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A CHAIN OF INEQUALITIES FOR SOME TYPES OF MULTIVARIATE DISTRIBUTIONS, WITH NINE SPECIAL CASES

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1. SUMMARY

In this paper we try to extract the core of the argument used by Y. L. Tong [15] for probabilities in multivariate equicorrelated normal distributions, and to generalize it as far as possible. The proof of the general inequalities is very simple, but these inequalities embrace a large number of interesting special cases. We give here nine illustrations: for multivariate equicorrelated normal, t, χ^2, Poisson, exponential distributions, for normal and rank statistics in comparing many treatments with one control, for order statistics used in estimating quantiles, and for characteristic roots of covariance matrices in certain multiple sampling.

2. GENERAL INEQUALITIES

Y. L. Tong in [15], Theorem 1, proved the following assertion: If a random variable X is non-negative with probability 1, then

$$EX^k \geqq (EX^{k/s})^s \geqq (EX)^k + [EX^{k/s} - (EX)^{k/s}]^s \quad \text{for} \quad k \geqq s \geqq 1 \,. \tag{1}$$

An immediate consequence is the following chain of inequalities.

Lemma. *If a random variable X is non-negative with probability* 1, *then*

$$\begin{aligned} EX^k &\geqq [EX^{k-1}]^{k/(k-1)} \geqq [EX^{k-2}]^{k/(k-2)} \geqq \ldots \geqq [EX^m]^{k/m} \geqq [EX^2]^{k/2} \geqq \\ &\geqq [EX]^k + [EX^2 - (EX)^2]^{k/2} \geqq [EX]^k \quad \text{for} \quad k \geqq m \geqq 2 \,. \end{aligned} \tag{2}$$

Namely, on putting $s = k/(k-1)$ in (1), we get the first inequality in (2); on changing k into $k-1$ in this first inequality, we get the second inequality in (2), etc. The last but one inequality in (2) follows from (1) on putting $s = k/2$, and the last inequality is clear because $EX^2 \geqq (EX)^2$.

Now we can prove the main result of this paper.

Theorem. *Let $\mathbf{X}_i = (X_{i1}, \dots, X_{ip})$, $i = 1, \dots, k$, be a sample of k independent p-dimensional random vectors with the same distribution (but where the p components of each vector may be dependent), and let $\mathbf{U} = (U_1, \dots, U_q)$ be another q-dimensional random vector (with possibly dependent components) which is independent of all X_{ij}'s. Further, let $f = f(x_1, \dots, x_p; u_1, \dots, u_q)$ be any measurable r-dimensional vector function, and A any measurable r-dimensional set. Denoting*

$$(3) \qquad \beta(k) = P\{f(X_{i1}, \dots, X_{ip}; U_1, \dots, U_q) \in A;\ i = 1, \dots, k\}\,,$$

we have the following chain of inequalities

$$(4) \qquad \begin{aligned} \beta(k) &\geqq [\beta(k-1)]^{k/(k-1)} \geqq [\beta(k-2)]^{k/(k-2)} \geqq \dots \geqq [\beta(m)]^{k/m} \geqq \\ &\geqq [\beta(2)]^{k/2} \geqq \beta^k(1) + [\beta(2) - \beta^2(1)]^{k/2} \geqq \beta^k(1) \\ &\text{for} \quad k \geqq m \geqq 2\,. \end{aligned}$$

The proof follows easily from the Lemma stated above on putting there $X = P\{f(X_{i1}, \dots, X_{ip}; U_1, \dots, U_q) \in A \mid U_1, \dots, U_q\}$, and its idea is extracted from the proof of Theorem 2 in Tong [15]. Namely, we have

$$\begin{aligned} \beta(k) &= P\{f(\mathbf{X}_i; \mathbf{U}) \in A;\ i = 1, \dots, k\} = EP\{f(\mathbf{X}_i; \mathbf{U}) \in A;\ i = 1, \dots, k \mid \mathbf{U}\} = \\ &= EP^k\{f(\mathbf{X}_i; \mathbf{U}) \in A \mid \mathbf{U}\} \geqq [EP^{k-1}\{f(\mathbf{X}_i; \mathbf{U}) \in A \mid \mathbf{U}]^{k/(k-1)} = \\ &= [EP\{f(\mathbf{X}_i; \mathbf{U}) \in A;\ i = 1, \dots, k-1 \mid \mathbf{U}\}]^{k/k-1} = \\ &= [P\{f(\mathbf{X}_i; \mathbf{U}) \in A;\ i = 1, \dots, k-1\}]^{k/k-1} = [\beta(k-1)]^{k/k-1}\,, \end{aligned}$$

and further inequalities in (4) can be proved similarly.

The inequalities (4) are analogues of those given in Theorem 2 in Tong [15]; however, additional inequalities can be clearly obtained by our method from the second inequality in (1).

Remark. C. G. Khatri in [7], Corollary 1 (ii), proved that $EX^k \geqq (EX^m)(EX^{k-m})$ provided $X \geqq 0$, $k \geqq m \geqq 0$. These inequalities can also be used in our method of proof in place of the above Lemma, and the resulting inequalities are clearly

$$(5) \qquad \begin{aligned} &\beta(k) \geqq \beta(k-1)\,\beta(1)\,, \\ &\beta(k) \geqq \beta(k-2)\,\beta(2), \dots, \\ &\beta(k) \geqq \beta(m)\,\beta(k-m) \quad \text{for} \quad k > m \geqq 1\,. \end{aligned}$$

A question now arises naturally which of the inequalities (5) or (4) yield closer bounds for $\beta(k)$. Clerly, in view of symmetry, we may restrict ourselves in (5) only to cases with $m \geqq k - m$, i.e. with $m \geqq \frac{1}{2}k$. However, for these cases (4) implies $\beta(m) \geqq [\beta(k - m)]^{m/(k-m)}$, which immediately gives

$$[\beta(m)]^{k/m} \geqq \beta(m)\,\beta(k - m)\,. \tag{6}$$

Therefore, if we know the probabilities $\beta(1), \beta(2), \ldots, \beta(m)$ up to dimension m, $[\beta(m)]^{k/m}$ in (4) gives the closest bound for $\beta(k)$ which can be obtained from (4) and (5).

3. SPECIAL CASES

The inequalities (4) contain a large number of interesting special cases. Nine of them will be mentioned here for illustration, but many more can be found. This flexibility is mainly due to the possibility of choosing arbitrarily the function f.

In each of the following cases we shall display the forms of the respective function f and of the probability $\beta(k)$, but we shall not repeat the inequalities (4) for these specific probabilities.

1. Equicorrelated normal distributions. Let $Z_1, \ldots, Z_k$ have a k-variate normal distribution with mean values 0, variances 1, and with all correlations equal to $\varrho \geqq 0$. It is then a well known and often used fact that such a distribution may be represented by the distribution of $Z_i = (1 - \varrho)^{1/2} X_i - \varrho^{1/2} U$, $i = 1, \ldots, k$, where $U, X_1, \ldots, X_k$ are mutually independent $N(0, 1)$ variables. Therefore, putting in our Theorem $p = q = r = 1$, $f(X_i; U) = (1 - \varrho)^{1/2} X_i - \varrho^{1/2} U$, and either $A = (-\infty, d)$ or $A = (-d, d)$, we get the probabilities either

$$\beta(k) = P\{Z_i < d;\ i = 1, \ldots, k\} \quad \text{or} \quad \beta(k) = P\{|Z_i| < d;\ i = 1, \ldots, k\}\,,$$

respectively. Thus, by this specialization, we get again the inequalities proved by Tong [15], Theorem 2. In this special case of equicorrelated variables, the inequalities (4) improve the general inequality $\beta(k) \geqq \beta^k(1)$ obtained by Slepian [12], and by Šidák [10] and Khatri [7], respectively.

2. Equicorrelated Student distributions. One type of a multivariate Student distribution is the distribution of $Z_1/S, \ldots, Z_k/S$, where $Z_1, \ldots, Z_k$ is as before in Case 1, and

$$S = \left(\nu^{-1} \sum_{j=1}^{\nu} W_j^2\right)^{1/2}$$

where $W_1, \ldots, W_\nu$ are independent $N(0, 1)$ variables, also independent of $Z_1, \ldots, Z_k$.

We use again the same representation of Z_i's as before, and put $p = r = 1$, $q = 2$, $(U_1, U_2) = (U, S)$,

$$f(X_i; U, S) = [(1 - \varrho)^{1/2} X_i - \varrho^{1/2} U]/S ,$$

and either $A = (-\infty, d)$ or $A = (-d, d)$. Then we have either

$$\beta(k) = P\{Z_i/S < d;\ i = 1, \ldots, k\} \quad \text{or} \quad \beta(k) = P\{|Z_i|/S < d;\ i = 1, \ldots, k\} ,$$

respectively. The relevant inequalities (4) are also due to Tong [15], Theorem 2.

Another type of a multivariate Student distribution is the distribution of $Z_1/S_1, \ldots$ $\ldots, Z_k/S_k$, where $Z_1, \ldots, Z_k$ is again as before, and

$$S_i = (\nu^{-1} \sum_{j=1}^{\nu} W_{ij}^2)^{1/2} , \quad i = 1, \ldots, k ,$$

where $\mathbf{W}_j = (W_{1j}, \ldots, W_{kj})$, $j = 1, \ldots, \nu$, is a random sample of vectors, which are mutually independent and independent of $Z_1, \ldots, Z_k$, and each of which has the same k-variate normal distribution with mean values 0 and variances 1; we shall also suppose that all correlations between W_{ij} and W_{hj} $(i, h = 1, \ldots, k;\ i \neq h)$ are equal to $\tau \geqq 0$. Clearly again as before, we can use a similar representation

$$Z_i = (1 - \varrho)^{1/2} X_{i0} - \varrho^{1/2} U_0 , \quad i = 1, \ldots, k ,$$

and

$$W_{ij} = (1 - \tau)^{1/2} X_{ij} - \tau^{1/2} U_j , \quad i = 1, \ldots, k;\ j = 1, \ldots, \nu ,$$

where all X_{i0}, X_{ij}, U_0, U_j are mutually independent $N(0, 1)$ variables. If we put now $p = q = \nu + 1$, $r = 1$,

$$f(X_{i0}, X_{i1}, \ldots, X_{i\nu}; U_0, U_1, \ldots, U_\nu) = \frac{(1 - \varrho)^{1/2} X_{i0} - \varrho^{1/2} U_0}{\{\nu^{-1} \sum_{j=1}^{\nu} [(1 - \tau)^{1/2} X_{ij} - \tau^{1/2} U_j]^2\}^{1/2}} ,$$

and either $A = (-\infty, d)$ or $A = (-d, d)$, we get the probability either

$$\beta(k) = P\{Z_i/S_i < d;\ i = 1, \ldots, k\} \quad \text{or} \quad \beta(k) = P\{|Z_i|/S_i < d;\ i = 1, \ldots, k\} ,$$

respectively. Concerning the latter probability, the inequalities (4) improve, in this special case of equicorrelated variables, the inequality $\beta(k) \geqq \beta^k(1)$ which was proved by Šidák [11] and Khatri [7] for more general covariance structures.

3. Equicorrelated χ^2 distributions. Let $\mathbf{Z}_j = (Z_{1j}, \ldots, Z_{kj})$ be a random vector having a k-variate normal distribution with mean values 0, variances 1, and with all correlations equal to $\varrho_j \geqq 0$; let j run through $1, 2, \ldots, \nu$, and let the vectors $\mathbf{Z}_j$ for different j's be independent. Similarly as in Case 1, we can use a representation

$Z_{ij} = (1 - \varrho_j)^{1/2} X_{ij} - \varrho_j^{1/2} U_j$, $i = 1, \ldots, k$; $j = 1, \ldots, v$, where all X_{ij}, U_j are mutually independent $N(0, 1)$ variables. Putting now in our Theorem $p = q = v$, $r = 1$,

$$f(X_{i1}, \ldots, X_{iv}; U_1, \ldots, U_v) = \sum_{j=1}^{v} [(1 - \varrho_j)^{1/2} X_{ij} - \varrho_j^{1/2} U_j]^2 ,$$

and $A = (d_1, d_2)$, we get the probability

$$\beta(k) = P\{d_1 < \sum_{j=1}^{v} Z_{ij}^2 < d_2;\ i = 1, \ldots, k\} .$$

It may be observed that the inequality $\beta(k) \geqq \beta^k(1)$ for such a probability was proved under different assumptions than ours by the following authors: Jensen [5] proved it for the case where the assumption of independence of the vectors $\boldsymbol{Z}_j$ was relaxed, but only for $k = 2$; Khatri [7] proved it under more general conditions on the correlation structure, but only for one-sided intervals (d_1, d_2), i.e. where either $d_1 = 0$ or $d_2 = \infty$. All of these inequalities can be clearly used for finding conservative confidence intervals for k variances simultaneously; for details cf. Jensen-Jones [6], p. 328, or Khatri [7], p. 1855.

4. **Poisson distributions.** A special case of multivariate Poisson variables $Z_1, \ldots, Z_k$ may be given by the model $Z_1 = X_1 + U, \ldots, Z_k = X_k + U$, where $X_1, \ldots, X_k$, U are independent Poisson variables. Note that this model includes, in particular, the bivariate Poisson distribution (cf. Haight [3], Section 3.12, or Holgate [4]). If we put $p = q = r = 1$ and $f(X_i; U) = X_i + U$, the probability $\beta(k)$ in the Theorem will become $\beta(k) = P\{Z_i \in A;\ i = 1, \ldots, k\}$.

5. **Exponential distributions.** Similarly, a special case of multivariate exponential variables $Z_1, \ldots, Z_k$ may be defined by the model $Z_1 = \min(X_1, U), \ldots, Z_k = \min(X_k, U)$, where $X_1, \ldots, X_k$, U are independent exponential variables. In particular, this model for $k = 2$ gives the bivariate exponential distribution (cf. Marshall-Olkin [8], Theorem 3.2). Thus, putting $p = q = r = 1$, $f(X_i; U) = \min(X_i, U)$, we obtain the probability $\beta(k) = P\{Z_i \in A;\ i = 1, \ldots, k\}$.

6. **Comparison of k groups with one control — normal variables.** Let us have a sample $Y_{01}, \ldots, Y_{0m}$ in a control group, and the samples (of the same size) $Y_{i1}, \ldots, Y_{in}$, $i = 1, \ldots, k$, in k experimental groups. Let each Y_{ij} have the normal distribution $N(\mu_i, \sigma^2)$ with the same known variance σ^2 and let all of them be mutually independent. The question is to find which μ_i, $i = 1, \ldots, k$, differ significantly from μ_0. For this purpose, Dunnett [2] (or, cf. also Miller [9], Section 2.5) proposed the test statistics

$$Z_i = \frac{\bar{Y}_i - \bar{Y}_0}{\sigma} \sqrt{\left(\frac{mn}{m + n}\right)}, \quad i = 1, \ldots, k ,$$

where

$$\bar{Y}_i = n^{-1} \sum_{j=1}^{n} Y_{ij} \quad \text{for} \quad i = 1, \ldots, k\,,$$

and

$$\bar{Y}_0 = m^{-1} \sum_{j=1}^{m} Y_{0j}\,.$$

If $\mu_1 = \mu_2 = \ldots = \mu_k$, we may put $p = n$, $q = m$, $r = 1$, $X_{i1} = Y_{i1}, \ldots, X_{in} = Y_{in}$ for $i = 1, \ldots, k$, $U_1 = Y_{01}, \ldots, U_m = Y_{0m}$, $f(X_{i1}, \ldots, X_{in};\ U_1, \ldots, U_m) = Z_i$, and either $A = (-\infty, d)$ or $A = (-d, d)$. Then

$$\beta(k) = P\{Z_i < d;\ i = 1, \ldots, k\} \quad \text{or} \quad \beta(k) = P\{|Z_i| < d;\ i = 1, \ldots, k\}\,,$$

respectively. The inequalities (4) in our Theorem can be used e.g. for approximating the critical values for the relevant test based on the variables $Z_1, \ldots, Z_k$; in fact, the inequality $\beta(k) \geqq [\beta(2)]^{k/2}$ was used for this purpose for the twosided test already by Dunnett [2] himself.

If the variance σ^2 in this problem is unknown (this problem was also considered by Dunnett [2]), everything is only slightly modified. Namely, in the definition of Z_i we need to replace σ by its estimate

$$s = \sqrt{\{(m + kn - k - 1)^{-1} [\sum_{j=1}^{m} (Y_{0j} - \bar{Y}_0)^2 + \sum_{i=1}^{k} \sum_{j=1}^{n} (Y_{ij} - \bar{Y}_i)^2]\}}\,,$$

and the remaining discussion is completely analogous.

7. **Comparison of k groups with one control — nonparametric tests.** Let us consider the same situation and the same question as in Case 6, except that the Y_{ij}'s may have any continuous distribution. For this problem, Steel [13] (or, cf. also Miller [9], Section 4.3) recommended the rank statistics

$$Z_i = \sum_{j=1}^{n} R_{ij}\,, \quad i = 1, \ldots, k\,,$$

where R_{ij} is the rank of the observation Y_{ij} in the pooled sample $Y_{i1}, \ldots, Y_{in}, Y_{01}, \ldots$ $\ldots, Y_{0m}$. (I.e. Z_i is the common Wilcoxon statistic for the i-th group and the control group.) If all observations Y_{ij} in all k experimental groups have the same distribution, we may again identify $p = n$, $q = m$, $r = 1$, $X_{i1} = Y_{i1}, \ldots, X_{in} = Y_{in}$ for $i = 1, \ldots, k$, $U_1 = Y_{01}, \ldots, U_m = Y_{0m}$, and put

$$f(X_{i1}, \ldots, X_{in}; U_1, \ldots, U_m) = \sum_{j=1}^{n} R_{ij}\,,$$

$A = (d_1, d_2)$. Then

$$\beta(k) = P\{d_1 < Z_i < d_2;\ i = 1, \ldots, k\}\,,$$

and the inequalities (4) can be again used for approximating the critical values of the corresponding test. (Note that the inequality $\beta(k) \geqq \beta^k(1)$ leads to approximating the critical values in question by those of the common Wilcoxon test.)

Further, if the problem is modified so that the observations are arranged in blocks, we can use the so-called many-one sign statistics (cf. Steel [14], or Miller [9], Section 4.1); the developments for this case are similar as before.

8. **Estimation of quantiles in equicorrelated normal distributions.** Let us have a sample of n independent vectors $\mathbf{Y}_j = (Y_{1j}, \ldots, Y_{kj})$, $j = 1, \ldots, n$, where each $\mathbf{Y}_j$ has a k-variate normal distribution with an unknown mean vector $(\mu_1, \ldots, \mu_k)$, an unknown vector of standard deviations $(\sigma_1, \ldots, \sigma_k)$, and with all correlations equal to $\varrho \geqq 0$. Let $y_{i\alpha}$ denote the α-quantile of the normal distribution $N(\mu_i, \sigma_i^2)$, and let $Y_i^{(1)} \leqq Y_i^{(2)} \leqq \ldots \leqq Y_i^{(n)}$ denote the ordered observations $Y_{i1}, Y_{i2}, \ldots, Y_{in}$. We are interested in finding a lower bound for the confidence coefficient of simultaneous confidence statements

$$Y_i^{(s)} < y_{i\alpha} < Y_i^{(t)}\,, \quad i = 1, \ldots, k\,,$$

for some fixed s, t, $1 \leqq s < t \leqq n$. To this end, define first the vectors $\mathbf{Z}_j = (Z_{1j}, \ldots, Z_{kj})$ by $Z_{ij} = (Y_{ij} - \mu_i)/\sigma_i$, $i = 1, \ldots, k$; $j = 1, \ldots, n$. Similarly as before, let $Z_i^{(1)} \leqq Z_i^{(2)} \leqq \ldots \leqq Z_i^{(n)}$ be the ordered observations $Z_{i1}, Z_{i2}, \ldots, Z_{in}$. Now, for these Z_{ij}, we can use (cf. Case 1 or 3) a representation $Z_{ij} = (1 - \varrho)^{1/2} X_{ij} - \varrho^{1/2} U_j$, where all X_{ij}, U_j are mutually independent $N(0, 1)$, variables. Further, if z_α is the α-quantile of the distribution $N(0, 1)$, then clearly $y_{i\alpha} = \sigma_i z_\alpha + \mu_i$. Finally, let $p = q = n$, $r = 2$, let the coordinates of the two-dimensional function f be

$$f_1(X_{i1}, \ldots, X_{in}; U_1, \ldots, U_n) = Z_i^{(s)}\,,$$

$$f_2(X_{i1}, \ldots, X_{in}; U_1, \ldots, U_n) = Z_i^{(t)}\,,$$

and let $A = (-\infty, z_\alpha) \times (z_\alpha, \infty)$. Then we obtain the probability

$$\beta(k) = P\{Z_i^{(s)} < z_\alpha < Z_i^{(t)};\ i = 1, \ldots, k\} =$$
$$= P\{(Y_i^{(s)} - \mu_i)/\sigma_i < z_\alpha < (Y_i^{(t)} - \mu_i)/\sigma_i;\ i = 1, \ldots, k\} =$$
$$= P\{Y_i^{(s)} < y_{i\alpha} < Y_i^{(t)};\ i = 1, \ldots, k\}\,,$$

and our Theorem may give us the desired lower bound for $\beta(k)$. If we specialize $y_{i\alpha}$ to be the medians, our result $\beta(k) \geqq \beta^k(1)$ may be compared with those presented by O. J. Dunn [1]: she showed that $\beta(2) \geqq \beta^2(1)$ for any bivariate population with continuous marginal distributions, but that $\beta(k) \geqq \beta^k(1)$ need not hold for $k \geqq 3$. Our result shows that, for a very special type of distributions, $\beta(k) \geqq \beta^k(1)$ does hold for any k.

9. **Characteristic roots of covariance matrices in a case of multiple sampling.** Clearly, our Theorem and its proof may be also modified so that the function f takes on for its values measurable r-dimensional sets, and that

$$\beta(k) = P\{f(X_{i1}, \ldots, X_{ip}; U_1, \ldots, U_q) \subset A;\ i = 1, \ldots, k\}\,.$$

The following case will illustrate this modification. Let us have $k + 1$ random samples of s-dimensional vectors: the 0-th sample consists of vectors $\mathbf{Y}_{0j} = (Y_{10j}, \ldots, Y_{s0j})$, $j = 1, \ldots, m$; for $i = 1, \ldots, k$, the i-th sample consists of vectors $\mathbf{Y}_{ij} = (Y_{1ij}, \ldots, Y_{sij})$, $j = 1, \ldots, n$; let all of these vectors be independent and possess the same distribution. Then, for each $i = 1, \ldots, k$, let us combine the 0-th sample and the i-th sample into one sample $\mathbf{Y}_{01}, \ldots, \mathbf{Y}_{0m}, \mathbf{Y}_{i1}, \ldots, \mathbf{Y}_{in}$, and let C_i denote the set of characteristic roots of the empirical covariance matrix of such a combined sample. We are interested in the probability that $C_i \subset A$, $i = 1, \ldots, k$. In our Theorem, we put $p = sn$, $q = sm$, $r = 2$,

$$\mathbf{X}_i = (Y_{1i1}, \ldots, Y_{si1}, \ldots, Y_{1in}, \ldots, Y_{sin}) \quad \text{for} \quad i = 1, \ldots, k\,,$$

$$\mathbf{U} = (Y_{101}, \ldots, Y_{s01}, \ldots, Y_{10m}, \ldots, Y_{s0m})\,, \quad f(X_{i1}, \ldots, X_{ip}; U_1, \ldots, U_q) = C_i\,,$$

and we obtain

$$\beta(k) = P\{C_i \subset A;\ i = 1, \ldots, k\}\,.$$

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Souhrn

ŘETĚZ NEROVNOSTÍ PRO JISTÉ TYPY MNOHOROZMĚRNÝCH ROZLOŽENÍ S DEVÍTI SPECIÁLNÍMI PŘÍPADY

ZBYNĚK ŠIDÁK

Článek se snaží extrahovat jádro důkazu, pomocí něhož Y. L. Tong [15] dostal jisté nerovnosti pro pravděpodobnosti v mnohorozměrném ekvikorelovaném normálním rozložení, a pak toto jádro co možno daleko zobecnit. Výsledkem je obecná věta ukazující, že pro pravděpodobnosti (3) platí řetěz nerovností (4). Důkaz je velmi jednoduchý, ale tyto nerovnosti v sobě obsahují mnoho zajímavých speciálních případů, z nichž devět je dále v článku uvedeno pro ilustraci: případy mnohorozměrného ekvikorelovaného normálního, t, χ^2 rozložení, mnohorozměrného Poissonova a exponenciálního rozložení, případy normálních a pořadových statistik při srovnávání více populací s jednou kontrolní, odhadování kvantilů pomocí pořádkových statistik v ekvikorelovaném normálním rozložení a případ charakteristických kořenů kovariančních matic při jistých mnohonásobných výběrech.

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