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TABLES FOR THE TWO-SAMPLE SAVAGE RANK TEST OPTIMAL FOR EXPONENTIAL DENSITIES

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Introduction. We present here the tables of scores and of upper-tail and lower-tail critical values for the so called Savage test for cases when the pooled sample size $m + n$ satisfies $6 \leqq m + n \leqq 20$ and the one-sided significance levels lie near 0·5%, 1%, 2·5%, 5%. This test is optimal (in a sense to be specified later) for the two-sample scale problem if the basic population densities are exponential.

In the sequel we shall use the terminology and the results from the book by Hájek-Šidák [1].

Description of the test. Let us consider the following experimental situation. We have two random samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ with the densities f_1 and f_2, respectively; suppose that $m \leqq n$, and put $N = m + n$. Our aim is to test the hypothesis H_0 that f_1 equals f_2 identically (but otherwise they may be arbitrary) against the alternatives that they differ in scale, i.e. $f_1(x) = \sigma^{-1} f((x - \mu)/\sigma)$, $f_2(x) = f(x - \mu)$, where f is some density, μ is a common location parameter, σ is the parameter to be tested.

Let $R_1, \ldots, R_m$ denote the ranks of $X_1, \ldots, X_m$ in the pooled sample of all observations $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ arranged in the increasing order of their magnitude. The theoretical so called Savage test (cf. [1], Section III.2.1) is based on the statistic $S' = \sum_{i=1}^{m} a'_N(R_i)$, where $a'_N(k)$ are the scores for the two-sample scale problem with the exponential densities, i.e. $a'_N(k) = \sum_{j=N-k+1}^{N} 1/j$. The test with the critical region $\{S' \geqq c_u\}$, c_u being some upper-tail critical value, is the locally most powerful rank test of H_0 against $\sigma > 1$ if the basic density f is of the exponential type; the test is also asymptotically optimum for such a problem. In particular, in practice we encounter most usually the problem with $\mu = 0$, $f(x) = \lambda^{-1} e^{-x/\lambda}$ for $x \geqq 0$, $f(x) = 0$ for

$x < 0$, which leads to

$$f_1(x) = (\sigma\lambda)^{-1}\, e^{-x/(\sigma\lambda)} \quad \text{for} \quad x \geqq 0\,,$$
$$f_2(x) = \lambda^{-1} e^{-x/\lambda} \quad \text{for} \quad x \geqq 0\,,$$
$$f_1(x) = f_2(x) = 0 \quad \text{for} \quad x < 0\,;$$

naturally, the Savage test is therefore optimal (in the above sense) for this specialized problem, too.

Of course, in practice we cannot use the precise scores $a'_N(k)$, but only some rounded off scores instead. However, the test remains unchanged if we multiply its statistic by any positive constant. Therefore, to be quite precise in our notation, we work in the present paper with the test statistic

$$S = \sum_{i=1}^{m} a_N(R_i)$$

where $a_N(k) = 100a'_N(k)$ rounded off to integer values. If our aim is to test against the one-sided alternative $\sigma > 1$ (or $\sigma < 1$) expressing that the first density f_1 has a larger (smaller) scale parameter than the second density f_2, we employ a one-sided critical region $\{S \geqq c_u\}$ (or $\{S \leqq c_l\}$) where c_u (c_l) is some upper-tail (lower-tail, respectively) critical value. The test against $\sigma \neq 1$ employs a two-sided critical region $\{S \geqq c_u \text{ or } S \leqq c_l\}$.

The test has its name after I. R. Savage, who derived a very closely related rank test of H_0 against Lehmann's alternatives in the paper [2] (for some details cf. also [1]). However, the tables for the practical use of the Savage test seem to be nowhere available.

Description of the tables. The scores $a_N(k)$ for the Savage test are given in Table 1 for $6 \leqq N \leqq 20$.

Table 2 consists of four double-columns for $\alpha_1 = 0{\cdot}5\%$, $\alpha_2 = 1\%$, $\alpha_3 = 2{\cdot}5\%$, $\alpha_4 = 5\%$; in its i-th double-column ($i = 1, 2, 3, 4$), in the upper line for each pair n, m it contains the upper-tail critical value c_{ui} of the statistic S and the corresponding exact probability $P\{S \geqq c_{ui}\}$ in per cents such that this probability is the closest possible to the value α_i ($i = 1, 2, 3, 4$); similarly, in the lower line it contains the lower-tail critical value c_{li} of the statistic S and the exact probability $P\{S \leqq c_{li}\}$ in per cents such that this probability is the closest possible to α_i ($i = 1, 2, 3, 4$). For example, precise rules and formulas for tabulating the upper-tail critical values c_{ui} are as follows: First, let there be two consecutive possible values $c^-_{ui} < c^+_{ui}$ of the statistic S such that $P\{S \geqq c^+_{ui}\} < \alpha_i \leqq P\{S \geqq c^-_{ui}\}$; then, if $P\{S \geqq c^-_{ui}\} - \alpha_i \geqq \alpha_i - P\{S \geqq c^+_{ui}\}$ we tabulate c^+_{ui} and $P\{S \geqq c^+_{ui}\}$; if, conversely, $P\{S \geqq c^-_{ui}\} - \alpha_i < \alpha_i - P\{S \geqq c^+_{ui}\}$ we tabulate c^-_{ui} and $P\{S \geqq c^-_{ui}\}$. Second, let there be no c^+_{ui} with $P\{S \geqq c^+_{ui}\} < \alpha_i$; then we take for c^-_{ui} the maximal possible value of S and tabulate this c^-_{ui} and $P\{S \geqq c^-_{ui}\}$ in the i-th double-column corresponding to the largest α_i (among $\alpha_1, \alpha_2, \alpha_3, \alpha_4$) for which $\alpha_i \leqq P\{S \geqq c_{ui}^-\}$ is satisfied; the double-columns for smaller α_j's (if there are any) are then filled by dashes, the double-columns for larger α_j's (if there are any) are filled according to the first rule. Third, if this second case occurs for α_1, the value c_{u1}^- is preceded by a star indicating that the tabulated c_{u1}^- is the largest possible value (so that it can be distinguished from the first case). As for the lower-tail critical values c_{li} contained in the lower line for each n, m, the rules for their tabulation are analogous, and we hope we need not write them in detail.

This table (except the maximal and the minimal values, i.e. except those preceded by dashes or by a star) can be applied in two somewhat different ways, and we shall describe it in the case of the upper-tail critical values: If we have no objections against a slight exceeding of the significance level α_i, we can use the critical region $\{S \geqq c_{ui}\}$ with the exact significance level $P\{S \geqq c_{ui}\}$ shown in Table 2. However, if we insist on performing a conservative test, i.e. on having a significance level not exceeding α_i while we find in the table $P\{S \geqq c_{ui}\} > \alpha_i$, we can base the test on the critical region $\{S > c_{ui}\}$ which has the significance level $< \alpha_i - (P\{S \geqq c_{ui}\} - \alpha_i)$.

Of course, the two-sided critical region $\{S \geqq c_{ui}$ or $S \leqq c_{li}\}$, used for testing against $\sigma \neq 1$, has the significance level $P\{S \geqq c_{ui}\} + P\{S \leqq c_{li}\}$.

Table 2 has been computed by a direct combinatorial procedure, so that the probabilities given in this table are exact (except, of course, their natural rounding off). The author would like to thank M. Nosál for programming necessary computations, and J. Hájek for suggesting the way of tabulation used here.

Example 1. Let the X sample be 2; 4; 11, and the Y sample 5; 12; 20; 35; 51; 78; 116; 149; 192; 256 so that $m = 3$, $n = 10$, $N = 13$. Our aim is to test H_0 against $\sigma < 1$ (i.e. against the one-sided alternative that the first density has a smaller scale parameter than the second one), and we wish to perform a conservative Savage test at a significance level not exceeding 1%. We would use a critical region $\{S \leqq c_l\}$, and in Table 2 we find $c_{l2} = 68$ with the exact significance level $P\{S \leqq 68\} = 1{\cdot}05\%$. Since we do not wish to exceed 1%, we use the critical region $\{S < 68\}$, whose level is certainly $<1\% - (1{\cdot}05\% - 1\%) = 0{\cdot}95\%$. The actual value of S is found to be $S = 59$, so that we reject H_0 at a level $<0{\cdot}95\%$.

Example 2. Let the X sample be 15; 27; 62; 124; and the Y sample 3; 8; 13; 44, so that $m = 4$, $n = 4$, $N = 8$. Our aim is to test H_0 against the two-sided alternative $\sigma \neq 1$ by means of the Savage test at some significance level between 5% and 10%. The test must use a two-sided critical region $\{S \geqq c_u$ or $S \leqq c_l\}$ and a reasonable principle may be to make both probabilities $P\{S \geqq c_u\}$ and $P\{S \leqq c_l\}$ at least roughly equal, which means for our case to make them lie roughly between 2·5% and 5%. In Table 2, in the third and fourth double-columns, we find the following four suitable one-sided critical regions: $\{S \geqq 629\}$ with the level 2·86%, $\{S \geqq 595\}$ with the level 5·71%, $\{S \leqq 171\}$ with the level 2·86%, $\{S \leqq 205\}$ with the level

5·71%. Since we do not wish to exceed 10%, we have the following three possibilities of combining these regions into a two-sided critical region: to choose either $\{S \geqq 629$ or $S \leqq 171\}$ with the level 2·86% + 2·86% = 5·72%, or $\{S \geqq 629$ or $S \leqq 205\}$ with the level 2·86% + 5·71% = 8·57%, or $\{S \geqq 595$ or $S \leqq 171\}$ with the level 5·71% + 2·86% = 8·57%. One of these critical regions must be chosen in advance, before the experiment. Now, the actual value of S in our example is $S = 595$, so that with the first or the second two-sided critical regions H_0 would not be rejected at the corresponding levels, whereas with the third region H_0 would be rejected at the level 8.57%.

Remark on asymptotic normality. It can be shown that the statistic S' has, under H_0, the mean value $\mathsf{E}S' = m$, the variance

$$\operatorname{var} S' = \frac{mn}{N-1}\left(1 - \frac{1}{N}\sum_{j=1}^{N}\frac{1}{j}\right),$$

and the standardized variable $(S' - \mathsf{E}S')(\operatorname{var} S')^{-1/2}$ has asymptotically the standard normal distribution whenever $m \to \infty$, $n \to \infty$ in an arbitrary way.

In Table 3 we illustrate the closeness of this normal approximation for all pairs m, n with $m + n = 20$. The arrangement of the table is analogous to that of Table 2, but the first numbers in each double-column are the approximate levels in per cents, while the second numbers are again the exact levels; i.e. in the upper line we tabulate $1 - \Phi((10^{-2}c_{ui} - \mathsf{E}S')/(\operatorname{var} S')^{1/2})$ in per cents (Φ being the standard normal distribution function, c_{ui} the critical value from Table 2), and $P\{S \geqq c_{ui}\}$ in per cents taken from Table 2; in the lower line we tabulate similarly $\Phi((10^{-2}c_{li} - \mathsf{E}S') :$ $: (\operatorname{var} S')^{1/2})$ and $P\{S \leqq c_{li}\}$.

Remark on ties. If the samples contain some equal observations, so that their ranks are not well defined, we can either randomize their ordering by means of an artificial supplementary experiment, or use the following method of average scores: Consider a set of τ equal observations, which would have the ranks (say) $r + 1, \ldots$ $\ldots, r + \tau$ if they were distinct but if they had otherwise the same position among the remaining observations; then we assign to each of these equal observations the average score $\tau^{-1} \sum_{i=1}^{\tau} a_N(r + i)$. If there are not many equal observations, our Table 2 can still serve as an approximation.

References

[1] *J. Hájek, Z. Šidák:* Theory of rank tests. Academia, Prague & Academic Press, New York-London 1967.

[2] *I. R. Savage:* Contributions to the theory of rank order statistics — the two-sample case. Ann. Math. Statist. 27 (1956), 590—615.

Table 1. Scores $a_N(k)$ for the Savage statistic S

k \ N	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17	14	13	11	10	9	8	8	7	7	6	6	6	5	5
2	37	31	27	24	21	19	17	16	15	14	13	12	11	11	10
3	62	51	43	38	34	30	27	25	23	22	20	19	18	17	16
4	95	76	63	55	48	43	39	35	32	30	28	26	24	23	22
5	145	109	88	75	65	57	51	46	42	39	36	34	32	30	28
6	245	159	122	100	85	74	65	59	53	49	45	42	39	37	35
7		259	172	133	110	94	82	73	66	60	55	51	48	44	42
8			272	183	143	119	102	90	80	73	66	61	57	53	49
9				283	193	152	127	110	97	87	79	72	67	62	58
10					293	202	160	135	117	103	93	85	78	72	67
11						302	210	168	142	123	110	99	90	83	77
12							310	218	175	148	130	116	105	95	88
13								318	225	182	155	136	121	110	100
14									325	232	188	161	141	126	115
15										332	238	194	166	146	131
16											338	244	200	171	151
17												344	250	205	176
18													350	255	210
19														355	260
20															360

Table 2. Upper-tail critical values c_{ui} and lower-tail critical values c_{li} of the Savage statistic S, and significance levels $P\{S \geq c_{ui}\}$ and $P\{S \leq c_{li}\}$ in per cents

n	m	0·5%		1·0%		2·5%		5·0%	
5	1	—	—	—	—	—	—	245	16·67
		—	—	—	—	—	—	17	16·67
4	2	—	—	—	—	—	—	390	6·67
		—	—	—	—	—	—	54	6·67
3	3	—	—	—	—	485	5·00	485	5·00
		—	—	—	—	116	5·00	116	5·00
6	1	—	—	—	—	—	—	259	14·29
		—	—	—	—	—	—	14	14·29
5	2	—	—	—	—	418	4·76	418	4·76
		—	—	—	—	45	4·76	45	4·76
4	3	—	—	—	—	527	2·86	494	5·71
		—	—	—	—	96	2·86	121	5·71
7	1	—	—	—	—	—	—	272	12·50
		—	—	—	—	—	—	13	12·50
6	2	—	—	—	—	444	3·57	444	3·57
		—	—	—	—	40	3·57	40	3·57
5	3	—	—	566	1·79	566	1·79	507	5·36
		—	—	83	1·79	83	1·79	119	5·36
4	4	—	—	654	1·43	629	2·86	595	5·71
		—	—	146	1·43	171	2·86	205	5·71
8	1	—	—	—	—	—	—	283	11·11
		—	—	—	—	—	—	11	11·11
7	2	—	—	—	—	466	2·78	416	5·56
		—	—	—	—	35	2·78	49	5·56
6	3	—	—	599	1·19	566	2·38	521	4·76
		—	—	73	1·19	90	2·38	110	4·76
5	4	* 699	0·79	699	0·79	654	2·38	623	4·76
		* 128	0·79	128	0·79	165	2·38	190	4·76
9	1	—	—	—	—	—	—	293	10·00
		—	—	—	—	—	—	10	10·00
8	2	—	—	486	2·22	486	2·22	436	4·44
		—	—	31	2·22	31	2·22	44	4·44
7	3	* 629	0·83	629	0·83	571	2·50	534	5·00
		* 65	0·83	65	0·83	92	2·50	109	5·00
6	4	739	0·48	714	0·95	677	2·38	636	5·24
		113	0·48	130	0·95	157	2·38	188	5·24
5	5	824	0·40	787	1·19	762	2·38	729	5·16
		178	0·40	215	1·19	240	2·38	273	5·16

Table 2 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
10	1	—	—	—	—	—	—	302	9·09
		—	—	—	—	—	—	9	9·09
9	2	—	—	504	1·82	504	1·82	421	5·45
		—	—	28	1·82	28	1·82	49	5·45
8	3	* 656	0·61	623	1·21	578	2·42	547	4·85
		* 58	0·61	71	1·21	85	2·42	106	4·85
7	4	750	0·61	730	0·91	686	2·42	647	5·15
		115	0·61	128	0·91	152	2·42	180	5·15
6	5	849	0·43	818	1·08	784	2·60	747	4·98
		175	0·43	202	1·08	233	2·60	264	5·19
11	1	—	—	—	—	—	—	310	8·33
		—	—	—	—	—	—	8	8·33
10	2	—	—	520	1·52	470	3·03	437	4·55
		—	—	25	1·52	35	3·03	44	4·55
9	3	680	0·45	647	0·91	585	2·73	547	5·00
		52	0·45	64	0·91	86	2·73	107	5·45
8	4	782	0·40	745	1·01	699	2·42	655	5·05
		103	0·40	125	1·01	148	2·42	177	5·05
7	5	864	0·51	834	1·01	799	2·53	758	5·05
		173	0·51	193	1·01	223	2·53	256	5·05
6	6	948	0·54	928	0·97	891	2·49	854	4·98
		250	0·54	270	0·97	307	2·49	344	4·98
12	1	—	—	—	—	—	—	318	7·69
		—	—	—	—	—	—	8	7·69
11	2	—	—	536	1·28	486	2·56	428	5·13
		—	—	24	1·28	33	2·56	43	5·13
10	3	704	0·35	646	1·05	596	2·45	559	4·90
		49	0·35	68	1·05	83	2·45	102	4·90
9	4	781	0·56	761	0·98	712	2·52	667	5·03
		108	0·56	122	1·12	148	2·52	173	5·03
8	5	887	0·47	860	1·01	816	2·49	771	5·05
		168	0·47	187	1·01	219	2·56	252	5·05
7	6	975	0·52	954	0·99	913	2·45	871	5·01
		244	0·52	264	0·99	299	2·45	338	5·01
13	1	—	—	—	—	—	—	325	7·14
		—	—	—	—	—	—	7	7·14
12	2	—	—	550	1·10	500	2·20	422	5·49
		—	—	22	1·10	30	2·20	47	5·49
11	3	692	0·55	647	1·10	603	2·47	564	4·95
		54	0·55	64	1·10	81	2·47	102	5·22

Table 2 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
10	4	805	0·50	767	1·00	722	2·50	672	5·00
		104	0·50	117	1·00	142	2·50	169	5·00
9	5	906	0·50	875	1·00	828	2·50	781	5·00
		164	0·50	183	1·00	214	2·50	246	4·95
8	6	1000	0·50	972	1·00	927	2·53	883	5·03
		233	0·50	257	1·00	294	2·53	331	5·03
7	7	1087	0·50	1061	0·99	1021	2·48	979	4·95
		312	0·50	338	0·99	378	2·48	420	4·95
14	1	—	—	—	—	—	—	332	6·67
		—	—	—	—	—	—	7	6·67
13	2	* 564	0·95	564	0·95	480	2·86	435	4·76
		* 21	0·95	21	0·95	36	2·86	44	4·76
12	3	712	0·44	662	1·10	613	2·42	563	5·05
		51	0·44	66	1·10	81	2·42	101	4·84
11	4	815	0·51	785	1·03	727	2·49	677	4·98
		103	0·51	117	1·03	143	2·56	169	5·05
10	5	922	0·53	891	1·00	839	2·50	791	5·00
		160	0·50	179	1·00	212	2·53	243	5·00
9	6	1024	0·50	993	1·00	943	2·52	895	5·00
		229	0·50	252	1·02	288	2·52	325	4·96
8	7	1114	0·50	1085	0·99	1039	2·49	994	4·99
		305	0·50	329	0·98	372	2·46	414	5·02
15	1	—	—	—	—	—	—	338	6·25
		—	—	—	—	—	—	6	6·25
14	2	* 576	0·83	576	0·83	493	2·50	431	5·00
		* 19	0·83	19	0·83	33	2·50	42	5·00
13	3	706	0·54	669	1·07	619	2·50	571	5·00
		54	0·54	62	1·07	79	2·50	99	5·00
12	4	824	0·49	792	0·99	736	2·47	685	5·00
		99	0·49	114	0·99	140	2·58	165	5·00
11	5	939	0·50	904	1·01	849	2·50	798	4·99
		156	0·50	176	1·03	207	2·50	240	5·01
10	6	1042	0·49	1008	1·00	955	2·51	905	4·98
		221	0·50	244	0·99	283	2·50	321	5·03
9	7	1135	0·51	1105	0·99	1054	2·52	1006	4·98
		296	0·49	321	0·99	365	2·53	407	5·03
8	8	1223	0·51	1195	0·99	1148	2·49	1102	5·00
		377	0·51	405	0·99	452	2·49	498	5·00
16	1	—	—	—	—	—	—	344	5·88
		—	—	—	—	—	—	6	5·88

Table 2 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
15	2	* 588	0·74	588	0·74	505	2·21	438	5·15
		* 18	0·74	18	0·74	31	2·21	45	5·15
14	3	724	0·44	674	1·03	622	2·50	572	5·00
		51	0·44	60	1·03	79	2·50	99	5·15
13	4	835	0·50	801	1·01	743	2·52	691	5·00
		98	0·50	112	1·01	137	2·48	164	4·96
12	5	952	0·50	916	1·00	859	2·49	805	5·01
		154	0·50	172	1·00	205	2·49	237	4·96
11	6	1057	0·50	1023	1·00	967	2·50	913	5·03
		218	0·49	241	1·01	279	2·51	317	5·00
10	7	1157	0·49	1123	1·00	1069	2·49	1017	5·01
		290	0·50	316	0·99	359	2·48	402	5·00
9	8	1249	0·50	1217	1·01	1165	2·51	1116	5·01
		368	0·49	398	1·01	445	2·48	492	5·02
17	1	—	—	—	—	—	—	350	5·56
		—	—	—	—	—	—	6	5·56
16	2	* 600	0·65	550	1·31	491	2·61	440	5·23
		* 17	0·65	24	1·31	30	2·61	43	5·23
15	3	721	0·49	690	0·98	628	2·45	573	5·02
		49	0·49	61	0·98	77	2·45	97	4·90
14	4	848	0·49	812	0·98	749	2·48	696	5·00
		96	0·49	111	1·01	137	2·52	162	5·00
13	5	967	0·49	927	1·00	868	2·50	812	4·98
		151	0·49	170	1·00	202	2·47	235	4·97
12	6	1074	0·50	1037	1·00	977	2·52	922	4·98
		214	0·50	237	1·00	276	2·52	314	5·02
11	7	1175	0·50	1140	1·00	1082	2·49	1027	5·01
		284	0·49	311	1·01	355	2·51	397	4·97
10	8	1271	0·49	1236	1·00	1181	2·49	1128	5·00
		361	0·49	391	1·01	439	2·51	486	5·02
9	9	1360	0·50	1327	1·00	1275	2·49	1225	4·98
		443	0·50	476	1·00	528	2·49	578	4·98
18	1	—	—	—	—	—	—	355	5·26
		—	—	—	—	—	—	5	5·26
17	2	* 610	0·58	560	1·17	501	2·34	438	5·26
		* 16	0·58	22	1·17	28	2·34	42	5·26
16	3	731	0·52	686	1·03	632	2·48	576	4·95
		51	0·52	59	1·03	77	2·37	97	5·06
15	4	857	0·49	819	0·98	757	2·48	698	5·03
		95	0·54	108	0·98	134	2·53	161	5·01
14	5	977	0·50	936	1·01	873	2·51	815	5·01
		148	0·50	167	1·01	200	2·52	233	5·04

Table 2 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
13	6	1086	0·51	1047	1·00	985	2·49	927	4·98
		210	0·50	233	1·00	272	2·50	310	5·00
12	7	1190	0·50	1152	1·00	1091	2·50	1034	4·98
		279	0·50	305	0·99	349	2·48	393	5·03
11	8	1288	0·50	1251	1·00	1192	2·50	1136	5·02
		354	0·50	383	0·99	432	2·49	479	4·98
10	9	1380	0·50	1345	1·00	1288	2·51	1235	5·00
		434	0·50	467	1·00	520	2·51	570	4·98
19	1	—	—	—	—	360	5·00	360	5·00
		—	—	—	—	5	5·00	5	5·00
18	2	* 620	0·53	570	1·05	491	2·63	437	5·26
		* 15	0·53	21	1·05	32	2·63	44	5·26
17	3	735	0·53	697	0·96	637	2·46	578	5·09
		49	0·53	57	0·96	76	2·54	96	5·09
16	4	865	0·50	824	0·99	761	2·50	702	5·02
		92	0·52	107	0·97	133	2·52	160	5·02
15	5	986	0·50	944	1·01	880	2·50	820	5·01
		146	0·50	165	0·99	198	2·53	230	4·94
14	6	1099	0·50	1057	1·01	993	2·50	933	5·00
		207	0·49	230	1·00	269	2·50	307	4·98
13	7	1205	0·50	1165	1·00	1101	2·49	1041	5·01
		274	0·49	301	1·00	346	2·51	389	5·01
12	8	1305	0·50	1266	1·00	1204	2·49	1145	5·01
		348	0·50	378	1·01	427	2·49	475	5·03
11	9	1400	0·50	1362	1·01	1302	2·51	1246	4·99
		427	0·50	460	1·01	513	2·49	564	4·99
10	10	1489	0·50	1454	1·00	1397	2·49	1343	4·97
		511	0·50	546	1·00	603	2·49	657	4·97

Table 3. Approximate and exact significance levels of the Savage test for $N = 20$

n	m	0·5%		1·0%		2·5%		5·0%	
19	1	—	—	—	—	—	—	0·20	5·00
		—	—	—	—	—	—	14·71	5·00
18	2	0·04	0·53	0·15	1·05	0·98	2·63	2·87	5·26
		6·89	0·53	7·55	1·05	8·88	2·63	10·53	5·26
17	3	0·17	0·53	0·37	0·96	1·15	2·46	3·05	5·09
		4·53	0·53	5·07	0·96	6·55	2·54	8·46	5·09
16	4	0·26	0·50	0·54	0·99	1·49	2·50	3·46	5·02
		3·19	0·52	3·89	0·97	5·41	2·52	7·44	5·02
15	5	0·35	0·50	0·68	1·01	1·73	2·50	3·76	5·01
		2·45	0·50	3·13	0·99	4·65	2·53	6·67	4·94
14	6	0·44	0·50	0·82	1·01	1·95	2·50	4·01	5·00
		1·95	0·49	2·60	1·00	4·11	2·50	6·19	4·98
13	7	0·54	0·50	0·95	1·00	2·15	2·49	4·26	5·01
		1·58	0·49	2·21	1·00	3·71	2·51	5·83	5·01
12	8	0·66	0·50	1·10	1·00	2·36	2·49	4·50	5·01
		1·32	0·50	1·91	1·01	3·35	2·49	5·51	5·03
11	9	0·78	0·50	1·27	1·01	2·59	2·51	4·71	4·99
		1·11	0·50	1·66	1·01	3·06	2·49	5·20	4·99
10	10	0·93	0·50	1·44	1·00	2·80	2·49	4·94	4·97
		0·93	0·50	1·44	1·00	2·80	2·49	4·94	4·97

Souhrn

TABULKY PRO DVOUVÝBĚROVÝ SAVAGEŮV POŘADOVÝ TEST OPTIMÁLNÍ PRO EXPONENCIÁLNÍ HUSTOTY

Zbyněk Šidák

Publikují se tabulky skórů a horních a dolních kritických hodnot pro tzv. Savageův test pro případy, kdy rozsah $m + n$ dvou spojených výběrů leží v mezích $6 \leqq m + n \leqq 20$ a jednostranná hladina významnosti leží blízko 0,5%, 1%, 2,5%, 5%. Tento test je optimální pro dvouvýběrový problém škály, jsou-li základní rozložení exponenciální.

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