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Tomáš Cipra

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IMPROVEMENT OF PREDICTION FOR A LARGER NUMBER OF STEPS IN DISCRETE STATIONARY PROCESSES

Tomáš Cipra

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In this paper conditions are investigated under which an additional process does not improve the prediction in a given discrete stationary process provided the "compound" process is ARMA (m, n) and the prediction is constructed for a general number of steps. So this paper develops some existing works devoted to the improvement of prediction for the single-step predictors and deals with the connection between the single-step prediction and the prediction for a greater number of steps. Moreover, some useful hints for the actual prediction in the multivariate ARMA (m, n) process are contained in the paper.

1. INTRODUCTION

Let $\{X_t\}$ be a vector discrete stationary process with zero mean value. Denote by $\hat{X}_t(a)$ the predictor of X_t based on $X_{t-a}, X_{t-a-1}, \ldots$ (i.e. the particular scalar components of $\hat{X}_t(a)$ are the best linear approximations of the corresponding components of X_t in the Hilbert space generated by all scalar components of all vectors $X_{t-a}, X_{t-a-1}, \ldots$, see e.g. [10]). It happens very frequently in practical situations that we are not satisfied with the accuracy of this predictor and therefore try to improve it by means of an additional (vector) process $\{Y_t\}$ (the dimensions of the processes $\{X_t\}$ and $\{Y_t\}$ may be different but finite). Analogously, denote by $\hat{X}_t(a, b)$ the new "better" predictor of X_t based on $X_{t-a}, X_{t-a-1}, \ldots, Y_{t-b}, Y_{t-b-1}, \ldots$.

The predictor $\hat{X}_t(a, b)$ also plays an important role in such cases when particular components of a vector process are observed through various time periods which is typical for meteorology, hydrology and other technical and economic disciplines (e.g., data on the rainfall are delivered with a certain delay after the data on the temperature).

The accuracy of the predictors is measured as usual by the matrices

$$\Delta_X(a) = \mathsf{E}[X_t - \hat{X}_t(a)]\,[X_t - \hat{X}_t(a)]' ,$$

$$\Delta_X(a, b) = \mathsf{E}[X_t - \hat{X}_t(a, b)]\,[X_t - \hat{X}_t(a, b)]' .$$

Obviously, it holds that

$$\Delta_X(a) \geqq \Delta_X(a, b) \geqq \Delta_X(a, c) \quad (b \geqq c), \tag{1}$$

where e.g. $\Delta_X(a) \geqq \Delta_X(a, b)$ denotes that the matrix $\Delta_X(a) - \Delta_X(a, b)$ is positive semidefinite. If $\Delta_X(a) = \Delta_X(a, b)$ we can say that the process $\{Y_t\}$ does not improve the prediction in the process $\{X_t\}$ for the given steps of the prediction.

There is another reason supporting the importance of the mentioned problem. The concept of causal relations (or causality) among time series may be based just on the improvement of prediction (see [8], [9] and for economic applications e.g. [11]). The basic definition of the causality is such that a time series $\{Y_t\}$ causes another time series $\{X_t\}$ if X_t can be predicted better by some values of $\{Y_t\}$ than by not doing so (various types of causality are distinguished with respect to the used values of $\{Y_t\}$).

Anděl [1], [2], [3] and Cipra [4], [5], [6] dealt with various problems of the improvement of prediction for the single-step predictor $\hat{X}_t(1)$. Provided the compound process $\{W_t\} = \{(X_t', Y_t')'\}$ is the process AR, MA or ARMA the conditions for the equality $\Delta_X(1) = \Delta_X(1,\mathrm{b})$ were formulated in the mentioned papers and it was shown that certain equalities of this type can imply that the processes $\{X_t\}$ and $\{Y_t\}$ are uncorrelated.

The problem of improvement of the predictor $\hat{X}_t(a)$ with a greater number of the predictive steps $(a \geqq 1)$ is much more complicated than the previous cases. In the present paper the equality $\Delta_X(a) = \Delta_X(a, a)$ is investigated because of its importance from the practical point of view. An interesting connection of this equality with the equality $\Delta_X(1) = \Delta_X(1, 1)$ is revealed.

2. PRELIMINARIES

A brief survey of the mathematical tools used is given in this section.

The following assertion is the well-known theorem on the inversion of a matrix divided into blocks:

Theorem 1. *Let* $\begin{pmatrix} K, & L \\ M, & N \end{pmatrix}$ *be a square regular matrix with square blocks* K *and* N. *If the block* N *is regular then the matrix* $K - LN^{-1}M$ *is also regular and*

$$\begin{pmatrix} K, & L \\ M, & N \end{pmatrix}^{-1} = \begin{pmatrix} (K - LN^{-1}M)^{-1}, & -(K - LN^{-1}M)^{-1}LN^{-1} \\ -N^{-1}M(K - LN^{-1}M)^{-1}, & N^{-1} + N^{-1}M(K - LN^{-1}M)^{-1}LN^{-1} \end{pmatrix}.$$

Now consider an r-dimensional ARMA (m, n) process $\{W_t\}$ defined by

$$\sum_{k=0}^{n} A_k W_{t-k} = \sum_{j=0}^{m} B_j Z_{t-j}, \tag{2}$$

where A_k and B_j are $r \times r$ matrices such that

$$\det\left(\sum_{k=0}^{n} A_k z^k\right) \neq 0 \quad \text{for} \quad |z| \leqq 1\,, \quad B_0 \neq 0 \tag{3}$$

and $\{Z_t\}$ is an r-dimensional white noise, i.e. $\mathsf{E}Z_t = 0$, $\operatorname{var} Z_t = I_{r \times r}$ (a unit matrix), $\operatorname{cov}(Z_s, Z_t) = 0$ for $s \neq t$. The assumption (3) is usual for the model ARMA (m, n).

Further, let $\{W_t\} = \{(X'_t, Y'_t)'\}$, where the process $\{X_t\}$ is p-dimensional and the process $\{Y_t\}$ is q-dimensional $(p + q = r)$. Denote

$$\sum_{k=0}^{n} A_k z^k = \begin{pmatrix} K(z), & L(z) \\ M(z), & N(z) \end{pmatrix}, \quad \sum_{j=0}^{m} B_j z^j = \begin{pmatrix} P(z), & Q(z) \\ R(z), & S(z) \end{pmatrix}, \tag{4}$$

$$v(z) = \det N(z)\,, \quad N_0(z) = \operatorname{adj} N(z)\,, \tag{5}$$

$$v(z)\,K(z) - L(z)\,N_0(z)\,M(z) = \sum_{k=0}^{n(q+1)} F_k z^k\,, \tag{6}$$

$$v(z)\,P(z) - L(z)\,N_0(z)\,R(z) = \sum_{j=0}^{nq+m} G_j z^j\,, \tag{7}$$

$$v(z)\,Q(z) - L(z)\,N_0(z)\,S(z) = \sum_{j=0}^{nq+m} H_j z^j\,, \tag{8}$$

where the blocks $K(z)$ and $P(z)$ are $p \times p$ matrices, $N(z)^{-1} = 1/v(z)\,N_0(z)$ and $p \times p$ matrices F_k, $p \times p$ matrices G_j and $p \times q$ matrices H_j do not depend on z (obviously, the matrices F_k, G_j and H_j are unambiguously defined by (6)–(8)).

The following theorem proved by Anděl [3] will be very useful for this paper:

Theorem 2. *Let the model* (2) *satisfy the assumption* (3) *and let*

$$\det N(z) \neq 0 \quad \textit{for} \quad |z| \leqq 1\,. \tag{9}$$

Consider the following model

$$\sum_{k=0}^{n(q+1)} F_k \xi_{t-k} = \sum_{j=0}^{nq+m} G_j \eta_{t-j} + \sum_{j=0}^{nq+m} H_j \zeta_{t-j}\,, \tag{10}$$

where $\{\eta_t\}$ is a p-dimensional white noise, $\{\zeta_t\}$ is a q-dimensional white noise and the processes $\{\eta_t\}$ and $\{\zeta_t\}$ are uncorrelated, i.e.

$$\mathsf{E}\eta_t = 0\,, \quad \mathsf{E}\zeta_t = 0\,, \quad \operatorname{var} \eta_t = I_{p\times p}\,, \quad \operatorname{var} \zeta_t = I_{q \times q}\,,$$

$$\operatorname{cov}(\eta_s, \eta_t) = 0 \;\textit{for}\; s \neq t\,, \quad \operatorname{cov}(\zeta_s, \zeta_t) = 0 \;\textit{for}\; s \neq t\,, \quad \operatorname{cov}(\eta_s, \zeta_t) = 0\,.$$

Then the relation

$$\det\left(\sum_{k=0}^{n(q+1)} F_k z^k\right) \neq 0 \quad \textit{for} \quad |z| \leqq 1 \tag{11}$$

holds in the model (10) *and there exists a p-dimensional process ξ_t defined by the* *model* (10) *such that*

$$\xi_t \in \mathrm{H}\{\eta_t, \eta_{t-1}, \ldots, \zeta_t, \zeta_{t-1}, \ldots\}. \tag{12}$$

Moreover, this process $\{\xi_t\}$ *is unambiguously defined by* (10) *and* (12) *and its spectral density matrix is equal to the spectral density matrix of the process* $\{X_t\}$.

The condition (12) indicates that each of the scalar components of ξ_t lies in the Hilbert space generated by all scalar components of all vectors $\eta_t, \eta_{t-1}, \ldots, \zeta_t, \zeta_{t-1}, \ldots$, (see [10]).

Since the processes $\{X_t\}$ and $\{\xi_t\}$ have the same spectral properties they must also have the same predictive properties. Therefore Theorem 2 provides a convenient method how to predict in the process $\{X_t\}$ when only the model (2) for the compound process $\{W_t\} = \{(X_t', Y_t')'\}$ is known. Using this method we take advantage of the explicit model

$$\sum_{k=0}^{n(q+1)} F_k X_{t-k} = \sum_{j=0}^{nq+m} G_j \eta_{t-j} + \sum_{j=0}^{nq+m} H_j \zeta_{t-j}, \tag{13}$$

when we predict in the process $\{X_t\}$ (i.e. when we look for $\hat{X}_t(a)$). It is possible to write the model (13) in the following more compact form:

$$\sum_{k=0}^{n(q+1)} F_k X_{t-k} = \sum_{j=0}^{nq+m} U_j \varepsilon_{t-j}, \tag{14}$$

where $U_j = (G_j, H_j)$ is a compound matrix and $\{\varepsilon_t\} = \{(\eta_t', \zeta_t')'\}$ is an r-dimensional white noise.

3. IMPROVEMENT OF PREDICTION FOR A LARGER NUMBER OF STEPS

We shall investigate the equality $\Delta_X(a) = \Delta_X(a, a)$ for the model (2) in this section. The main result is contained in Theorem 4 where a general sufficient condition for this equality is given. Theorem 4 has a consequence that is important from the practical point of view. This consequence is formulated in Theorem 5. The following Lemma 3 has only an auxiliary character:

Lemma 3. *Consider the model* (14) (*i.e. specially, the relation* (11) *holds*). *Let* $T_0, T_1, \ldots$ *be* $p \times r$ *matrices such that*

$$F(z)\, T(z) = U(z) \quad \textit{for} \quad |z| \leqq 1, \tag{15}$$

where

$$F(z) = \sum_{k=0}^{n(q+1)} F_k z^k, \quad U(z) = \sum_{j=0}^{nq+m} U_j z^j, \quad T(z) = \sum_{i=0}^{\infty} T_i z^i. \tag{16}$$

Let a be an arbitrary fixed natural number. Further, let there exist $p \times p$ *matrices* $C_0 = I, C_1, \ldots, C_{nq+m}$ *such that*

$$\det\left(\sum_{j=0}^{nq+m} C_j z^j\right) \neq 0 \quad \textit{for} \quad |z| \leqq 1 \tag{17}$$

and

(18) $$U_j = C_j U_0\,, \quad j = 0, 1, \ldots, nq + m\,.$$

Then

(19) $$\Delta_X(a) = T_0 T_0' + T_1 T_1' + \ldots + T_{a-1} T_{a-1}'\,.$$

Remark. The matrices $T_0, T_1, \ldots$ exist and are unambiguously defined by (15) and (16). This follows from the assumption (11).

Proof of Lemma 3. Put

(20) $$U_0 \varepsilon_t = v_t\,.$$

According to (18) and (20) we can write the model (14) in the following form:

(21) $$\sum_{k=0}^{n(q+1)} F_k X_{t-k} = \sum_{j=0}^{nq+m} C_j v_{t-j}\,,$$

where $\mathsf{E} v_t = 0$, $\operatorname{var} v_t = U_0 U_0'$ and $\operatorname{cov}(v_s, v_t) = 0$ for $s \neq t$. The assumption (17) guarantees that the model (21) is invertible (see e.g. [7]), i.e.

(22) $$v_t \in \mathrm{H}\{X_t, X_{t-1}, \ldots\}\,.$$

The inverse relation

(23) $$X_t \in \mathrm{H}\{v_t, v_{t-1}, \ldots\}$$

holds according to (11).

Further, introduce $p \times p$ matrices $\tilde{T}_0, \tilde{T}_1, \ldots$ by the relation

(24) $$F(z)\, \tilde{T}(z) = C(z)\,,$$

where

(25) $$C(z) = \sum_{j=0}^{nq+m} C_j z^j\,, \quad \tilde{T}(z) = \sum_{i=0}^{\infty} \tilde{T}_i z^i$$

i.e. matrices $\tilde{T}_0, \tilde{T}_1 \ldots$ are defined analogously to the matrices $T_0, T_1, \ldots$ (see Remark). Obviously, the relation $U(z) = C(z)\, U_0$ holds so that

(26) $$T_i = \tilde{T}_i U_0\,, \quad i = 0, 1, \ldots,$$

since $F(z)\, T(z) = C(z)\, U_0$, $F(z)\, \tilde{T}(z) = C(z)$ and the matrices T_i and $\tilde{T}_i$ are unambiguously defined.

According to (24) it follows from the model (21) that

(27) $$X_t = \sum_{i=0}^{\infty} \tilde{T}_i v_{t-i} = \sum_{i=0}^{a-1} \tilde{T}_i v_{t-i} + \sum_{i=a}^{\infty} \tilde{T}_i v_{t-i}\,.$$

The condition (22) implies that $\sum_{i=a}^{\infty} \tilde{T}_i v_{t-i} \in \mathrm{H}\{X_{t-a}, X_{t-a-1}, \ldots\}$ and the condition (23) implies that $\sum_{i=0}^{a-1} \tilde{T}_i v_{t-i}$ is orthogonal to $\mathrm{H}\{X_{t-a}, X_{t-a-1}, \ldots\}$. Therefore

(28) $$X_t - \hat{X}_t(a) = \sum_{i=0}^{a-1} \tilde{T}_i v_{t-i}$$

and we can write

$$\Delta_X(a) = \mathsf{E}[X_t - \hat{X}_t(a)]\,[X_t - \hat{X}_t(a)]' =$$

$$= \mathsf{E}(\sum_{i=0}^{a-1} \tilde{T}_i v_{t-i})\,(\sum_{i=0}^{a-1} \tilde{T}_i v_{t-i})' = \sum_{i=0}^{a-1} \tilde{T}_i \{\mathsf{E}(v_{t-i} v_{t-i}')\}\, \tilde{T}_i' =$$

$$= \sum_{i=0}^{a-1} \tilde{T}_i U_0 U_0' \tilde{T}_i' = \sum_{i=0}^{a-1} T_i T_i' ,$$

making use of the relation (26). So the formula (19) is proved.

Theorem 4. *Consider an r-dimensional* ARMA (m, n) *process* $\{W_t\}$ *defined by*

(29) $$\sum_{k=0}^{n} A_k W_{t-k} = \sum_{j=0}^{m} B_j Z_{t-j} ,$$

where A_k *and* B_j *are* $r \times r$ *matrices such that*

(30) $$\det(\sum_{k=0}^{n} A_k z^k) \neq 0 , \quad \det(\sum_{j=0}^{m} B_j z^j) \neq 0 , \quad \det N(z) \neq 0 \quad \text{for} \quad |z| \leqq 1$$

(the matrix $N(z)$ *is defined in* (4)) *and* $\{Z_t\}$ *is an r-dimensional white noise. Let* $\{W_t\} = \{(X_t', Y_t')'\}$, *where the process* $\{X_t\}$ *is p-dimensional and the process* $\{Y_t\}$ *is q-dimensional* $(p + q = r)$. *Let the denotation* (4)–(8) *hold. Finally, let there exist* $p \times p$ *matrices* $D_0 = I, D_1, \ldots, D_{nq+m}$ *such that*

(31) $$(G_j, H_j) = D_j(G_0, H_0) , \quad j = 0, 1, \ldots, nq + m .$$

Then $\Delta_X(a) = \Delta_X(a, a)$ *holds for an arbitrary natural number* a.

Proof. It is convenient to divide the proof into several parts:

(i) In the first part the matrix

(32) $$\Delta(a) = \mathsf{E}[W_t - \hat{W}_t(a)]\,[W_t - \hat{W}_t(a)]'$$

will be calculated, where $\hat{W}_t(a)$ is the predictor of W_t based on $W_{t-a}, W_{t-a-1}, \ldots$ (i.e. $\Delta(a)$ measures the accuracy of this predictor). If $r \times r$ matrices $V_0, V_1, \ldots$ are defined by

(33) $$A(z)\,V(z) = B(z) ,$$

where

(34) $$A(z) = \sum_{k=0}^{n} A_k z^k , \quad B(z) = \sum_{j=0}^{m} B_j z^j , \quad V(z) = \sum_{i=0}^{\infty} V_i z^i ,$$

then it holds that

(35) $$\Delta(a) = V_0 V_0' + V_1 V_1' + \ldots + V_{a-1} V_{a-1}' .$$

The formula (35) follows easily from the relation

$$W_t - \hat{W}_t(a) = \sum_{i=0}^{a-1} V_i Z_{t-i}\,, \tag{36}$$

which can be derived similarly as the formula (28) for the model (21) in the proof of Lemma 3. Clearly, the matrix $\Delta_X(a, a)$ that is of interest to us is equal to the upper left $p \times p$ block of the matrix $\Delta(a)$.

(ii) In the second part of the proof we shall prove that the matrices $D_0, D_1, \ldots$ $\ldots, D_{nq+m}$, the existence of which is assumed, must moreover fulfil the following relation:

$$\det D(z) \neq 0 \quad \text{for} \quad |z| \leqq 1\,, \tag{37}$$

where

$$D(z) = \sum_{j=0}^{nq+m} D_j z^j\,. \tag{38}$$

According to (7) and (8) we obtain that

$$v(z)\,P(z) - L(z)\,N_0(z)\,R(z) = D(z)\,G_0\,, \tag{39}$$

$$v(z)\,Q(z) - L(z)\,N_0(z)\,S(z) = D(z)\,H_0\,. \tag{40}$$

Further, according to Theorem 1,

$$\begin{pmatrix} K(z), & L(z) \\ M(z), & N(z) \end{pmatrix}^{-1} \begin{pmatrix} P(z), & Q(z) \\ R(z), & S(z) \end{pmatrix} = \tag{41}$$

$$= \begin{pmatrix} [K(z)-L(z)N(z)^{-1}M(z)]^{-1}[P(z)-L(z)N(z)^{-1}R(z)]\,, & [K(z)-L(z)N(z)^{-1}M(z)]^{-1}[Q(z)-L(z)N(z)^{-1}S(z)] \\ * & * \end{pmatrix}$$

holds, where the asterisks replace the blocks that are not important for the proof. According to the assumption (30) the both matrices on the left-hand side of the equality (41) are regular for $|z| \leqq 1$ so that the matrix

$$\begin{pmatrix} [K(z) - L(z)\,N(z)^{-1}\,M(z)]^{-1}\,[P(z) - L(z)\,N(z)^{-1}\,R(z)], & [K(z) - L(z)\,N(z)^{-1}\,M(z)]^{-1}\,[Q(z) - L(z)\,N(z)^{-1}\,S(z)] \end{pmatrix}$$

with p rows and r columns must have rank p for $|z| \leqq 1$. It is possible to reduce this matrix in the following way:

$$\begin{aligned}
&[K(z) - L(z)\,N(z)^{-1}\,M(z)]^{-1}\,(P(z) - L(z)\,N(z)^{-1}\,R(z),\, Q(z) - L(z)\,N(z)^{-1}\,S(z)) = \\
&\quad = v^{-1}(z)\,[K(z) - L(z)\,N(z)^{-1}\,M(z)]^{-1} \times \\
&\quad \times (v(z)\,P(z) - L(z)\,N_0(z)\,R(z),\, v(z)\,Q(z) - L(z)\,N_0(z)\,S(z)) = \\
&\quad = v^{-1}(z)\,[K(z) - L(z)\,N(z)^{-1}\,M(z)]^{-1}\,D(z)\,(G_0, H_0)\,,
\end{aligned}$$

where we took advantage of (39) and (40). Therefore the matrix $D(z)$ must be regular for $|z| \leqq 1$.

(iii) In this part of the proof we shall derive the formula for $\Delta_X(a)$. According to Theorem 2 we can use the model (14) for this purpose. Further, according to the assumption (31) and to the previous part of the proof, there exist matrices $D_0 = I$, $D_1, \ldots, D_{nq+m}$ such that

$$U_j = D_j U_0\,, \quad j = 0, 1, \ldots, nq + m \tag{42}$$

and (37) are fulfilled simultaneously.

Therefore, the assumptions of Lemma 3 are fulfilled provided we write D_j instead of C_j. Hence $\Delta_X(a) = T_0 T_0' + \ldots + T_{a-1} T_{a-1}'$, where the denotations (15)–(16) are used.

(iv) In this part we shall complete the proof since we shall show that the matrix $\Delta_X(a) = T_0 T_0' + \ldots + T_{a-1} T_{a-1}'$ is equal to the upper left $p \times p$ block of the matrix $\Delta(a) = V_0 V_0' + \ldots + V_{a-1} V_{a-1}'$.

From (15) we obtain

$$T(z) = F(z)^{-1}\, U(z) = \tag{43}$$

$$= [K(z) - L(z)\, N(z)^{-1}\, M(z)]^{-1}\, (P(z) - L(z)\, N(z)^{-1}\, R(z),\, Q(z) - L(z)\, N(z)^{-1}\, S(z))$$

and from (33) we obtain

$$V(z) = A(z)^{-1}\, B(z) = \tag{44}$$

$$= \begin{pmatrix} [K(z) - L(z) N(z)^{-1} M(z)]^{-1} [P(z) - L(z) N(z)^{-1} R(z)], & [K(z) - L(z) N(z)^{-1} M(z)]^{-1} [Q(z) - L(z) N(z)^{-1} S(z)] \\ *\,, & * \end{pmatrix}$$

for $|z| \leqq 1$. Comparing (43) and (44) we can conclude that

$$T_i = \bar{V}_i\,, \quad i = 0, 1, \ldots\,, \tag{45}$$

where the $p \times r$ matrices $\bar{V}_i$ are formed by the first p rows of the matrices V_i. Hence the matrix $T_i T_i'$ is equal to the upper left $p \times p$ block of the matrix $V_i V_i'$ and therefore the matrix $\Delta_X(a) = T_0 T_0' + \ldots + T_{a-1} T_{a-1}'$ is equal to the upper left $p \times p$ block of the matrix $\Delta(a) = V_0 V_0' + \ldots + V_{a-1} V_{a-1}'$, i.e. to the matrix $\Delta_X(a, a)$.

Anděl [3] proved that for $a = 1$ the condition (31) is also necessary for the equality $\Delta_X(1) = \Delta_X(1,1)$. Combining this result with that of Theorem 4 we obtain the following important conclusion:

Theorem 5. *Let $\{W_t\}$ be the process from Theorem 4. Then the following implication holds*:

$$[\Delta_X(1) = \Delta_X(1, 1)] \Rightarrow [\Delta_X(a) = \Delta_X(a, a),\; a = 1, 2, \ldots]\,. \tag{46}$$

In other words, provided the process $\{Y_t\}$ did not improve the single-step predictor in the process $\{X_t\}$ it has no sense to use the process $\{Y_t\}$ when we wish to improve the prediction in $\{X_t\}$ for a larger number of steps. This conclusion seems to be useful for practical forecasting.

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Souhrn

ZPŘESŇOVÁNÍ PREDIKCE O VĚTŠÍ POČET KROKŮ V DISKRÉTNÍCH STACIONÁRNÍCH PROCESECH

TOMÁŠ CIPRA

Nechť $\{X_t\}$ je vektorový diskrétní stacionární proces s nulovou střední hodnotou a nechť $\hat{X}_t(a)$ označuje predikci veličiny X_t založenou na veličinách $X_{t-a}, X_{t-a-1}, \ldots$. V praktických situacích se často stává, že nejsme spokojeni s přesností této predikce a snažíme se ji zlepšit přidáním dalšího (vektorového) procesu $\{Y_t\}$, takže vlastně používáme predikci $\hat{X}_t(a, b)$ veličiny X_t založenou na $X_{t-a}, X_{t-a-1}, \ldots, Y_{t-b}, Y_{t-b-1}, \ldots$. Přesnost predikcí je měřena, jak je zvykem, pomocí rozptylových matic $\Delta_X(a) = \mathsf{E}[X_t - \hat{X}_t(a)][X_t - \hat{X}_t(a)]'$ a $\Delta_X(a, b) = \mathsf{E}[X_t - \hat{X}_t(a, b)][X_t - \hat{X}_t(a, b)]'$. Jestliže $\Delta_X(a) = \Delta_X(a, b)$, pak lze říci, že proces $\{Y_t\}$ nezlepší predikci v procesu $\{X_t\}$ pro příslušné kroky predikce. Pro případ, že složený proces $\{(X_t', Y_t')'\}$ je typu

ARMA (m, n), je v článku odvozena obecná postačující podmínka pro rovnost $\Delta_X(a) = \Delta_X(a, a)$ (a je libovolné přirozené číslo), neboť tento typ zpřesňování predikce je v praxi nejčastější. Jako důsledek se zajímavým praktickým dosahem je dále ukázáno, že rovnost $\Delta_X(1) = \Delta_X(1, 1)$ již implikuje rovnost $\Delta_X(a) = \Delta_X(a, a)$ pro všechna přirozená čísla a. Navíc mohou být z práce získány některé užité návody pro skutečnou konstrukci predikcí různého typu ve vektorovém procesu ARMA (m, n).

Author's address: RNDr. *Tomáš Cipra*, CSc., Matematicko-fyzikální fakulta University Karlovy, Sokolovská 83, 186 00 Praha 8.