the resulting double period solution must be Δ-symmetric.*

Proof. Let Γ_μ be the trajectory of the double period solution bifurcated from the (HS) in question. It is sufficient to prove the existence of points $\mathbf{x} \in \Gamma_\mu$ and $g(\mathbf{x}) \in \Gamma_\mu$, $g(\mathbf{x}) \neq \mathbf{x}$. It is well-known that after a period doubling bifurcation two fixed points of P^2_μ arise, let us denote then $\mathbf{x}_1(\mu)$ and $\mathbf{x}_2(\mu)$. Then

$$P_\mu(\mathbf{x}_1(\mu)) = \mathbf{x}_2(\mu) \quad \text{and} \quad P_\mu(\mathbf{x}_2(\mu)) = \mathbf{x}_1(\mu) \,.$$

The relation (8) yields (the latter μ is omitted)

$$P(g(\mathbf{x}_1)) = g(P(\mathbf{x}_1)) = g(\mathbf{x}_2) \,,$$
$$P(g(\mathbf{x}_2)) = g(P(\mathbf{x}_2)) = g(\mathbf{x}_1) \,,$$

hence

$$g(\mathbf{x}_1) = P(g(\mathbf{x}_2)) = P(P(g(\mathbf{x}_1))) = P^2(g(\mathbf{x}_1))$$

and $g(\mathbf{x}_1)$ is a fixed point of P^2. Then $g(\mathbf{x}_1) = \mathbf{x}_1$ or $g(\mathbf{x}_1) = \mathbf{x}_2$. The first equality is not possible: if $g(\mathbf{x}_1) = \mathbf{x}_1$ then $\mathbf{x}_1 \in \Delta$, hence $\Gamma_\mu \subset \Delta$ which is impossible – a period doubling bifurcation cannot arise in the two-dimensional diagonal Δ. Thus the second equality $g(\mathbf{x}_1) = \mathbf{x}_2$ holds and the points $\mathbf{x}_1$ and $\mathbf{x}_2 = g(\mathbf{x}_1) \neq \mathbf{x}_1$ lie on Γ_μ, QED.

3. THE PERIOD DOUBLING BIFURCATION OF THE Δ-SYMMETRIC SOLUTION

3.1. Let γ_μ be the trajectory of a Δ-symmetric solution of the equation (4) with a period ω_μ. Let us denote the cross-section which transversally intersects the trajectory γ_μ at a point $\mathbf{x}_\mu^0$ by Σ and let $P_\mu(\mathbf{x})$ be the corresponding Poincaré map. Under our assumption, the point $g(\mathbf{x}_\mu^0) \neq \mathbf{x}_\mu^0$ must lie on γ_μ. Then $\tilde{\Sigma} = g(\Sigma)$ is the cross-section of the trajectory γ_μ at the point $g(\mathbf{x}_\mu^0)$. Let us denote by $\tilde{P}_\mu : \tilde{\Sigma} \to \tilde{\Sigma}$ the corresponding Poincaré map. It is known that the maps P_μ and $\tilde{P}_\mu$ are locally conjugate, *see* [7]. In our special case the following lemma is valid.

Lemma 4. *For the maps P_μ and $\tilde{P}_\mu$ defined above we have*

$$\tilde{P}_\mu = g \circ P_\mu \circ g^{-1} = g \circ P_\mu \circ g \,, \tag{11}$$

whenever $P_\mu \circ g$ is defined.

Proof. We express the maps P and $\tilde{P}$ by the flow T^t: for $\mathbf{x} \in \Sigma$ we put $P(\mathbf{x}) = T^{\omega(\mathbf{x})}(\mathbf{x})$ and for $\mathbf{y} \in \tilde{\Sigma}$ we put $\tilde{P}(\mathbf{y}) = T^{\tilde{\omega}(\mathbf{y})}(\mathbf{y})$. By an argument fully analogous to the one used before (cf. *Theorem* 1), we obtain the equality

$$\tilde{\omega}(g(\mathbf{x})) = \omega(\mathbf{x}) \,. \tag{12}$$

Then for arbitrary $\mathbf{x} \in \Sigma$ we have $g(\mathbf{x}) = \mathbf{y} \in \tilde{\Sigma}$ and

$$\tilde{P}(g(\mathbf{x})) = T^{\tilde{\omega}(g(\mathbf{x}))}(g(\mathbf{x})) = g(T^{\tilde{\omega}(g(\mathbf{x}))}(\mathbf{x})) = g(T^{\omega(\mathbf{x})}(\mathbf{x})) = g(P(\mathbf{x}))\,,$$

hence the relation (11) holds.

3.2. Theorem 2. *The Δ-symmetric solution cannot bifurcate by the period doubling bifurcation.*

Proof. Let us suppose that for $\mu = \mu_0$ the "*double*" trajectory Γ_μ arose from the Δ-symmetric trajectory γ_{μ_0} by the period doubling bifurcation. Hence the two fixed points $\mathbf{x}_1(\mu)$, $\mathbf{x}_2(\mu)$ of the mapping P_μ^2 lie on the trajectory Γ_μ and $P_\mu(\mathbf{x}_1) = \mathbf{x}_2$, $P_\mu(\mathbf{x}_2) = \mathbf{x}_1$. The points $\mathbf{y}_1 = g(\mathbf{x}_1)$ and $\mathbf{y}_2 = g(\mathbf{x}_2)$, however, are also fixed points of the mapping $\tilde{P}_\mu^2$ for

$$\tilde{P}(\mathbf{y}_1) = (g \circ P \circ g)(\mathbf{y}_1) = g(P(\mathbf{x}_1)) = g(\mathbf{x}_2) = \mathbf{y}_2$$

and

$$\tilde{P}(\mathbf{y}_2) = (g \circ P \circ g)(\mathbf{y}_2) = g(P(\mathbf{x}_2)) = g(\mathbf{x}_1) = \mathbf{y}_1$$

Hence the trajectory Γ_μ is Δ-symmetric, because both the points $\mathbf{x}_1$ and $g(\mathbf{x}_1) \neq \mathbf{x}_1$ lie on Γ_μ.

Let Ω_μ be the period of the double period solution corresponding to the trajectory Γ_μ. The points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1, \mathbf{y}_2$ lie on the trajectory Γ_μ in the order $\mathbf{x}_1, \mathbf{y}_1, \mathbf{x}_2, \mathbf{y}_2, \mathbf{x}_1$ or in the order $\mathbf{x}_1, \mathbf{y}_2, \mathbf{x}_2, \mathbf{y}_1, \mathbf{x}_1$. According to Lemma 2 the phase shift between $\mathbf{x}_1$ and $\mathbf{y}_1$ and also between the points $\mathbf{x}_2$ and $\mathbf{y}_2$ is $\frac{1}{2}\Omega_\mu$. Hence the parts of Γ_μ between the points $\mathbf{x}_2, \mathbf{y}_1$ and $\mathbf{x}_1, \mathbf{y}_2$ *have no "moving" time*. This is in contradiction with our assumption about the existence of a period doubling bifurcation.

Corollary 1. *The Δ-symmetric solution cannot bifurcate into an n-periodic solution.*

The proof is fully analogous to that of Theorem 2.

Let us suppose that an *invariant torus* $\mathbf{T}^2_\mu$ arises from the Δ-symmetric solution by the *Neimark-Sacker* bifurcation. This occurs in the system (1), *see* [9]. Then Corollary 1 yields:

Corollary 2. *The rotation number $\varrho(\mu)$ of the flow T^t_μ on the torus $\mathbf{T}^2_\mu$ is irrational for all $\mu \in O(\mu_0)$, where $O(\mu_0)$ is sufficiently small, and hence $\varrho(\mu)$ is constant.*

4. CLOSING REMARKS

It seems proper to make the following remarks.

1. Systems of the forms (4) often arise when two identical oscillators are coupled and the coupling between them is symmetrical, see for example [11].

2. The system (1) serves as an example. It has a periodic (HS) which arises by the *Hopf* bifurcation from the steady state solution $[A, B/A, A, B/A]$, *see* [3] [10].

3. The Δ-symmetric solution of (1) arises by the second *Hopf* bifurcation from the steady state solution $[A, B/A, A, B/A]$ when the second pair of eigenvalues crosses the imaginary axis, *see* [10], [11].

4. Δ-symmetric solution has been found numerically in [9]. This solution bifurcated from the (HS) by a period doubling bifurcation.

APPENDIX

THE BIFURCATION OF (HS) WHEN ONE MULTIPLIER IS EQUAL TO +1

The cross section Σ of the orbit γ_{μ_0} can be chosen in such a way that

1. Σ is a hyperplane of the codimension 1.
2. The hyperplane Σ contains the origin $\mathbf{0} \in \mathbb{R}^4$.

3. Σ intersects transversally the orbit $\gamma_{\mu 0}$ at the point $\mathbf{x}_{\mu 0}$.

4. $g(\Sigma) = \Sigma$.

Then the Poincaré map $P_{\mu 0}$ corresponding to the orbit $\gamma_{\mu 0}$ is defined on $\Sigma(\mathbf{x}_{\mu 0}) = \sum \cap B_\varepsilon(\mathbf{x}_{\mu 0})$, where $B_\varepsilon(\mathbf{x}_{\mu 0})$ is an appropriate neighbourhood of the point $\mathbf{x}_{\mu 0}$ in $\mathbb{R}^4$. The map $P_{\mu 0}$ commutes with the diffeomorphism g, see Lemma 3. The equation of the hyperplane Σ is

$$x_1 + by_1 + x_2 + by_2 = 0 \,. \tag{13}$$

The value of the coefficient b is determined by the coordinates of the point $\mathbf{x}_{\mu 0}$. The intersection $\Sigma \cap \Delta$ is the straight line p and $\mathbf{x}_{\mu 0} \in p$. The straight line p has the directional vector $\boldsymbol{u} = b\mathbf{e}_1 - \mathbf{e}_2 = [b, -1, b, -1]$.

Now we introduce a new coordinate system in the hyperplane Σ. The origin of this coordinate system is located at the point $\mathbf{x}_{\mu 0}$. The directions of the coordinate axes x, y, z are given by the vectors $\mathbf{e}_3$, $\mathbf{e}_4$, $\boldsymbol{u}$. The axis z is identical with the straight line p. The following lemma is obvious.

Lemma 5. *In the above coordinate system on Σ, the restriction $g|_\Sigma$ is a linear mapping with the matrix*

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

that is,

$$g|_\Sigma(x, y, z) = (-x, -y, z) \,. \tag{14}$$

Let us consider the one-parameter system of the Poincaré maps P_μ corresponding to the orbits γ_μ for the values of the parameter μ sufficiently close to μ_0. Thus the map P_μ commutes with the mapping $g|_\Sigma$ on $\Sigma(\mathbf{x}_{\mu 0})$ for all μ sufficiently close to μ_0. Let us denote the multipliers of the orbit $\gamma_{\mu 0}$ by $\lambda_1(\mu_0)$, $\lambda_2(\mu_0)$, $\lambda_3(\mu_0)$. It is well known that these multipliers are the eigenvalues of the matrix $\mathsf{D}_{\mathbf{x}}\, P_{\mu 0}(\mathbf{x}_{\mu 0})$ i.e. of the Jacobi matrix of the map $P_{\mu 0}$ at the point $\mathbf{x}_{\mu 0}$. Let us assume that $\mathsf{D}_{\mathbf{x}}\, P_{\mu 0}(\mathbf{x}_{\mu 0})$ has only one simple eigenvalue $+1$ on the unit circle, viz. $\lambda_1(\mu_0) = +1$.

Now we can apply Theorem 1.1 from [8] (or Theorem 7.1 in [7], Chap. 7). The one-dimensional eigenspace E^0 corresponding to the eigenvalue $+1$ is invariant under the mapping (14). Two distinct cases are possible.

Case I. The eigenspace E^0 is spanned by the vector $\boldsymbol{u} = b\mathbf{e}_1 - \mathbf{e}_2$. The corresponding *center manifold* is the axis z. The restriction of the mapping (14) on the axis z is the identity map and therefore the restriction of the Poincaré map P_μ on the z-axis commutes only with the identity. This is a standard situation, *see* [5], in which two orbits exist for μ on one side of the critical value μ_0 while no orbit exists on the other side of μ_0. In terms of the *solution diagram*, there exists a *turning point* for $\mu = \mu_0$.

Case II. The eigenspace E^0 is spanned by a vector $\alpha\mathbf{e}_3 + \beta\mathbf{e}_4$, α, β are some real numbers. The corresponding center manifold is invariant under the mapping (14) and we may assume without loss of generality that this center manifold is the axis x The restriction of the mapping (14) on the x-axis has the form

$$\tilde{g}(x) = -x\,. \tag{15}$$

Then the restriction of the Poincaré map P_μ on the x-axis must commute with the mapping (15), hence this mapping has the form, *see* [5],

$$P_\varepsilon(x) = (\varepsilon + 1)\,x \pm x^3\,, \tag{16}$$

where $\varepsilon = \mu - \mu_0$. It is easy to check the validity of the relation

$$P_\varepsilon(\tilde{g}(x)) = \tilde{g}(P_\varepsilon(x))\,. \tag{17}$$

From the form (16) of the Poincaré map we can deduce that on one side of the criticality one orbit exists while on the other side three orbits exist. In terms of the solution diagram, the well-known *pitchfork* bifurcation appears.

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Souhrn

BIFURKACE ZDVOJENÍ PERIODY V MODELU DVOU SPŘAŽENYCH TANKŮ S REAKCÍ TYPU „BRUSSELATOR"

ALOIS KLÍČ

Budiž dána soustava diferenciálních rovnic $\dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}, \mu)$, kde $\mathbf{x} \in \mathbb{R}^4$ a $\mu \in \mathbb{R}$ je parametr. Vektorové pole $\mathbf{v}$ je invariantní vůči jistému lineárnímu difeomorfizmu $g : \mathbb{R}^4 \to \mathbb{R}^4$. Jsou dokázány dvě věty o bifurkacích periodických řešení dané soustavy. V dodatku je analyzován vliv symetrie na bifurkační jevy pro případ, kdy jeden multiplikátor orbity je roven $+1$.

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