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NUMERICAL ALGORITHM FOR THE COMPUTATION OF THE SLIP-LINE FIELD

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If we consider the plane strain problem of a rigid-perfectly plastic material, the characteristic lines of the partial differential equations of equilibrium are slip-lines. The geometry of the slip-line field is very well known [1], [3].

Since many metal-forming processes can be considered as the plane strain problem, the determination of the stress and velocity fields of the formed material is an important base for the dimensioning of dies. The existing methods of geometric construction of the slip-line field appear to be toilsome, slow, not always precise enough and it is not possible to program them for a computer; therefore the following paragraphs are devoted to a numerical algorithm which can be very easily programmed for a computer.

We assume that the physical conditions are given so that we know one point and two slip-lines α_0, β_0, each of one family, which pass through this point (see Fig. 1). Along each curve we choose the division so that the differences $\Delta\varphi_\alpha$ and $\Delta\varphi_\beta$ are constant and we deduce for a general quadrangle $(n, m), (n + 1, m), (n, m + 1)$, $(n + 1, m + 1)$ the recurrence procedure for calculation of coordinates of the point $(n + 1, m + 1)$, if we know the coordinates of the other three points.

Now we choose the local coordinate system u, v with the origin at point P (see Fig. 1) and transform this system by rotating it around the point P into the coordinate system $\bar{u}$, $\bar{v}$, which is formed by tangents to the slip-line sat the point P. Differentiating the transformation equations and making use of the orthogonality of the slip-lines, we obtain

$$\text{(1)} \qquad \begin{aligned} &\text{along an } \alpha \text{ line:} \quad \mathrm{d}\bar{u} = \bar{v}\,\mathrm{d}\varphi\,,\\ &\text{along a } \beta \text{ line:} \quad \mathrm{d}\bar{v} = \bar{u}\,\mathrm{d}\varphi\,. \end{aligned}$$

If we substitute into this relations the differences for the differentials and if we introduce for the increasing angle φ sign $\Delta\varphi = +1$ and for the decreasing angle sign $\Delta\varphi = -1$, we obtain from (1) the system of difference equations

$$\bar{u}_{n+1,m+1} - \bar{u}_{n+1,m} = \tfrac{1}{2}(\bar{v}_{n+1,m+1} + \bar{v}_{n+1,m})(\pm 1)\,\Delta\varphi_\alpha\,,$$
$$\bar{v}_{n+1,m+1} - \bar{v}_{n,m+1} = -\tfrac{1}{2}(\bar{u}_{n+1,m+1} + \bar{u}_{n,m+1})(\pm 1)\,\Delta\varphi_\beta\,,$$

which gives

$$(2) \qquad \bar{u}_{n+1,m+1} = \frac{1}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}\,\bar{u}_{n+1,m} + \frac{(\frac{1}{2})(\pm 1)\,\Delta\varphi_\alpha}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}(\bar{v}_{n+1,m} + \bar{v}_{n,m+1}) - \frac{(\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}\,\bar{u}_{n,m+1}$$

$$\bar{v}_{n+1,m+1} = \frac{1}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}\,\bar{v}_{n,m+1} - \frac{(\frac{1}{2})(\pm 1)\,\Delta\varphi_\beta}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}(\bar{u}_{n,m+1} + \bar{u}_{n+1,m}) - \frac{(\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}{1 + (\frac{1}{4})(\pm 1)\,\Delta\varphi_\alpha(\pm 1)\,\Delta\varphi_\beta}\,\bar{v}_{n+1,m}\,.$$

Relations (2) give in the local coordinate system $\bar{u}, \bar{v}$ the coordinates $\bar{u}_{n+1,m+1}$; $\bar{v}_{n+1,m+1}$ if we know $\bar{u}_{n,m+1}$; $\bar{v}_{n,m+1}$, $\bar{u}_{n+1,m}$; $\bar{v}_{n+1,m}$ and differences $\Delta\varphi_\alpha$ and $\Delta\varphi_\beta$.

From these relations (2), which are true in the local coordinate system of one curvilinear quadrangle, we derive the numerical algorithm for calculation of nodal point coordinates of the slip-line field.

We choose the coordinate system x, y as in Figure 1 and the division of the slip-lines α_0, β_0 so that $|\Delta\varphi_\alpha| = |\Delta\varphi_\beta| = \Delta\varphi$, where $\Delta\varphi$ is constant. Following Hencky's first theorem for the slip-line field we can write relations (2) for a general quadrangle *PQRS* in the form

$$\bar{u}_{n+1,m+1} = \frac{1}{1 - (\frac{1}{4})(\Delta\varphi)^2}\,\bar{u}_{n+1,m} + \frac{(\frac{1}{2})\,\Delta\varphi}{1 - (\frac{1}{4})(\Delta\varphi)^2}(\bar{v}_{n+1,m} + \bar{v}_{n,m+1}) + \frac{(\frac{1}{4})(\Delta\varphi)^2}{1 - (\frac{1}{4})(\Delta\varphi)^2}\,\bar{u}_{n,m+1}\,,$$

$$\bar{v}_{n+1,m+1} = \frac{1}{1 - (\frac{1}{4})(\Delta\varphi)^2}\,\bar{v}_{n,m+1} + \frac{(\frac{1}{2})\,\Delta\varphi}{1 - (\frac{1}{4})(\Delta\varphi)^2}(\bar{u}_{n,m+1} + \bar{u}_{n+1,m}) + \frac{(\frac{1}{4})(\Delta\varphi)^2}{1 - (\frac{1}{4})(\Delta\varphi)^2}\,\bar{v}_{n+1,m}\,.$$

If we further denote

$$A = \frac{1}{1 - (\frac{1}{4})(\Delta\varphi)^2}, \quad B = \frac{(\frac{1}{2})\,\Delta\varphi}{1 - (\frac{1}{4})(\Delta\varphi)^2}, \quad C = \frac{(\frac{1}{4})(\Delta\varphi)^2}{1 - (\frac{1}{4})(\Delta\varphi)^2},$$

where $1 - (\frac{1}{4})(\Delta\varphi)^2 \neq 0$, it follows that

(3)
$$\bar{u}_{n+1,m+1} = A\bar{u}_{n+1,m} + B(\bar{v}_{n+1,m} + \bar{v}_{n,m+1}) + C\bar{u}_{n,m+1},$$
$$\bar{v}_{n+1,m+1} = A\bar{v}_{n,m+1} + B(\bar{u}_{n,m+1} + \bar{u}_{n+1,m}) + C\bar{v}_{n+1,m}.$$

Assume that we know in the coordinate system x, y the coordinates of the points

$$P(x_{n,m}, y_{n,m}), \quad Q(x_{n,m+1}, y_{n,m+1}), \quad R(x_{n+1,m}, y_{n+1,m}).$$

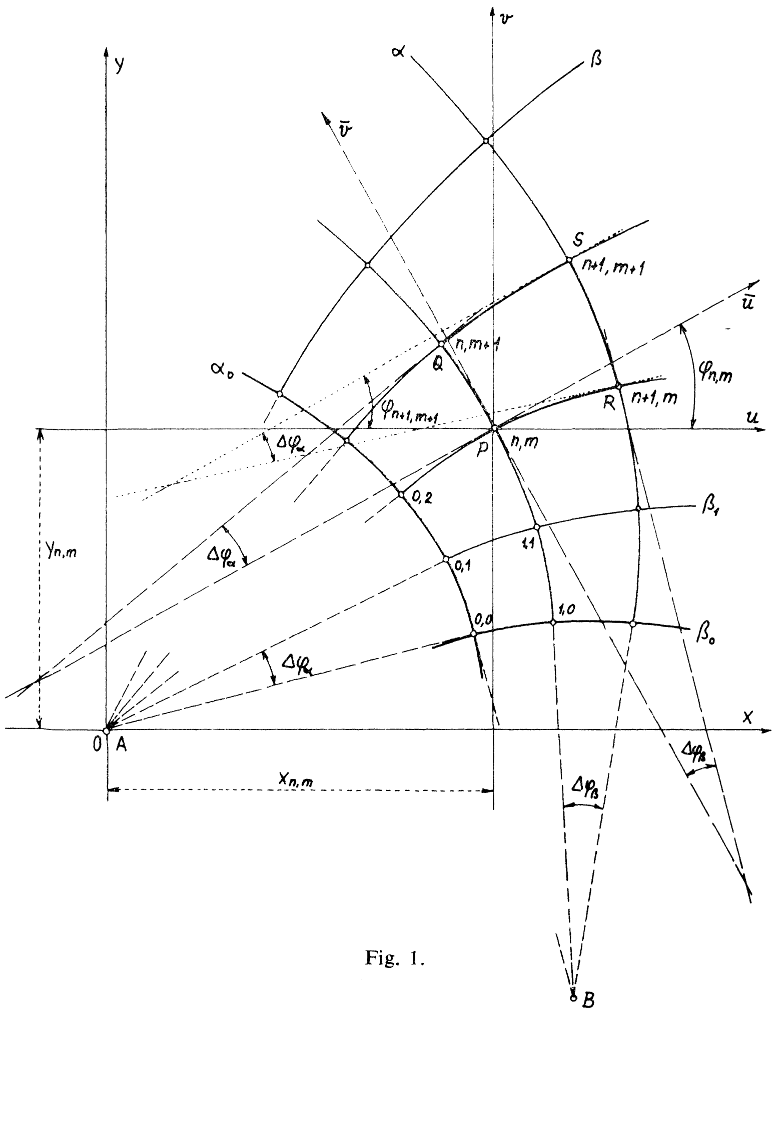

Fig. 1.

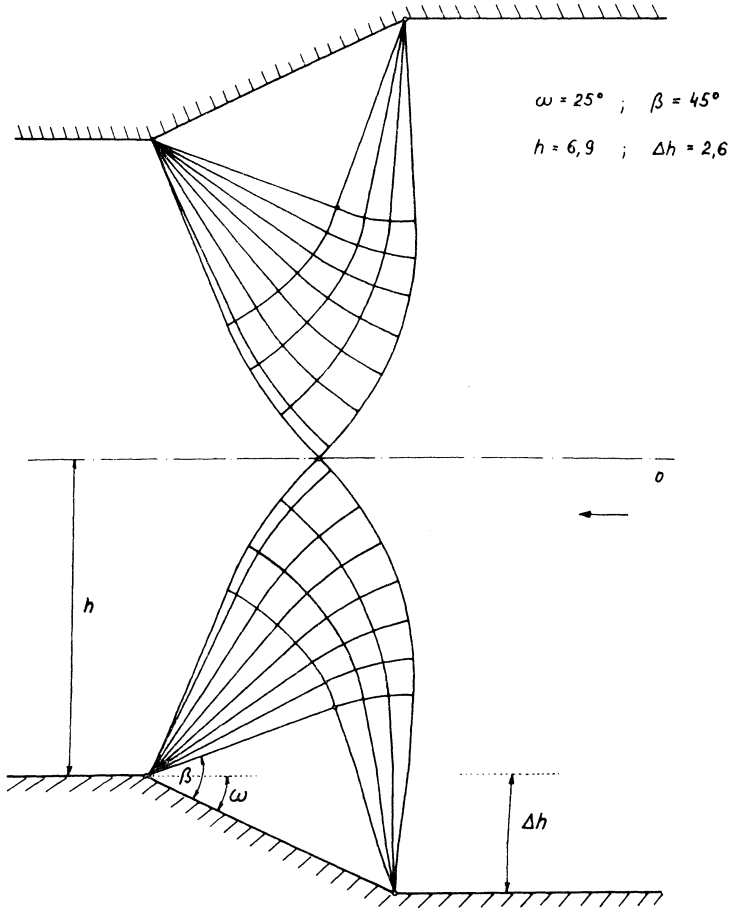

Fig. 2.

The transformation equations between systems $(x, y), (u, v), (\bar{u}, \bar{v})$, their inverse transformations and the relations (3) imply that the coordinates $(x_{n+1,m+1}; y_{n+1,m+1})$ of the point S can be written in the form

$$
\begin{aligned}
(4) \qquad x_{n+1,m+1} = {} & A(\cos \varphi_{n,m} - \tfrac{1}{2}\Delta\varphi \sin \varphi_{n,m})^2\, x_{n+1,m} + \\
& + A(\sin \varphi_{n,m} - \tfrac{1}{2}\Delta\varphi \cos \varphi_{n,m})^2\, x_{n,m+1} - \\
& - B(\Delta\varphi - 2 \sin 2\varphi_{n,m})\, x_{n,m} + \\
& + (B \cos 2\varphi_{n,m} + \tfrac{1}{2} \sin 2\varphi_{n,m})\, y_{n+1,m} + \\
& + (B \cos 2\varphi_{n,m} - \tfrac{1}{2} \sin 2\varphi_{n,m})\, y_{n,m+1} - 2B \cos 2\varphi_{n,m}\, y_{n,m}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1,m+1} = {} & \left(B \cos 2\varphi_{n,m} + \tfrac{1}{2} \sin 2\varphi_{n,m}\right) x_{n+1,m} + \\
& + \left(B \cos 2\varphi_{n,m} - \tfrac{1}{2} \sin 2\varphi_{n,m}\right) x_{n,m+1} - \\
& - 2B \cos 2\varphi_{n,m} x_{n,m} + A\left(\sin \varphi_{n,m} + \tfrac{1}{2}\Delta\varphi \cos \varphi_{n,m}\right)^2 y_{n+1,m} + \\
& + A\left(\cos \varphi_{n,m} + \tfrac{1}{2}\Delta\varphi \sin \varphi_{n,m}\right)^2 y_{n,m+1} - \\
& - B\left(\Delta\varphi + 2 \sin 2\varphi_{n,m}\right) y_{n,m} .
\end{aligned}
$$

TABLE OF NODAL POINT COORDINATES AND OF TANGENT ANGLES

	$x_{n,m}$	$y_{n,m}$	$\varphi_{n,m}$
The basic α slip-line	4,08786	1,48786	0,34907
	3,85869	2,00870	0,47997
	3,56349	2,49518	0,61087
	3,20731	2,93896	0,74176
	2,79626	3,33246	0,87266
	2,33737	3,66893	1,00356
	1,83848	3,94263	1,13446
The first α slip-line	4,634	1,647	0,21817
	4,452	2,269	0,34907
	4,186	2,872	0,47997
	3,838	3,444	0,61087
	3,412	3,973	-0,74176
	2,912	4,450	0,87266
	2,346	4,864	1,00356
The second α slip-line	5,196	1,734	0,08727
	5,085	2,453	0,21817
	4,874	3,174	0,34907
	4,561	3,882	0,47997
	4,147	4,562	0,61087
	3,633	5,201	0,74176
	3,022	5,784	0,87266
The boundary α slip-line	5,76	1,75	−0,04219
	5,74	2,55	0,08871
	5,61	3,39	0,21961
	5,36	4,23	0,35051
	4,99	5,07	0,48141
	4,49	5,89	0,61230
	3,86	6,66	0,74320
The boundary β slip-line	1,67	4,02	1,17666
	2,15	4,98	1,04576
	2,81	5,96	0,91486
	3,64	6,89	0,78540

Relations (4) give the recurrence procedure for computation of coordinates $(x_{n+1,m+1}; y_{n+1,m+1})$ if we know the coordinates $(x_{n+1,m}; y_{n+1,m})$, $(x_{n,m+1}; y_{n,m+1})$, $(x_{n,m}; y_{n,m})$ and the difference $\Delta\varphi$.

The accuracy of computation with the step $\Delta\varphi$ can be checked by comparing it with the computation using the half step.

If we know nodal point coordinates along two basic slip-lines α_0, β_0 which pass through the same point, the recurrence relations (4) give a numerical algorithm for computation of the slip-line field. Further physical conditions give the boundary slip-lines of the field.

This numerical algorithm was applied to the case when the basic slip-lines α_0, β_0 are circles. In the case of an axial section through the cylinder die with a cone, a program for computer MINSK 22 was prepared. This program, which is essentially formed by a numerical algorithm for the slip-line field, is used to compute the stress field and the forming force.

Figure 2 shows the slip-line field computed by this program. The step for computation was $\Delta\varphi = 0{,}043633$.

In the table on p. 87 the numerical values of nodal point coordinates and tangent angles for each third point accurate to three decimal bits are given. The boundary slip-line coordinates are accurate to two decimal bits.

References:

[1] *R. Hill:* Mathematical Theory of Plasticity. Oxford 1950.

[2] *W. Johnson, P. B. Mellor:* Plasticity for Mechanical Engineers. London 1962.

[3] *A. Mendelson:* Plasticity — Theory and Application. London 1968.

Souhrn

NUMERICKÝ ALGORITMUS PRO VÝPOČET POLE KLUZOVÝCH ČAR

VĚRA RADOCHOVÁ

V článku je odvozen numerický algoritmus pro výpočet uzlových bodů pole kluzových čar, známe-li uzlový bod a dvě kluzové čáry různých systémů, které jím procházejí. Tento algoritmus je snadno naprogramovatelný na počítač a bylo ho použito pro výpočet rozložení napětí v osovém řezu válcovou zápustkou s kuželovým přechodem.

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