Aplikace matematiky

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Aplikace matematiky, Vol. 21 (1976), No. 3, 168–172

Persistent URL: http://dml.cz/dmlcz/103636

A NOTE ON A PAPER BY GOVIL AND KUMAR

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(Received April 1, 1974)

In an earlier paper, "On the behaviour of an intermittently working system with three types of components" published in Aplikace matematiky 16 (1971), 1–9, it was assumed that when the system works under reduced efficiency, it is immediately stopped for repairs. Hence a transition (see fig. 1) from $P_{R_j,m}(t)$ to $P_{W_i,m}(t)$ was not allowed. But, if it were so, where is the need to have a reduced efficiency class. The main purpose of such a class is to carry the work to a certain end without effecting the output of the system. Therefore, to be more realistic, such a transition must occur. In view of this, equation for $P_{R_j,m}(t)$; $P_{W_i,m}(t)$ and $P_{W_k}(t)$ need to be rewritten. Moreover, it is assumed that each time when *the repair of a component* of class L_2 takes place, the system goes to idle state [see $P_{I,m}(t)$] i.e., idle period of the system begins. In other words, system is taken to be doubly idle, first because of the repair of a component (of class L_2) and then, once the repair is done, its idle period begins.

Therefore, keeping in mind the above points we would rewrite the assumptions involved and redefine the various probability states whenever necessary.

ASSUMPTIONS

(i) After a failure of a component in the reduced efficiency class L_2, the system is allowed to work for a requisite time before the repair facilities are available.

(ii) After the repair of the first failure in L_2 (when an other one has not taken place) the system immediately starts operating with normal efficiency.

(iii) After the major repair of a failed component of class L_1 or all the failed components of L_3, the idle period of the system starts. Major repair in L_1 and L_2 includes the repair of a failed component of class L_2.

(iv) In case of a second failure in class L_2, when the system is working with reduced efficiency and waiting for the repair facilities, the system stops working and after the repair of the first component, the repair of the second component begins immediately. So long as the repairs of both the components of class L_2 are not completed, the system remains inoperative. Once the repairs are completed, the idle period of the system begins, as above in (iii).

Other assumptions regarding failure, waiting and repair time distributions in classes L_1 and L_3 are the same as those in the earlier paper.

In class L_2, failure, waiting and repairs follow exponential time distributions with rates λ'_j, θ_j and ϕ_j respectively $(1 \leqq j \leqq M)$. Define:

$P^W_{j,m}(t)$ = the probability that at time t, the system is operating with reduced efficiency while waiting for the facilities to repair the j^{th} component of class L_2 while m components of class L_3 are in working order;

$P_{j,m}(t)$ = the probability that at time t, the system is stopped when the j^{th} component of class L_2 is being repaired while m components of class L_3 and all the components of L_1 are in working order;

$P^W_{j,k,m}(t)$ = the joint probability that at time t, the system is waiting in idle state for the facilities to repair (j^{th}, k^{th}) components in L_2 while m components of class L_3 are in working order.

$P_{j,k,m}(t)$ = the joint probability that at time t, the system is inoperative due to the repair of the j^{th} component of class L_2 and the k^{th} component of the same class is waiting for repairs while m components of class L_3 are in working order.

$P_{k,j_r,m}(t)$ = the joint probability that at time t, the system is inoperative due to the repair of the k^{th} component of class L_2 after completing the repair of the j^{th} component of the same class while m components of class L_3 are in working order.

The definitions for the probability states $P_{0,m}(t)$; $P_{I,m}(t)$; $P_{W_i,m}(t)$; $P_{r_i,m}(t)$; $P_{W_K}(t)$ and $P_{R_K}(t)$ are taken to be the same as in the earlier paper.

We assume that initially the system is operating with normal efficiency, i.e. $P_{0,K}(0) = 1$ so that all other probabilities at $t = 0$ are zero.

The Laplace transforms of the equations of various probability states are given by:

$$(1) \quad [s + \lambda + \lambda' + \lambda'' + \alpha]\, \bar{P}_{0,m}(s) = \lambda''\, \bar{P}_{0,m+1}(s) + \beta\, \bar{P}_{I,m}(s) + \sum_{j=1}^{M} \phi_j\, \bar{P}_{j,m}(s)$$
$$(1 \leqq m \leqq K - 1)\,,$$

$$(2) \quad [s + \lambda + \lambda' + \lambda'' + \alpha]\, \bar{P}_{0,K}(s) = \beta\, \bar{P}_{I,K}(s) + \sum_{j=1}^{M} \phi_j\, \bar{P}_{j,K}(s)\,.$$

$$(3) \quad [s + \beta]\, \bar{P}_{I,m}(s) = \alpha\, \bar{P}_{0,m}(s) + \sum_{i=1}^{N} \eta_i\, \bar{P}_{r_i,m}(s) + \sum_{j=1}^{M} \sum_{k=1}^{M} \phi_k\, \bar{P}_{k,j_r,m}(s)$$
$$(1 \leqq m \leqq K - 1)\,,$$

(4) $$[s + \beta]\,\bar{P}_{I,K}(s) = \alpha\,\bar{P}_{0,K}(s) + \sum_{i=1}^{N} \eta_i\,\bar{P}_{r_i,K}(s) + \mu\,\bar{P}_{R_K} + \sum_{j=1}^{M}\sum_{k=1}^{M} \phi_k\,\bar{P}_{k,j_r,K}(s),$$

(5) $$[s + \lambda + \lambda' + \lambda'' + \alpha + \theta_j]\,\bar{P}^W_{j,m}(s) = \lambda'_j\,\bar{P}_{0,m}(s) + \lambda''\,\bar{P}^W_{j,m+1}(s)$$
$$(1 \leqq m \leqq K - 1),$$

(6) $$[s + \lambda + \lambda' + \lambda'' + \alpha + \theta_j]\,\bar{P}^W_{j,K}(s) = \lambda'_j\,\bar{P}_{0,K}(s),$$

(7) $$[s + \phi_j]\,\bar{P}_{j,m}(s) = \theta_j\,\bar{P}^W_{j,m}(s) \quad (1 \leqq m \leqq K),$$

(8) $$[s + \theta_j]\,\bar{P}^W_{j,k,m}(s) = \lambda'_k\,\bar{P}^W_{j,m}(s) \quad (1 \leqq m \leqq K),$$

(9) $$[s + \phi_j]\,\bar{P}_{j,k,m}(s) = \theta_j\,\bar{P}^W_{j,k,m}(s) \quad (1 \leqq m \leqq K),$$

(10) $$[s + \phi_k]\,\bar{P}_{k,j_r,m}(s) = \phi_j\,\bar{P}_{j,k,m}(s) \quad (1 \leqq m \leqq K),$$

(11) $$[s + \alpha'_i]\,\bar{P}_{W_i,m}(s) = \lambda_i[\bar{P}_{0,m}(s) + \sum_{j=1}^{M} \bar{P}^W_{j,m}(s)] \quad (1 \leqq m \leqq K),$$

(12) $$[s + \eta_i]\,\bar{P}_{r_i,m}(s) = \alpha'_i\,\bar{P}_{W_i,m}(s) \quad (1 \leqq m \leqq K),$$

(13) $$[s + \alpha'']\,\bar{P}_{W_K}(s) = \lambda''[\bar{P}_{0,1}(s) + \sum_{j=1}^{M} \bar{P}^W_{j,1}(s)],$$

(14) $$[s + \mu]\,\bar{P}_{R_K}(s) = \alpha''\,\bar{P}_{W_K}(s),$$

Simplifying relations (1) and (2) one obtains

(15) $$A\,\bar{P}_{0,m}(s) = \lambda''\,\bar{P}_{0,m+1}(s) + \sum_{j=1}^{M}\sum_{k=1}^{M} [a + b_j + c_{jk}]\,\bar{P}^W_{j,m}(s)$$
$$(1 \leqq m \leqq K - 1)$$

and

(16) $$B\,\bar{P}_{0,K}(s) = 1 + \frac{\mu\alpha''\lambda''\beta}{(s + \mu)(s + \alpha'')(s + \beta)}\,[\bar{P}_{0,1}(s) + \sum_{j=1}^{M} \bar{P}^W_{j,1}(s)],$$

where

$$a = \beta\left[\sum_{i=1}^{N} \frac{\lambda_i\alpha'_i\eta_i}{(s + \alpha'_i)(s + \eta_i)}\right] \Big/ [s + \beta],$$

$$A = [s + \lambda + \lambda' + \lambda'' + \alpha - a - \alpha\beta/(s + \beta)],$$

$$b_j = \theta_j\phi_j/[s + \phi_j],$$

$$c_{jk} = \lambda'_k\phi_k\theta_j\phi_j\beta/[(s + \theta_j)(s + \phi_j)(s + \phi_k)(s + \beta)]$$

and

$$B = \left[A - \sum_{j=1}^{M} \sum_{k=1}^{M} \frac{\lambda_j'[a + b_j + c_{jk}]}{[s + \lambda + \lambda' + \lambda'' + \alpha + \theta_j]}\right].$$

Define the generating functions

$$G(\varkappa, s) = \sum_{m=0}^{K-1} \bar{P}_{0,K-m}(s)\,\varkappa^m \quad \text{and} \quad H_j(\varkappa, s) = \sum_{m=0}^{K-1} \bar{P}^W_{j,k-m}(s)\,\varkappa^m$$
$$(1 \leqq j \leqq M).$$

Multiplying relations (15) and (16) by appropriate powers of x and summing over m, we obtain

(17) $$[A - \lambda''\varkappa]\, G(\varkappa, s) = \sum_{j=1}^{M} \sum_{k=1}^{M} [a + b_j + c_{jk}]\, H_j(\varkappa, s) + B\, \bar{P}_{0,K}(s) - \lambda''\varkappa^K\, \bar{P}_{0,1}(s)$$

and

(18) $$H_j(\varkappa, s) = \lambda_j' f_j(\varkappa)\, G(\varkappa, s) - \lambda''\varkappa^K f_j(\varkappa)\, \bar{P}^W_{j,1}(s) \quad (1 \leqq j \leqq M),$$

where

$$f_j(\varkappa) = [s + \lambda + \lambda' + \lambda'' + \alpha + \theta_j - \lambda''\varkappa]^{-1}.$$

Therefore

(19) $$H_j(\varkappa, s) = B\lambda_j' f_j(\varkappa)\, p(\varkappa)\, \bar{P}_{0,K}(s) - \lambda''\varkappa^K g_j(\varkappa)$$

and

(20) $$G(\varkappa, s) = B\, p(\varkappa)\, \bar{P}_{0,K}(s) - \lambda''\varkappa^K f(\varkappa)$$

where

$$p(\varkappa) = [A - \lambda''\varkappa - \sum_{j=1}^{M} \sum_{k=1}^{M} [a + b_j + c_{jk}]\, \lambda_j' f_j(\varkappa)]^{-1},$$

$$f(\varkappa) = p(\varkappa)\, [\bar{P}_{0,1}(s) + \sum_{j=1}^{M} \sum_{k=1}^{M} [a + b_j + c_{jk}]\, f_j(\varkappa)\, \bar{P}^W_{j,1}(s)]$$

and

$$g_j(\varkappa) = f_j(\varkappa)\, [\lambda_j' f(\varkappa) + \bar{P}^W_{j,1}(s)].$$

Using Maclaurin's Theorem in relations (19) and (20), we have

(21) $$\bar{P}_{0,K-m}(s) = \frac{B}{m!}\, \bar{P}_{0,K}(s) \left[\frac{\partial^m}{\partial \varkappa^m}\, p(\varkappa)\Big|_{\varkappa=0}\right] \quad (1 \leqq m \leqq K - 1)$$

and

(22) $$\bar{P}^W_{j,K-m}(s) = \frac{B\lambda_j'}{m!}\, \bar{P}_{0,K}(s) \left[\frac{\partial^m}{\partial \varkappa^m}\, (f_j(\varkappa)\, p(\varkappa))\Big|_{\varkappa=0}\right] \quad (1 \leqq m \leqq K - 1).$$

Using relations (21) and (22) in relation (16), one obtains

$$\bar{P}_{0,K}(s) =$$

$$= \left[B - \frac{B\mu\alpha''\lambda''\beta}{(K-1)!\,(s+\alpha'')(s+\mu)(s+\beta)} \left\{\frac{\partial^{K-1}}{\partial\varkappa^{K-1}} (p(\varkappa)\,[1 + \sum_{j=1}^{M} \lambda'_j f_j(\varkappa)])\Big|_{\varkappa=0}\right\}\right]^{-1}.$$

Similarly, the Laplace Transforms of other state probabilities could be evaluated.

Acknowledgement. The authors wish to express their deep sense of gratitude to their friends Sudesh Sachdev, a former Research Fellow and Komal for their moral assistance. Thanks are also due to the Referee for his valuable comments.

Souhrn

POZNÁMKA K ČLÁNKU GOVILA A KUMARA

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V článku jsou modifikovány předpoklady článku "On the behaviour of an intermittently working system with three types of components", Apl. mat 16 (1871), 1–9, jako diskuse vlastností uvažovaného systému.

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