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*Aplikace matematiky*, Vol. 26 (1981), No. 6, 418–425

Persistent URL: <http://dml.cz/dmlcz/103931>

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ON WEIGHTED ENTROPY OF TYPE  $(\alpha, \beta)$   
AND ITS GENERALIZATIONS

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(Received October 11, 1979)

1. INTRODUCTION

Consider the following model for a finite random experiment  $S$ ,

$$(1.1) \quad S = \begin{pmatrix} a_1, a_2, \dots, a_n \\ p_1, p_2, \dots, p_n \\ w_1, w_2, \dots, w_n \end{pmatrix} = \begin{pmatrix} A \\ P \\ W \end{pmatrix},$$

where  $A = (a_1, a_2, \dots, a_n)$  is the alphabet,  $P = (p_1, p_2, \dots, p_n)$  the complete probability distribution and  $W = (w_1, w_2, \dots, w_n)$  is the utility distribution, where  $w_i$ 's are non-negative real numbers.

Belis and Guiaşu [1] introduced the function

$$(1.2) \quad H(P; W) = - \sum_{k=1}^n p_k w_k \log p_k, \quad w_k \geq 0, \quad \sum_{k=1}^n p_k = 1,$$

which they considered a satisfactory measure for the average number quantity of valuable or useful information provided by a source letter.

The measure  $H(P; W)$  is additive in the following sense:

$$(1.3) \quad H(P^*Q; W^*V) = \bar{V} H(P; W) + \bar{W} H(Q; V),$$

where  $Q = (q_1, \dots, q_m)$ ,  $0 < q_j \leq 1$ ,  $\sum_{j=1}^m q_j = 1$ , and  $V = (v_1, \dots, v_m)$  is the utility distribution associated with  $Q$ ,  $\bar{W} = \sum_{k=1}^n p_k w_k$ ,  $\bar{V} = \sum_{j=1}^m v_j q_j$ ,

$$W^*V = \{w_i v_j / w_i \in W, v_j \in V\},$$

$$P^*Q = \{p_i q_j / p_i \in P, q_j \in Q\}.$$

\*) Supported by CNPq

In this paper, we introduce and characterize a weighted entropy of type  $(\alpha, \beta)$  given by

$$(1.4) \quad H_n(P; W; \alpha, \beta) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^n w_k (p_k^\alpha - p_k^\beta), \quad \alpha \neq \beta, \quad \alpha, \beta > 0.$$

Some interesting properties of this measure and its further generalizations are also presented.

## 2. CHARACTERIZATION OF WEIGHTED ENTROPY OF TYPE $(\alpha, \beta)$

Let a function  $H_n(P; W; \alpha, \beta)$  for all  $P = (p_1, \dots, p_n)$ ,  $p_k \geq 0$ ,  $\sum_{k=1}^n p_k = 1$ ,  $\alpha \neq \beta$ ,  $\alpha, \beta > 0$ , satisfy the following axioms:

- (I) *Continuity*:  $H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta)$  is a continuous function of its arguments.
- (II) *Symmetry*:  $H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta)$  is symmetric with respect to the couples  $(p_k, w_k)$ ,  $k = 1, 2, \dots, n$ .
- (III) *Generalized Branching*:

If

$$p_n = p' + p'', \quad w_n = \frac{p'w' + p''w''}{p' + p''},$$

then

$$\begin{aligned} & H_{n+1}(p_1, \dots, p_{n-1}, p', p''; w_1, \dots, w_{n-1}, w', w''; \alpha, \beta) = \\ & = H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta) + \\ & + \frac{A_\alpha}{A_\alpha - A_\beta} p_n^\alpha H_2(p'|p_n, p''|p_n; w', w''; \alpha, 1) + \\ & + \frac{A_\beta}{A_\beta - A_\alpha} p_n^\beta H_2(p'|p_n, p''|p_n; w', w''; 1, \beta), \end{aligned}$$

where  $A_\alpha = (2^{1-\alpha} - 1)$ ,  $A_\beta = (2^{1-\beta} - 1)$ ,  $\alpha \neq \beta$ ,  $\alpha, \beta > 0$ .

- (IV) *Uniformity*:

$$H_n(1/n, \dots, 1/n; w_1, \dots, w_n; \alpha, \beta) = \sum_{k=1}^n (w_k/n) a(n; \alpha, \beta).$$

- (V) *Normality and Decisivity*:

$$a(2; \alpha, \beta) = 1; \quad H_2(1, 0; w_1, w_2; \alpha, \beta) = 0.$$

**Theorem.** *The function satisfying the above axioms (I)–(V) is given by*

$$(2.1) \quad H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{k=1}^n w_k (p_k^\alpha - p_k^\beta),$$

where  $A_\alpha = (2^{1-\alpha} - 1)$ ,  $A_\beta = (2^{1-\beta} - 1)$ ,  $\alpha \neq \beta$ ,  $\alpha, \beta > 0$ .

Before proving the theorem, we shall prove some intermediate results.

Result 1. If  $q_j \geq 0$ ,  $j = 1, 2, \dots, m$ ,  $\sum_{j=1}^m q_j = p_k > 0$  and  $w_k = \sum_{j=1}^m q_j u_j / \sum_{j=1}^m q_j$ , then

$$(2.2) \quad H_{n+m-1}(p_1, \dots, p_{k-1}, q_1, \dots, q_m, p_{k+1}, \dots, p_n; w_1, \dots, w_{k-1}, u_1, \dots, u_m, w_{k+1}, \dots, w_n; \alpha, \beta) = H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta) + \\ + \frac{A_\alpha}{A_\alpha - A_\beta} p_k^\alpha H_m(q_1/p_k, \dots, q_m/p_k; u_1, \dots, u_m; \alpha, 1) + \\ + \frac{A_\beta}{A_\beta - A_\alpha} p_k^\beta H_m(q_1/p_k, \dots, q_m/p_k; u_1, \dots, u_m; 1, \beta).$$

This result can be easily proved by induction (cf. [4]).

Result 2. If

$$q_{kj} \geq 0, \quad j = 1, 2, \dots, m_k, \quad \sum_{j=1}^{m_k} q_{kj} = p_k > 0, \quad k = 1, 2, \dots, n, \\ \sum_{k=1}^n p_k = 1, \quad w_k = \frac{q_{k1}u_{k1} + \dots + q_{km_k}u_{km_k}}{p_k},$$

then

$$(2.3) \quad H_{m_1+\dots+m_n}(q_{11}, q_{12}, \dots, q_{1m_1}; \dots; q_{n1}, q_{n2}, \dots, q_{nm_n}; u_{11}, u_{12}, \dots, u_{1m_1}; \dots; u_{n1}, u_{n2}, \dots, u_{nm_n}; \alpha, \beta) = \\ = H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta) + \\ + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^n p_k^\alpha H_{m_k}(q_{k1}/p_k, \dots, q_{km_k}/p_k; u_{k1}, \dots, u_{km_k}; \alpha, 1) + \\ + \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^n p_k^\beta H_{m_k}(q_{k1}/p_k, \dots, q_{km_k}/p_k; u_{k1}, \dots, u_{km_k}; 1, \beta).$$

Proof follows from Axiom (III).

Result 3.

$$(2.4) \quad a(n; \alpha, \beta) = \frac{A_\alpha}{A_\alpha - A_\beta} a(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} a(n; 1, \beta),$$

where

$$(2.5) \quad a(n; \alpha, 1) = A_\alpha^{-1}(n^{1-\alpha} - 1), \quad \alpha \neq 1,$$

and

$$(2.6) \quad a(n; 1, \beta) = A_\beta^{-1}(n^{1-\beta} - 1), \quad \beta \neq 1.$$

Proof. In (2.3), put  $m_1 = m_2 = \dots = m_n = m$  and  $q_{kj} = 1/nm$ ,  $k = 1, 2, \dots, n$ ;  $j = 1, 2, \dots, m$ , then we get  $p_k = 1/n$  and  $w_k = \sum_{j=1}^m u_{kj}/m$ ,  $k = 1, 2, \dots, n$ , hence

$$(2.7) \quad \begin{aligned} & H_{nm}(1/nm, \dots, 1/nm; u_{11}, u_{12}, \dots, u_{nm}; \alpha, \beta) = \\ & = H_n(1/n, \dots, 1/n; w_1, \dots, w_n; \alpha, \beta) + \\ & + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^n (1/n)^\alpha H_m(1/m, \dots, 1/m; u_{k1}, \dots, u_{km}; \alpha, 1) + \\ & + \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^n (1/n)^\beta H_m(1/m, \dots, 1/m; u_{k1}, \dots, u_{km}; 1, \beta). \end{aligned}$$

By Axiom (IV), (2.7) reduces to

$$(2.8) \quad \begin{aligned} & a(nm; \alpha, \beta) \sum_{k=1}^n \sum_{j=1}^m \frac{u_{kj}}{nm} = a(n; \alpha, \beta) \sum_{k=1}^n \frac{w_k}{n} + \\ & + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^n (1/n)^\alpha a(m; \alpha, 1) \sum_{j=1}^m u_{kj}/m + \\ & + \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^n (1/n)^\beta a(m; 1, \beta) \sum_{j=1}^m u_{kj}/m \end{aligned}$$

or

$$\begin{aligned} a(nm; \alpha, \beta) & = a(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/n)^{\alpha-1} a(m; \alpha, 1) + \\ & + \frac{A_\beta}{A_\beta - A_\alpha} (1/n)^{\beta-1} a(m; 1, \beta). \end{aligned}$$

By symmetry,

$$(2.9) \quad \begin{aligned} a(nm; \alpha, \beta) & = a(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/m)^{\alpha-1} a(n; \alpha, 1) + \\ & + \frac{A_\beta}{A_\beta - A_\alpha} (1/m)^{\beta-1} a(n; 1, \beta). \end{aligned}$$

Putting  $m = 1$  in (2.9) and using Axioms (V) and (III), we get (2.4).

Also from (2.8) and (2.9), we have

$$(2.10) \quad \begin{aligned} & a(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/m)^{\alpha-1} a(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} (1/m)^{\beta-1} a(n; 1, \beta) = \\ & = a(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/n)^{\alpha-1} a(m; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} (1/n)^{\beta-1} a(m; 1, \beta). \end{aligned}$$

The expression (2.10) together with (2.4) gives

$$(2.11) \quad A_x \{ a(m; \alpha, 1) [1 - (1/n)^{\alpha-1}] + [(1/m)^{\alpha-1} - 1] a(n; \alpha, 1) \} = \\ = A_\beta \{ [1 - (1/n)^{\beta-1}] a(m; 1, \beta) + [(1/m)^{\beta-1} - 1] a(n; 1, \beta) \}.$$

Put  $n = 2$  in (2.11), then

$$(2.12) \quad A_x \{ (1 - 2^{1-\alpha}) a(m; \alpha, 1) - (1 - m^{1-\alpha}) = \\ = A_\beta \{ (1 - 2^{1-\beta}) a(m; 1, \beta) - (1 - m^{1-\beta}) \} = C \text{ (say).}$$

For  $m = 1$ , (2.12) gives  $C = 0$ , which immediately gives (2.5) and (2.6). (2.4) together with (2.5) and (2.6) gives

$$(2.13) \quad a(n; \alpha, \beta) = \frac{A_x}{A_x - A_\beta} \frac{n^{1-\alpha} - 1}{A_x} + \frac{A_\beta}{A_\beta - A_x} \frac{n^{1-\beta} - 1}{A_\beta} = \\ = \frac{n^{1-\alpha} - n^{1-\beta}}{A_x - A_\beta}, \quad \alpha \neq \beta.$$

Result 4.

$$(2.14) \quad H_2(p_1, p_2; w_1, w_2; \alpha, \beta) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^2 w_k (p_k^\alpha - p_k^\beta), \quad \alpha \neq \beta,$$

for all  $p_1, p_2 \in [0, 1]$  with  $p_1 + p_2 = 1$ .

Proof. In (2.3) set  $n = 2$ ,  $m_1 = r$ ,  $m_2 = s - r$ ,  $q_{kj} = 1/s$ ,  $k = 1, 2$ ;  $j = 1, 2, \dots, m$ ,

$$p_1 = \sum_{j=1}^{m_1} p_{kj} = \sum_{j=1}^r p_{kj} = \frac{r}{s}, \quad p_2 = \frac{s-r}{s}, \\ w_1 = \frac{1}{p_1} \sum_{j=1}^{m_1} q_{1j} u_{1j} = \frac{1}{r} \sum_{j=1}^r u_{1j}, \quad w_2 = \frac{1}{s-r} \sum_{j=1}^{s-r} u_{2j};$$

then

$$(2.15) \quad H_s(1/s, \dots, 1/s; u_{11}, \dots, u_{1r}, u_{21}, \dots, u_{2,s-r}; \alpha, \beta) = \\ = H_2(p_1, p_2; w_1, w_2; \alpha, \beta) + \\ + \frac{A_x}{A_x - A_\beta} \sum_{k=1}^2 p_k^\alpha H_{m_k}(q_{k1}/p_k, \dots, q_{km_k}/p_k; u_{k1}, \dots, u_{km_k}; \alpha, 1) + \\ + \frac{A_\beta}{A_\beta - A_x} \sum_{k=1}^2 p_k^\beta H_{m_k}(q_{k1}/p_k, \dots, q_{km_k}/p_k; u_{k1}, \dots, u_{km_k}; 1, \beta) = \\ = H_2(p_1, p_2; w_1, w_2; \alpha, \beta) + \\ + \frac{A_x}{A_x - A_\beta} [p_1^\alpha H_r(1/r, \dots, 1/r; u_{11}, \dots, u_{1r}; \alpha, 1) +$$

$$\begin{aligned}
& + p_2^\alpha H_{s-r}(1/(s-r), \dots, 1/(s-r); u_{21}, \dots, u_{2,s-r}; \alpha, 1) + \\
& + \frac{A_\beta}{A_\beta - A_\alpha} [p_1^\beta H_r(1/r, \dots, 1/r; u_{11}, \dots, u_{1r}; 1, \beta) + \\
& + p_2^\beta H_{s-r}(1/(s-r), \dots, 1/(s-r); u_{21}, \dots, u_{2,s-r}; 1, \beta)], \\
H_2(p_1, p_2; w_1, w_2; \alpha, \beta) & = \frac{\sum_{j=1}^r u_{1j} + \sum_{j=1}^{s-r} u_{2j}}{s} a(s; \alpha, \beta) - \\
& - \frac{A_\alpha}{A_\alpha - A_\beta} [p_1^\alpha \sum_{j=1}^r (u_{1j}/r) a(r; \alpha, 1) + p_2^\alpha \sum_{j=1}^{s-r} (u_{2j}/(s-r)) a(s-r; \alpha, 1)] - \\
& - \frac{A_\beta}{A_\beta - A_\alpha} [p_1^\beta \sum_{j=1}^r (u_{1j}/r) a(r; 1, \beta) + p_2^\beta \sum_{j=1}^{s-r} (u_{2j}/(s-r)) a(s-r; 1, \beta)] = \\
& = \frac{a(s; \alpha, \beta)}{s} [r w_1 + (s-r) w_2] - \\
& - \frac{A_\alpha}{A_\alpha - A_\beta} [p_1^\alpha w_1 a(r; \alpha, 1) + p_2^\alpha w_2 a(s-r; \alpha, 1)] - \\
& - \frac{A_\beta}{A_\beta - A_\alpha} [p_1^\beta w_1 a(r; 1, \beta) + p_2^\beta w_2 a(s-r; 1, \beta)].
\end{aligned}$$

The expression (2.15) together with (2.4), (2.5) and (2.6) gives (2.14) for rationals. By Continuity Axiom (I), the above result is valid for all reals  $p_1$  and  $p_2 \in [0, 1]$ . With the help of Axiom (III) it is easy to prove that

$$H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{k=1}^n (p_k^\alpha - p_k^\beta) w_k, \quad \alpha \neq \beta,$$

which completes the proof of Theorem.

### 3. PROPERTIES OF $H_n(P; W; \alpha, \beta)$

1. *Nonnegativity:*  $H_n(P; W; \alpha, \beta) \geq 0$  for  $\alpha, \beta > 0$ .
2. If  $w_1 = w_2 = \dots = w_n = w$ , then

$$H_n(P; W; \alpha, \beta) = w(2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^n (p_k^\alpha - p_k^\beta) = H_n(P; \alpha, \beta),$$

which is an entropy of type  $(\alpha, \beta)$  (cf. Sharma and Taneja [3], Taneja [4]).

3.

$$H_{n+1}(p_1, \dots, p_n, 0; w_1, \dots, w_n, w_{n+1}; \alpha, \beta) = H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta)$$

whatever the weights  $w_1, \dots, w_n, w_{n+1}$  and the probabilities  $p_1, \dots, p_n$  are.

4. The measure  $H_n(P; W; \alpha, \beta)$  is a convex  $\cap$  function of the probability distribution  $(p_1, \dots, p_n)$ , provided one of the parameters  $\alpha$  and  $\beta$  ( $> 0$ ) is greater than unity and the other is less than or equal to unity, i.e., either  $\alpha > 1, 0 < \beta \leq 1$  or  $\beta > 1, 0 < \alpha \leq 1$  (cf. Sharma and Taneja [3], Taneja [4]).

5. *Linearity*: For  $\alpha, \beta > 0$ , the function  $H_n(P; W; \alpha, \beta)$  is linear with respect to the utilities  $W$ , i.e., if  $W' \in R^+, W'' \in R^+, \lambda_1 \in R^+, \lambda_2 \in R^+$ , then  $H_n(P; \lambda_1 W' + \lambda_2 W''; \alpha, \beta) = \lambda_1 H_n(P; W'; \alpha, \beta) + \lambda_2 H_n(P; W''; \alpha, \beta)$ .

#### 4. GENERALIZED MEASURES OF WEIGHTED ENTROPY

The measure  $H(P; W)$  satisfies the additivity in the sense

$$(4.1) \quad H(P^*Q; W^*V) = \bar{V}H(P; W) + \bar{W}H(Q; V),$$

where  $\bar{W} = \sum_{k=1}^n w_k p_k, \bar{V} = \sum_{j=1}^m v_j q_j$ .

In general, regard  $\bar{W}$  and  $\bar{V}$  as functions of probabilities and utilities and consider the generalized additivity in the sense

$$(4.2) \quad H(P^*Q; W^*V) = G(Q; V)H(P; W) + G(P; W)H(Q; V),$$

where  $P, Q, W, V, P^*Q$  and  $U^*V$  have their usual meaning.

Write

$$(4.3) \quad H(P; W) = \sum_{k=1}^n h(p_k, w_k), \quad G(P; W) = \sum_{k=1}^n g(p_k, w_k),$$

where  $h$  and  $g$  are continuous real functions defined on the set  $[0, 1] \times [0, 1]$ .

The expressions (4.2) and (4.3) together give

$$(4.4) \quad \sum_{k=1}^n \sum_{j=1}^m h(p_k q_j, w_k v_j) = \sum_{k=1}^n \sum_{j=1}^m g(p_k, w_k) h(q_j, v_j) + \sum_{k=1}^n \sum_{j=1}^m g(q_j, v_j) h(p_k, w_k).$$

Real continuous solutions of the functional equation (4.4) (cf. Sharma and Gupta [2]) are the following three sets of solutions:

- (i)  $h(p, w) = p^\alpha w^\beta (c_1 \log p + c_2 \log w), g(p, w) = p^\alpha w^\beta,$
- (ii)  $h(p, w) = (1/2k)(p^\alpha w^\beta - p^\gamma w^\delta), g(p, w) = \frac{1}{2}(p^\alpha w^\beta + p^\gamma w^\delta),$  and
- (iii)  $h(p, w) = (1/R) p^\alpha w^\beta \sin(\gamma \log p + \delta \log w),$   
 $g(p, w) = p^\alpha w^\beta \cos(\gamma \log p + \delta \log w),$

where  $\alpha > 0, \beta \geq 0, \gamma, \delta, c_1, c_2, k$  and  $R$  are real constants.



Now take  $c_2 = 0$  in (i),  $\beta = \delta$  in (ii),  $\delta = 0$  in (iii), only for the function  $h$ , and introduce the boundary condition  $h(1, \frac{1}{2}) = 1$ . Then the above three sets of solutions with regard to (4.3) reduce to

$$(4.5) \quad H_{\rho}(P; W) = -2^{\alpha-1} \sum_{k=1}^n p_k^{\alpha} w_k^{\beta} \log p_k,$$

$$(4.6) \quad H_{\rho}(P; W) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^n w_k^{\beta} (p_k^{\alpha} - p_k^{\beta}), \quad \alpha \neq \beta, \quad \alpha, \beta > 0,$$

$$(4.7) \quad H_{\rho}(P; W) = -\frac{2^{\alpha-1}}{\sin \gamma} \sum_{k=1}^n p_k^{\alpha} w_k^{\beta} \sin(\gamma \log p_k), \quad \gamma \neq 0.$$

The limiting case of both the measures (4.6) and (4.7) when  $\gamma \rightarrow \alpha$  in (4.6) and  $\gamma \rightarrow 0$  in (4.7) is the measure (4.5). Thus, (4.5) is the weighted entropy or useful information for  $\alpha = \beta = 1$ .

These measures (4.5), (4.6) and (4.7) are the measures parallel to that studied earlier by Sharma and Taneja [3]. A more detailed study of these measures will appear elsewhere.

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#### Souhrn

### ENTROPIE TYPU $(\alpha, \beta)$ S VAHOU A JEJÍ ZOBECNĚNÍ

GUR DIAL, INDER JEET TANEJA

Belis a Guiasu studovali zobecnění Shannonovy entropie, tj. entropii s vahou nebo užitečnou entropii. V tomto článku je definována a charakterizována entropie typu  $(\alpha, \beta)$  s vahou a jsou studovány její vlastnosti. Je pojednáno též o dalších zobecněních, která zahrnují entropii s vahou s více parametry.

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