Aplikace matematiky

Vladimír Fiřt

Surfaces of characteristic curvature

Aplikace matematiky, Vol. 19 (1974), No. 1, 36–48

Persistent URL: `http://dml.cz/dmlcz/103512`

SURFACES OF CHARACTERISTIC CURVATURE

VLADIMÍR FIŘT

(Received March 28, 1973)

1. DEFINITION AND BASIC RELATIONS

Let us define the surface of characteristic curvature to be a surface whose principal curvatures, viz. $\varkappa_1$ and $\varkappa_2$, satisfy

$$F(\varkappa_1, \varkappa_2) = 0\,, \tag{1.1}$$

where F is a given function.[1] We shall call the relation (1.1) the characteristic curvature of the surface. This relation yields also the relation between Gauss' curvature $K = \varkappa_1\varkappa_2$ and the mean curvature of the surface $H = \frac{1}{2}(\varkappa_1 + \varkappa_2)$, as

$$\varkappa_{1,2} = H \pm (H^2 - K)^{1/2}\,. \tag{1.2}$$

We shall deduce the equation of the surface of characteristic curvature from (1.1) using (1.2) for the principal curvatures and the formulas

$$K = \frac{h_{11}h_{22} - h_{12}^2}{g_{11}g_{22} - g_{12}^2}\,, \tag{1.3}$$

$$H = \frac{1}{2}\,\frac{g_{11}h_{22} - 2g_{12}h_{12} + g_{22}h_{11}}{g_{11}g_{22} - g_{12}^2}\,,$$

where g_{ij} are the coefficients of the first basic form and h_{ij} the coefficients of the second basic form of the surface [2].

In the case of a surface with an orthogonal system of curvilinear coordinates ξ and η which are identical with the principal directions of the surfaces $g_{12} = h_{12} = 0$

[1]) The relation (1.1) is the necessary and sufficient condition for the surface to be a W-surface (Weingarten's surface). The number of W-surfaces, some properties of which are given in [1], thus includes the surfaces of characteristic curvature as well as rotary surfaces with the exception of the plane and the spherical surface.

and

$$\varkappa_1 = \frac{1}{R_1} = \frac{h_{11}}{g_{11}}, \quad \varkappa_2 = \frac{1}{R_2} = \frac{h_{22}}{g_{22}}, \tag{1.4}$$

where R_1 and R_2 are the principal radii. This means that for the six functions

$$\varkappa_1, \varkappa_2, g_{11}, g_{12}, h_{11}, h_{22} \tag{1.5}$$

of the two variables ξ and η six independent equations are valid, three of which are the Codazzi-Gauss equations, the other three being expressed by the relations (1.1) and (1.4). The surface of characteristic curvature is thus determined unambiguously.

Let us note the determination of a surface of characteristic curvature is based on the defined state (1.1) and not on the defined functions g_{ij} and h_{ij} which unambiguously determine the form and the magnitude of the surface (Bonnet's theorem).

In the particular case when the characteristic curvature (1.1) has the form of

$$\frac{\varkappa_1}{\varkappa_2} = k, \tag{1.6}$$

where k is a constant (real number), we obtain, after substituting (1.2) into (1.6) and using the formulas (1.3), the following equation of a surface of characteristic curvature:

$$(g_{11}g_{22} - g_{12}^2)(h_{11}h_{22} - h_{12}^2) - \tag{1.7}$$
$$- \frac{k}{(1+k)^2}(g_{11}h_{11} - 2g_{12}h_{12} + g_{22}h_{11})^2 = 0.$$

For the surface which is determined in Cartesian coordinates x, y, z by the explicit equation

$$z = f(x, y), \tag{1.8}$$

it holds [2] that

$$g_{11} = 1 + f_x^2, \quad g_{12} = f_x f_y, \quad g_{22} = 1 + f_y^2, \tag{1.9}$$
$$h_{11} = \frac{1}{\omega} f_{xx}, \quad h_{12} = \frac{1}{\omega} f_{xy}, \quad h_{22} = \frac{1}{\omega} f_{yy}^2,$$

where

$$\omega^2 = 1 + f_x^2 + f_y^2. \tag{1.10}$$

By substituting (1.9) into (1.7) we obtain the following nonlinear partial differential equation of a surface of characteristic curvature:

$$(1 + f_x^2 + f_y^2)(f_{xx}f_{yy} - f_{xy}^2) - \tag{1.11}$$
$$- \frac{k}{(1+k)^2}[(1 + f_x^2)f_{yy} - 2f_x f_y f_{xy} + (1 + f_y^2)f_{xx}]^2 = 0.$$

2. SOME PROPERTIES OF SURFACES OF CHARACTERISTIC CURVATURE

In this section we give some properties of the surface for which Eq. (1.6) is valid.

Theorem. *The ratio of the principal curvature and the normal curvature in the direction deviating from the principal direction of the surface by the same angle is constant at all the points of the surface of characteristic curvature* $\varkappa_1/\varkappa_2 = k$.

Proof. According to Euler's theorem the normal curvature $\varkappa$ of the normal section whose tangent forms an angle φ with the principal tangent equals [2], [3]

$$\varkappa = \varkappa_1 \cos^2 \varphi + \varkappa_2 \sin^2 \varphi \,. \tag{2.1}$$

After substituting (1.6) into (2.1) for a selected point P of the surface we have

$$^P\varkappa = {}^P\varkappa_2(k \cos^2 \varphi + \sin^2 \varphi) \tag{2.2}$$

and for another point R on the same surface we have

$$^R\varkappa = {}^R\varkappa_2(k \cos^2 \varphi + \sin^2 \varphi) \,. \tag{2.3}$$

From Eqs. (2.2) and (2.3) we obtain the relation

$$\frac{^P\varkappa_2}{^P\varkappa} = \frac{^R\varkappa_2}{^R\varkappa} \,, \tag{2.4}$$

which proves the above theorem.

The consequence of the theorem is the relation

$$\frac{^P\varkappa}{^R\varkappa} = \frac{^P\varkappa_2}{^R\varkappa_2} \,, \tag{2.5}$$

which yields the following property of the surface:

At two arbitrary points of the surface of characteristic curvature $\varkappa_1/\varkappa_2 = k$ the ratio of normal curvatures of normal sections of the surface is constant in the directions deviating from the principal directions of the surface by the same angle.

By substituting (1.6) into expressions for K and H we obtain

$$K = k\varkappa_2^2 \,, \quad H = \tfrac{1}{2}(1 + k)\, \varkappa_2 \,. \tag{2.6}$$

After squaring the second equation and dividing both equations (2.6) by one another we obtain

$$\frac{K}{H^2} = \frac{4k}{(1 + k)^2} \,, \tag{2.7}$$

consequently, at all points of the surface of characteristic curvature $\varkappa_1/\varkappa_2 = k$ the ratio of Gauss' curvature and the square of the mean curvature is constant and equals $4k/(1 + k)^2$.

From the first expression of (2.6) and (1.6) we obtain

$$\frac{K}{\varkappa_1^2} = \frac{1}{k}, \quad \frac{K}{\varkappa_2^2} = k, \tag{2.8}$$

so that at all points of the surface of characteristic curvature $\varkappa_1/\varkappa_2 = k$ the ratios of Gauss' curvature and the squares of the principal curvatures are constant and equal $1/k$ and k, respectively.

3. DIFFERENTIAL EQUATION OF THE MERIDIAN OF THE ROTARY SURFACE OF CHARACTERISTIC CURVATURE

In this section we shall deduce the differential equation of the meridian of a rotary surface for which Eq. (1.1) has the form

$$\frac{\varkappa_1^m}{\varkappa_2^n} = k, \tag{3.1}$$

where m, n, k are constants.

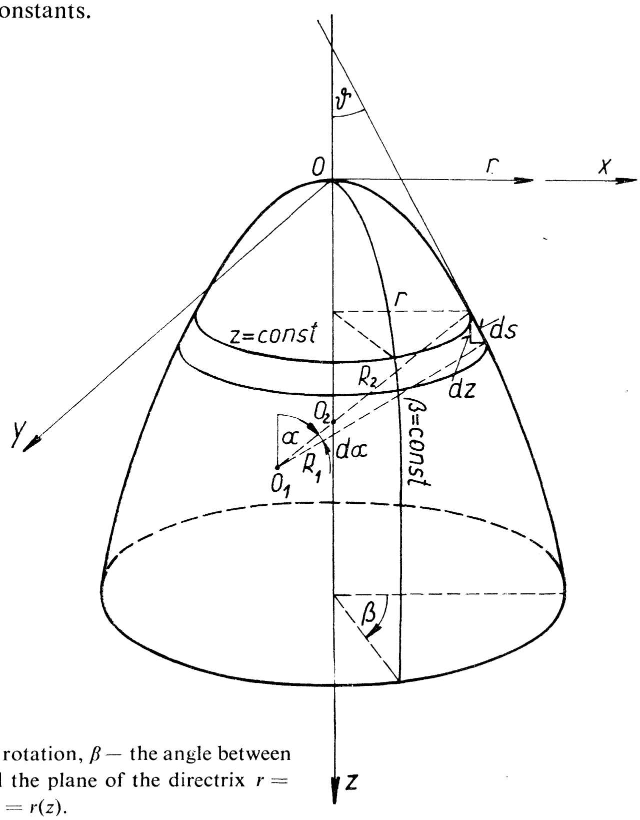

Fig. 1, z — is the axis of rotation, β — the angle between the plane of $y = 0$ and the plane of the directrix $r = r(z)$.

For a rotary surface defined by the parametric equations (Fig. 1)

$$x = r(z)\cos\beta\,, \quad y = r(z)\sin\beta\,, \quad z = z\,, \tag{3.2}$$

where r is the radius of a parallel circle in section $z = \text{const}$, the principal curvatures are [4]

$$\varkappa_1 = -\frac{r''}{(1 + r'^2)^{3/2}}\,, \quad \varkappa_2 = \frac{1}{r(1 + r'^2)^{1/2}}\,, \tag{3.3}$$

where $r' = \mathrm{d}r/\mathrm{d}z$, $r'' = \mathrm{d}^2r/\mathrm{d}z^2$,

$\varkappa_1$ – curvature of the rotary surface along the meridian,

$\varkappa_2$ – curvature of the rotary surface along the parallel circle.

By substituting the expressions (3.3) into Eq. (3.1) and rearranging them we obtain the following ordinary differential equation of the second order for the meridian (directrix)

$$r^n r''^m - (-1)^m\, k(1 + r'^2)^{(3m-n)/2} = 0\,. \tag{3.4}$$

In the case that $m = n$ and $(-1)^m = -1$, we can give Eq. (3.4) the form

$$rr'' + ar'^2 + a = 0\,, \tag{3.5}$$

where

$$a = k^{1/n}\,. \tag{3.6}$$

If $m = n$ and $(-1)^m = 1$, then $a = -k^{1/n}$.

For $n = 3m$ we obtain from (3.4) the following differential equation of the meridian

$$r^3 r'' = -k^{1/m}\,. \tag{3.7}$$

If $m = 0$, Eq. (3.4) changes into a first-order differential equation

$$r^2 r'^2 + r^2 = k^{2/n}\,. \tag{3.8}$$

For $n = 0$ Eq. (3.4) acquires the form

$$r''^2 + k^{2/m}(1 + r'^2)^3 = 0 \tag{3.9}$$

and for $k = 0$ we obtain $(r \neq 0)$

$$r'' = 0\,, \tag{3.10}$$

which is the differential equation of the meridian of conical and cylindrical surfaces $(r = C_1 z + C_2)$.

The first integral of Eq. (3.5) is determined by the introduction of a new function $y = y(r)$ which is defined by the relations [5]

$$y = p^2 , \quad p(r) = r'(z) . \tag{4.1}$$

According to (4.1) we have

$$r'^2(z) = y ,$$

$$r'' = p'(r)\, r' = p'p = \frac{1}{2} \frac{d}{dr} [p^2(r)] = \frac{1}{2} y' , \tag{4.2}$$

where

$$p' = dp/dr , \quad r' = dr/dz , \quad y' = dy/dr .$$

Substituting (4.2) into (3.5) we obtain for the function y a linear equation of Euler type

$$ry' + 2ay + 2a = 0 , \tag{4.3}$$

whose solution has the form

$$y = C_1 r^{-2a} - 1 , \tag{4.4}$$

where C_1 is the integration constant.

By substituting the first expression of (4.2) into (4.4) we obtain the following first integral of Eq. (3.5)

$$r^{2a} r'^2 + r^{2a} = C_1 . \tag{4.5}$$

The separation of variables in Eq. (4.5) and integration yield the common integral of Eq. (3.5)

$$z = \int (C_1 r^{-2a} - 1)^{-1/2} \, dr + C_2 , \tag{4.6}$$

where C_2 is the second integration constant.

If $k = -1$ and n is an odd number, then according to (3.6) $a = -1$ and from (4.6) we obtain the following equation of the meridian in the catenary form

$$z = \frac{1}{\sqrt{C_1}} \ln [r \sqrt{C_1} + \sqrt{(C_1 r^2 - 1)}] , \tag{4.7}$$

whose rotation about the axis z yields a catenoid.

If $k = 1$, then – according to (3.6) – $a = 1$ and from (4.6) we obtain

$$r^2 + (z - C_2)^2 = C_1 , \tag{4.8}$$

which is the equation of a semicircle of the radius of $R = C_2 = \sqrt{C_1}$.

For $a = -\frac{1}{2}$ ($k = (-\frac{1}{2})^n$) we obtain from (4.6) the equation of a parabola

$$r = \frac{1}{C_1} + \frac{C_1^2}{4}(z - C_2)^2 \tag{4.9}$$

and for a $= {}^1/_2$ we obtain

$$z = -2(\tfrac{1}{5}r^2 - \tfrac{4}{3}C_1 r + C_2^2)(C_1 - r)^{1/2}\,, \tag{4.10}$$

which is the equation of a cycloid (See Section 6).

Now we shall deal with the solution of Eq. (3.7) by means of the substitution

$$p(r) = r'\,, \quad r'' = p'r' = p'p = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r}(p^2)\,. \tag{4.11}$$

By substituting (4.11) into (3.7), separation of variables and integration we obtain

$$\frac{1}{r^2}k^{1/m} = p^2 + C_1 \tag{4.12}$$

and the application of the first relation of (4.11) yields

$$r'^2 = \frac{1}{r^2}k^{1/m} - C_1 \tag{4.13}$$

and thus

$$r' = \frac{1}{r}(k^{1/m} - C_1 r^2)^{1/2}\,. \tag{4.14}$$

Hence, after the separation of variables and integration we obtain a general solution of Eq. (3.7) in the form of

$$(C_1 z + C_2)^2 + C_1 r^2 - k^{1/m} = 0\,. \tag{4.15}$$

If in Eq. (4.13) we put $C_1 = 0$, then

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}z}(r^2) = k^{1/2m}\,, \tag{4.16}$$

from which we obtain the following solution of Eq. (3.7):

$$r^2 = 2k^{1/2m}z + C\,. \tag{4.17}$$

The solution of Eq. (3.8) has the form of (4.8),

$$r^2 + (z - C)^2 = k^{1/n}\,. \tag{4.18}$$

5. PARAMETRIC EQUATIONS OF THE MERIDIAN

In the structural design of membrane structure in the form of a rotary surface, Eq. (3.2) is often replaced by parametric equations in the form

$$x = r(\alpha)\cos\beta\,, \quad y = r(\alpha)\sin\beta\,, \quad z = z(\alpha)\,, \tag{5.1}$$

where the parameter α represents the angle contained by the normal of the surface and the rotation axis z (Fig. 1).

In this section we shall deduce the parametric equations $r = r(\alpha)$, $z = z(\alpha)$ of the meridian of the rotary surface of characteristic curvature for which the differential equation (3.5) is valid.

According to Fig. 1

$$r = R_2 \sin\alpha\,, \quad r' = \operatorname{tg}\vartheta = \operatorname{cotg}\alpha\,, \tag{5.2}$$

where $R_2 = 1/\varkappa_2$ is the principal radius of the surface in the section $\alpha = \text{const.}$

By substituting (5.2) into the first integral (4.5) of Eq. (3.5) we obtain

$$R_2^{2a}\sin^{2a}\alpha(1 + \operatorname{cotg}^2\alpha) = C_1\,,$$

which yields, after rearrangement,

$$R_2 = C\sin^{(1-a)/a}\alpha\,, \quad C = C_1^{1/a}\,. \tag{5.3}$$

According to (3.1) and (3.6) $\varkappa_1/\varkappa_2 = a$ and hence

$$R_1 = \frac{R_2}{a} = \frac{C}{a}\sin^{(1-a)/a}\alpha\,, \tag{5.4}$$

where $R_1 = 1/\varkappa_1$ is the principal radius of the surface in the section $\beta = \text{const.}$

Fig. 1 makes it obvious that

$$R_1\,\mathrm{d}\alpha = \mathrm{d}s = \frac{\mathrm{d}r}{\cos(\alpha + \mathrm{d}\alpha)}\,, \quad \frac{\mathrm{d}z}{\mathrm{d}s} = \sin\alpha \tag{5.5}$$

and, consequently, $(\cos(\alpha + \mathrm{d}\alpha) = \cos\alpha)$

$$\frac{\mathrm{d}r}{\mathrm{d}\alpha} = R_1\cos\alpha\,, \quad \frac{\mathrm{d}z}{\mathrm{d}\alpha} = R_1\sin\alpha\,. \tag{5.6}$$

From the first relation of (5.2) and (5.6) we obtain the Codazzi - Gauss equation

$$R_1\cos\alpha = \frac{\mathrm{d}}{\mathrm{d}\alpha}(R_2\sin\alpha)\,.$$

After the substitution of (5.4) into the first equation (5.6) and integration we obtain

$$r = \frac{C}{a} \int \sin^{(1-a)/a} \alpha \cos \alpha \, d\alpha + D_1 , \tag{5.7}$$

hence

$$r = C \sin^{1/a} \alpha + D_1 , \tag{5.8}$$

where C and D_1 are integration constants.

If we substitute (5.4) into the second relation of (5.6), we obtain

$$z = \frac{C}{a} \int \sin^{1/a} \alpha \, d\alpha + D_2 , \tag{5.9}$$

where D_2 is another integration constant. The integral on the right hand side of Eq. (5.9) can be expressed, for some values of parameter a, by goniometric functions and their logarithms, in other cases by elliptic integrals.

If l is a natural number and $1/a = 2l$, we obtain from (5.9) the expression [6]

$$z = \frac{C}{a}\left[\frac{1}{2^{2l}}\binom{2l}{l}\alpha + \frac{(-1)^l}{2^{2l-1}}\sum_{j=0}^{l-1}(-1)^j\binom{2l}{j}\frac{\sin(2l-2j)\,\alpha}{2l-2j}\right] + D_2 . \tag{5.10}$$

For $1/a = 2l + 1$,

$$z = \frac{C}{a}\left[\frac{1}{2^{2l}}(-1)^{l+1}\sum_{j=0}^{l}(-1)^j\binom{2l+1}{j}\frac{\cos(2l+1-2j)\,\alpha}{2l+1-2j}\right] + D_2 . \tag{5.11}$$

For $1/a = -2l$ the relation (5.9) can be expressed as follows [6]:

$$z = -\frac{C}{a}\frac{\cos\alpha}{2l-1}\left[\operatorname{cosec}^{2n-1}\alpha + \sum_{j=1}^{l-1}\frac{2^j(l-1)(l-2)\dots(l-j)}{(2l-3)(2l-5)\dots(2l-2j-1)}\operatorname{cosec}^{2l-2j-1}\alpha\right] + D_2 \tag{5.12}$$

and for $1/a = -(2l + 1)$,

$$z = -\frac{C}{a}\frac{\cos\alpha}{2l}\left[\operatorname{cosec}^{2l}\alpha + \sum_{j=1}^{l=1}\frac{(2l-1)(2l-3)\dots(2l-2j+1)}{2^j(l-1)(l-2)\dots(l-j)}\operatorname{cosec}^{2l-2j}\alpha\right] + \frac{C}{a}\frac{(2l-1)!!}{2^l l!}\ln\operatorname{tg}\frac{\alpha}{2} + D_2 . \tag{5.13}$$

Eq. (5.8) and (5.9) or (5.10), (5.11), (5.12) and (5.13) are parametric equations of the meridian of the rotary surface of characteristic curvature (3.1) for $m = n$.

6. APPLICATION

In air-supported membrane structures in the form of a rotary surface the membrane force T_1 in the direction of the meridian tangent and the membrane force T_2 in the direction of the tangent to the parallel circle due to internal overpressure q equal [7]

$$T_1 = \frac{1}{2} R_2 q\,, \quad T_2 = R_2 q \left(1 - \frac{R_2}{2R_1}\right). \tag{6.1}$$

The membrane forces T_1 and T_2 are referred to unit length in the middle surface of the membrane and have the dimension of kp/cm, kp/m, Mp/m, etc.

The skin of these structures consists in an air-tight technical fabric capable of transferring tensile forces only; for this reason it should hold that

$$T_1 > 0\,, \quad T_2 > 0\,. \tag{6.2}$$

If one of the conditions (6.2) is not complied with in a major vicinity of any point of the membrane, undersirable folding of the membrane takes place and large shearing forces originate which can break the membrane (fabric) or reduce its life. Apart from that, the membrane in such a place is considerably deformed by the wind and snow loads which considerably reduce the overall rigidity of the air-supported structure and its structural and useful function as well as its resistance to wind loads [8], [9].

For the surfaces with positive Gauss' curvature the following geometric condition follows from the second relation (6.1) and (6.2)

$$\frac{\varkappa_1}{\varkappa_2} < 2 \quad (2R_1 > R_2)\,. \tag{6.3}$$

In rotary surfaces of characteristic curvature we can always comply with the condition (6.2) in advance by a suitable choice of the relation between the principal curvatures $\varkappa_1$ and $\varkappa_2$.

For example, if we make in (3.1) $m = n = 1$ and $k = \frac{1}{2}$, then

$$\frac{\varkappa_1}{\varkappa_2} = \frac{1}{2} < 2 \tag{6.4}$$

which complies with the condition (6.3). The parametric equations of the meridian of the rotary surface of characteristic curvature (6.4) have, according to (5.8) and (5.10) the form ($a = \frac{1}{2}$, $l = 1$)

$$\begin{aligned} r &= C \sin^2 \alpha + D_1\,, \\ z &= C(\alpha - \tfrac{1}{2} \sin 2\alpha) + D_2\,. \end{aligned} \tag{6.5}$$

If we identify the origin of axes z, r with the intersection point of the meridian with the axis z, then $r = 0$ and $z = 0$ for $\alpha = 0$, and from (6.5) we obtain $D_1 = D_2 = 0$. The integration constant C represents the maximum value r_{max} of the radius of the parallel circle ($C = r_{max}$) and, consequently, Eqs. (6.5) can be written in the form

$$r = r_{max} \sin \alpha\,, \quad z = r_{max}(\alpha - \tfrac{1}{2} \sin 2\alpha)\,, \tag{6.6}$$

which are parametric equations of the cycloid (Fig. 2). We come to the conclusion that the rotation of the cycloid characterized by Eqs. (6.6) about the axis z yields a rotary surface of characteristic curvature (6.4). Its principal radii are, according to (5.3) and (5.4),

$$R_1 = 2r_{max} \sin \alpha\,, \quad R_2 = r_{max} \sin \alpha\,. \tag{6.7}$$

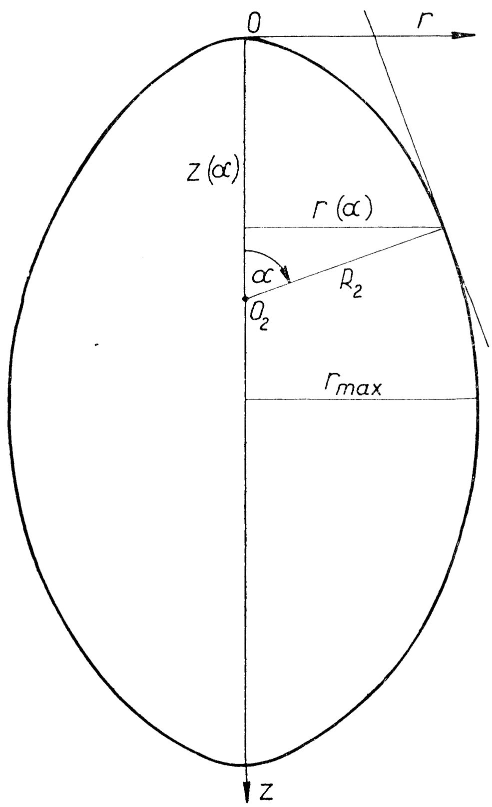

Fig. 2.

From the relations (6.1) for this surface we obtain

$$T_1 = \tfrac{1}{2}R_2 q = \tfrac{1}{2}r_{\max} q \sin\alpha\,, \qquad (6.8)$$
$$T_2 = \tfrac{3}{4}R_2 q = \tfrac{3}{4}r_{\max} q \sin\alpha\,.$$

Consequently, the conditions (6.2) are complied with at all points of the rotary cycloidal surface with the exception of both poles, i.e., the points corresponding to $\alpha = 0$ and $\alpha = \pi$, where $T_1(0) = T_2(0) = 0$ and $T_1(\pi) = T_2(\pi) = 0$.

The ratio of the principal membrane forces due to inner overpressure which characterizes, to a certain extent, the spatial rigidity of the membrane structure, is constant in the given case, viz.

$$\frac{T_1}{T_2} = \frac{2}{3}\,. \qquad (6.9)$$

This means that any part of the rotary cycloid surface is a suitable form for the skin of an air-supported structure in the same way as a part of a cylindrical tube with $T_1/T_2 = \frac{1}{2}$ or a part of a spherical surface with $T_1/T_2 = 1$.

In conclusion let us note that the surfaces generated by the rotation of quadratic curves about the axis z are the surfaces of characteristic curvature

$$\frac{\varkappa_1}{\varkappa_2^3} = R_0^2 = \text{const} \qquad (6.10)$$

and their principal radii can be expressed by the formulas [10]

$$R_1 = \frac{R_0}{(1 + \gamma \sin^2\alpha)^{3/2}}\,, \quad R_2 = \frac{R_0}{(1 + \gamma \sin^2\alpha)^{1/2}}\,, \qquad (6.11)$$

where R_0 is the value of the principal radii at the pole, i.e. for $\alpha = 0$. The value of $\gamma = 0$ corresponds to a spherical surface, $\gamma = -1$ to a paraboloid, $\gamma > -1$ to ellipsoids and $\gamma < -1$ to hyperboloids.

The surfaces of characteristic curvature (6.10) and those of characteristic curvature

$$(\varkappa_1 - \varkappa_2)\frac{1}{\sin^2\alpha} = \text{const} \qquad (6.12)$$

are investigated in [10].

References

[1] *Eisenhart, L. P.:* An Introduction to Differential Geometry with Use of the Tensor Calculus. Princeton Univ. Press, Princeton 1947.

[2] *Budinský, B., Kepr, B.:* Základy diferenciální geometrie s technickými aplikacemi (Fundamentals of differential geometry with technical applications). SNTL, Praha 1970.

[3] *Hostinský, B.:* Diferenciální geometrie křivek a ploch (Differential geometry of curves and surfaces). Přírodovědecké nakladatelství, Praha 1950.

[4] *Власов, В. З.:* Избранные труды, том 1. Изд. Академии наук СССР, Москва 1962.

[5] *Камке, Э.:* Справочник по обыкновенным дифференциальным уравнениям. Изд. "Наука", Москва 1971.

[6] *Градштейн, И. С., Рыжик, И. М.:* Таблицы интегралов сумм, рядов и произведений. Физматгиз, Москва 1962.

[7] *Отто, Ф., Тростель, Р.:* Пневматические строительные конструкции. Стройиздат Москва 1972.

[8] *Fiřt V.:* Vývoj membránových konstrukcí (Development of membrane structures), Inženýrské stavby 20, 1972, č. 8—9, 321—326.

[9] *Fiřt, V.:* Air-supported structures with automatic adjustment of inner overpressure, Int. Symp. on Pneumatic Structures, Proc., Vol. 2, Delft 1972.

[10] *Novožilov, B. B.:* Теория тонких оболочек. Судпромгиз, Ленинград 1962.

Souhrn

PLOCHY S CHARAKTERISTICKOU KŘIVOSTÍ

VLADIMÍR FIŘT

V práci jsou odvozeny rovnice ploch, mezi jejimiž hlavními křivostmi platí definovaný vztah (1.1), který představuje charakteristickou křivost plochy.

Diferenciální rovnice plochy s charakteristickou křivostí (1.6) má v kartézských souřadnicích tvar (1.11). Některé vlastnosti této plochy jsou uvedeny v odst. 2.

V odst. 3 je odvozena diferenciální rovnice (3.4) meridiánu rotační plochy s charakteristickou křivostí (3.1). Zvláštní případy této rovnice (3.5), (3.7) a (3.8) jsou řešeny v odst. 4.

V odst. 5 autor odvozuje parametrické rovnice meridiánu rotační plochy užitím prvého integrálu (4.5) rovnice (3.5) a v odst. 6 uvádí jejich použití v technické praxi.

Author's address: Ing. *Vladimír Fiřt*, CSc., Ústav teoretické a aplikované mechaniky ČSAV, Vyšehradská 49, 128 49 Praha 2.