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ELECTRIC NETWORK ANALYSIS BY THE GENERALIZED CUT-SET MATRIX METHOD

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1. INTRODUCTION

The nodal analysis method makes certain difficulties when applied to networks with mutual inductance branches. Further complications arise when the given network contains branches with ideal voltage sources whose both nodes are independent nodes of the network. A generalization of the nodal analysis being applicable to networks with magnetic couplings was described in [1]; a further generalization for networks containing ideal voltage sources in addition to ideal current sources was described in [2]. Another very effective method for the analysis of networks containing both kinds of sources is a modified nodal analysis called the cut-set method. In comparison with the nodal analysis this method has the advantage of greater simplicity and clearness and especially the necessary numerical calculations are much simple for certain networks. However, just as the nodal method, the basic version of the cut-set method too, fails to work in case of networks including the magnetic coupling. In this work we shall develop the articles [1] and [2] and generalize the cut-set

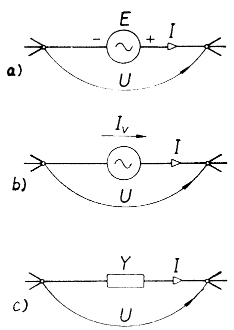

Fig. 1. Three kinds of branches which may occur in the network being analysed.

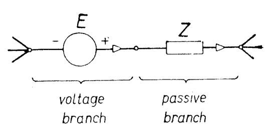

Fig. 2. Example: voltage source with internal impedance as a series connection of voltage and passive branch.

method so as to make it applicable to linear networks which contain ideal voltage sources together with ideal current sources and whose branches may be mutually magnetically coupled in any manner.

In the following treatment we shall divide the branches of the given network in such a way that each of them may be classified as one of the three kinds of branches, namely, the branches containing only ideal voltage sources (Fig. 1a) – they will be called "voltage branches", further those containing only ideal current sources (Fig. 1b) – we shall call them "current branches" and finally those containing only passive elements (Fig. 1c) which will be called "passive branches". For example, if a network contains a voltage source which is not ideal (i.e. which has a non-zero internal impedance), it must be considered as a voltage branch and a passive branch connected in series (Fig. 2).

The cut-set method was introduced into the theory of electric network by Guillemin [3] who followed the results from topology published by Whitney [4]. The possibilities of a matrix formulation of the cut-set method were disclosed by Seshu and Reed [5].

2. THE CUT-SET MATRIX ANALYSIS OF NETWORKS CONTAINING VOLTAGE SOURCES AND CURRENT SOURCES

Let us first formulate the cut-set matrix method for the case when the network includes ideal voltage and current sources but has no magnetic couplings between its branches.

Let the network considered have k nodes, s separate parts and l branches, consisting of voltage branches, q current branches and r passive branches ($l = p + q + r$). The number of tree branches will be denoted by $m\,(= k - s)$ and the number of link branches by $n\ (= l - m)$.

First we shall characterize the topological structure of the network. The graph of the network considered is oriented as follows: the voltage and current branches are oriented in agreement with the polarity of the voltage and current sources, respectively (according to Fig. 1a, b) and the passive branches are oriented arbitrarily. In the graph of the network we choose a tree containing all the voltage branches, no current branches and an arbitrary number of passive branches[1]). If we assign to every branch of this tree one "cut-set"[2]) incident to the respective tree branch, not

[1]) As we know (see e.g. [3]) there exists at least one such tree for any network having a physical sense.

[2]) Let us explain the term "cut-set". Into a connected network we introduce an arbitrary continuous simply closed surface (in the case of planar networks it will be sufficient to introduce any simply closed curve), incident at only one of its points to the network branches but to none of the network nodes. From the geometric viewpoint this surface is a topological sphere (or, for planar networks, a topological circle called the Jordan curve). Thus the introduction of a topo-

incident to the respective tree branch, not incident to another branch of the tree but possibly incident to any link branch[3]), we obtain a set of m linearly independent cut-sets which we shall call the "basic set of cut-sets". If all the tree branches are oriented in such a sense that they are directed to the outside of the spatial area closed by its corresponding cut-set, we obtain a cut-set called the "well oriented set of basic cut-sets" (see [7]).

When numbering the network branches we shall proceed in the following order:

- voltage tree branches
- passive tree branches
- passive link branches and
- current link branches.

When numbering the cut-sets we number first the cut-sets belonging to the voltage tree branches and then those belonging to the passive tree branches.

The oriented network graph numbered in this manner is algebraically represented by the third incidence matrix (or cut-set matrix) $\mathbf{H}$. This matrix, which describes the topological structure of the network, is of the type $(l;\, m)$ and expresses the incidence of the branches and the cut-sets of the oriented graph of the network considered. The rows of matrix $\mathbf{H}$ correspond to the graph branches and its columns to the cut-sets. Its elements are: $h_{ij} = +1$ if the oriented i-th branch is directed to the outside of the j-th cut-set, or $h_{ij} = -1$ if the orientation of the i-th branch is opposite, or $h_{ij} = 0$ if the i-th branch is not incident to the j-th cut-set. In the book [7] it was shown that for a well oriented basic set of cut-sets the third incidence matrix $\mathbf{H}$ may be expressed as a partitioned matrix consisting of a square submatrix $\mathbf{H}_I(m)$ and a rectangular submatrix $\mathbf{H}_{II}(n;\, m)$, the matrix $\mathbf{H}_I$ being a unit matrix:

(1)
$$\mathbf{H} = \underbrace{\begin{bmatrix} \mathbf{H}_I \\ \hline \mathbf{H}_{II} \end{bmatrix}}_{m} \begin{matrix} \}m \\ \}n \end{matrix} = \underbrace{\begin{bmatrix} \mathbf{J} \\ \hline \mathbf{H}_{II} \end{bmatrix}}_{m} \begin{matrix} \}m \\ \}n \end{matrix}$$

The physical structure of the network analysed is characterized as in the

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logical sphere into an electric network is uniquely determined by the branches to which this sphere is incident. A set of these branches is called the "cut-set".

From the above definition of the cut-set it follows that if in a given connected network we interrupt the branches of its cut-set the network will split into two separate parts. If any one of these branches is connected again the network will be restored to a connected one.

[3]) It is useful (but not necessary) to introduce the cut-sets in such a way that they be incident to the relevant branches just once. The reason is that the equations of the network will thus be formally simplified. If we assign to each tree branch just one cut-set, we can say that this cut-set represents a set of a minimum number of branches whose interruption causes the network to split into two separate parts.

method of node voltages (see [1], [7]), i.e. by the diagonal matrix of branch admittances

(2) $$\mathbf{Y}(l) = \text{diag}\,[Y_{11}, \ldots, Y_{ll}]$$

and by the column matrix of branch current sources $\mathbf{I}_v(l; 1)$, by the column matrix of branch voltages $\mathbf{U}(l; 1)$, by the column matrix of branch currents $\mathbf{I}(l; 1)$ and furthermore by the column matrix of branch voltage sources $\mathbf{E}(l; 1)$.

For the elements of the matrix of branch admittances, corresponding to the voltage branches, there evidently holds

$$\lim Y_{ii} = +\infty \quad \text{for} \quad i, \ldots, p$$

and for its elements corresponding to the current branches we have

$$\lim Y_{ii} = 0 \quad \text{for} \quad i = l - q + 1, \ldots, l$$

The elements of matrices $\mathbf{U}$ and $\mathbf{I}$ corresponding to voltage and current branches, are given by the known values of the sources

(3a) $$U_i = -E_i \quad \text{for} \quad i = 1, \ldots, p$$

and

(3b) $$I_i = I_{vi} \quad \text{for} \quad i = l - q + 1, \ldots, l$$

In further applications it will be advantageous if we express the matrices $\mathbf{I}_v(l; 1)$, $\mathbf{U}(l; 1)$, $\mathbf{I}(l; 1)$ and $\mathbf{E}(l; 1)$ as partitioned matrices:

(4)
$$\mathbf{I}_v = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I}_{vIII} \end{bmatrix} \begin{matrix} \}m \\ \}n-q \\ \}q \end{matrix}, \quad \mathbf{U} = \begin{bmatrix} \mathbf{U}_I \\ \mathbf{U}_{II} \\ \mathbf{U}_{III} \end{bmatrix} \begin{matrix} \}p \\ \}m-p \\ \}n \end{matrix}, \quad \mathbf{I} = \begin{bmatrix} \mathbf{I}_I \\ \mathbf{I}_{II} \\ \mathbf{I}_{III} \end{bmatrix} \begin{matrix} \}m \\ \}n-q \\ \}q \end{matrix}, \quad \mathbf{E} = \begin{bmatrix} \mathbf{E}_I \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \begin{matrix} \}p \\ \}m-p \\ \}n \end{matrix}$$

According to Eqs. (3) it is evident that

(5) $$\mathbf{U}_I = -\mathbf{E}_I \quad \text{and} \quad \mathbf{I}_{III} = \mathbf{I}_{vIII}$$

When analysing the network we know the matrices $\mathbf{H}$, $\mathbf{Y}$ and the submatrices $\mathbf{I}_{vIII}$ and $\mathbf{E}_I$ and search for matrices $\mathbf{U}$ and $\mathbf{I}$, more exactly, with regard to Eq. (5), only for their submatrices $\mathbf{U}_{II}$, $\mathbf{U}_{III}$, $\mathbf{I}_I$ and $\mathbf{I}_{II}$. It will apparently suffice if we find the submatrix $\mathbf{U}_{II}$ (whose elements are the voltages across the passive tree branches) or the submatrix $\mathbf{I}_{II}$ (whose elements are the currents in the passive link branches). From these submatrices it is then easy to find the remaining submatrices $\mathbf{U}_{III}$ and $\mathbf{I}_I$ (see e.g. [6], [7], [8]).

We start from the equation

$$\mathbf{I} = \mathbf{Y}\mathbf{U} + \mathbf{I}_v \tag{6}$$

which follows from Ohm's law and Kirchhoff's current law. Now we use the generalized Kirchhoff's current law according to which the sum of all the currents incident to any cut-set of the network is identically equal to zero [7]. In the matrix form it may be expressed by the equation

$$^t\mathbf{H}\mathbf{I} = \mathbf{0} \tag{7}$$

Multiplying Eq. (6) by the matrix $^t\mathbf{H}$ and using Eq. (7) we obtain the equation

$$^\sim\mathbf{I}_v = -{}^\sim\mathbf{Y}{}^\sim\mathbf{U} \tag{8}$$

This so-called "cut-set matrix equation" expresses the application of the generalized Kirchhoff's current law to the set of m cut-sets in the network considered. Matrices $^\sim\mathbf{I}_v(m;\,1)$ and $^\sim\mathbf{Y}(m)$ are determined by the relations

$$^\sim\mathbf{I}_v(m;\,1) = {}^t\mathbf{H}\mathbf{I}_v \tag{9}$$

and

$$^\sim\mathbf{Y}(m) = {}^t\mathbf{H}\mathbf{Y}\mathbf{H} \tag{10}$$

(the matrix $^\sim\mathbf{Y}$ is apparently a symmetric one) and the so-called "matrix of cut-set voltages" $^\sim\mathbf{U}(m;\,1)$ is related to matrix $\mathbf{U}$ by a simple relation

$$\mathbf{U} = \mathbf{H}{}^\sim\mathbf{U} \tag{11}$$

Since we have chosen a well oriented basic set of cut-sets, for which the third incidence matrix $\mathbf{H}$ assumes the form given by Eq. (1), we have, according to Eqs. (4) and (5):

$$^\sim\mathbf{U} = \begin{array}{|c|}\hline -\mathbf{E}_I \\ \hline \mathbf{U}_{II} \\ \hline\end{array} \begin{array}{l}\}\, p \\ \}\, m - p\end{array} \tag{12}$$

i.e. the cut-set voltages are the voltages across the corresponding tree branches.

If the network considered contains only current sources ($q \neq 0$) and no voltage sources ($p = 0$) its analysis is comparatively easy: by solving the cut-set equation (8) we determine the voltages across all the tree branches:

$$\mathbf{U}_{II} = -{}^\sim\mathbf{Y}^{-1}\,{}^\sim\mathbf{I}_v \tag{13}$$

Let us now investigate a more general case when the network contains voltage sources in addition to current sources ($p \neq 0$, $q \neq 0$). Again, we are interested in voltages across all the tree branches; it will apparently suffice to calculate the submatrix $\mathbf{U}_{II}$ from the cut-set equation (8). We divide the matrices ${}^{\sim}\mathbf{I}_v$ and ${}^{\sim}\mathbf{Y}$ into the corresponding submatrices

$$
{}^{\sim}\mathbf{I}_v = {}^{t}\mathbf{H}_{II} \begin{array}{|c|l}\hline \mathbf{0} & \}\, n-q \\ \hline \mathbf{I}_{vIII} & \}\, q \\ \hline \end{array} = \begin{array}{|c|l}\hline {}^{\sim}\mathbf{I}_{vI} & \}\, p \\ \hline {}^{\sim}\mathbf{I}_{vII} & \}\, m-p \\ \hline \end{array} \tag{14}
$$

and

$$
{}^{\sim}\mathbf{Y}(m) = \begin{array}{|c|c|l}\hline {}^{\sim}\mathbf{Y}_{11} & {}^{\sim}\mathbf{Y}_{12} & \}\, p \\ \hline {}^{\sim}\mathbf{Y}_{21} & {}^{\sim}\mathbf{Y}_{22} & \}\, m-p \\ \hline \multicolumn{1}{c}{\underbrace{\qquad}_{p}} & \multicolumn{1}{c}{\underbrace{\qquad}_{m-p}} & \end{array} \tag{15}
$$

After a simple rearrangement we obtain

$$
{}^{\sim}\mathbf{I}_{vII} = {}^{\sim}\mathbf{Y}_{21}\mathbf{E}_I - {}^{\sim}\mathbf{Y}_{22}\mathbf{U}_{II} \tag{16}
$$

which is a matrix representation of the generalized Kirchhoff's current law applied to $m - p$ linearly independent cut-sets. From this equation we obtain

$$
\mathbf{U}_{II} = -{}^{\sim}\mathbf{Y}_{22}^{-1}({}^{\sim}\mathbf{I}_{vII} - {}^{\sim}\mathbf{Y}_{21}\mathbf{E}_I) \tag{17}
$$

The element in the i-th row of the matrix ${}^{\sim}\mathbf{I}_{vII}(m - p;\, 1) = [{}^{\sim}\mathbf{I}_{vIIi}]$ is the sum of currents on the current branches, incident to the i-th linearly independent cut-set ($i = p + 1, \ldots, m$), the element in the i-th row of matrix $\mathbf{U}_{II}(m - p;\, 1) = [U_{IIi}]$ is represented by the voltage along the i-th passive branch ($i = p + 1, \ldots, m$) and finally the element in the i-th row of matrix $\mathbf{E}_I(p;\, 1) = [E_i]$ is the value of the source voltage of the i-th voltage branch ($i = 1, \ldots, p$).

Let us note that the elements $\mathbf{Y}_{ii}$ ($i = 1, \ldots, p$) given by Eqs. (2) cannot appear in the submatrices ${}^{\sim}\mathbf{Y}_{22}$ and ${}^{\sim}\mathbf{Y}_{12}$ (this follows from the knowledge concerning the values of elements of matrix ${}^{\sim}\mathbf{Y}$ described in [7]), so that the fact that the voltage sources considered are ideal (i.e. that $\lim Y_{ii} = +\infty$) makes no difficulties in the solution.

If we know the matrix ${}^{\sim}\mathbf{U}$, then using Eq. (11) we can easily determine the matrix $\mathbf{U}$ whose elements are the voltages across all the remaining branches of the network considered.

The currents in the link branches are given by the submatrices $\mathbf{I}_{II}$ and $\mathbf{I}_{III}$. With regard to Eq. (5) the submatrix $\mathbf{I}_{III}$ is known and the submatrix $\mathbf{I}_{II}$ is calculated from the relation

$$
\mathbf{I}_{II} = \operatorname{diag}[Y_{m+1,m+1}, \ldots, Y_{l-q,l-q}] \tag{18}
$$

If we know the currents in the link branches (they represent the elements of the submatrices $\mathbf{I}_{II}$ and $\mathbf{I}_{III}$) we can easily determine (see e.g. [6], [7], [8]) the currents in the tree branches (i.e. the submatrix $\mathbf{I}_I$).

Note. At the beginning of this chapter we have expressed the assumption that the network considered may contain both current sources and voltage sources, but later we used Eq. (6) which holds if the network includes current sources only. Now we shall show that this procedure does not lead to wrong results.

Let us admit that the network voltage sources are not ideal but have internal impedances Z_{ii} $(i = 1, \ldots, p)$. Then they can be replaced by equivalent current sources and Eq. (6) is justified. Instead of Eq. (3a) it holds

$$I_i = Z_{ii}I_i - E_i$$

If we put $\lim Z_{ii} = 0$, then Eq. (3a) holds as well as Eqs. (5) and (12) and the fact that this limit implies the validity of $\lim Y_{ii} = +\infty$ doesn't matter, as already stated.

Summarizing the results attained, the practical procedure in the cut-set network analysis may be formulated in the following six steps:

1. We orientate the network in the described manner, choose one of its trees containing all the voltage sources, number the network branches in a prescribed order and introduce a well oriented set of basic cut-sets.Then we form the matrix $\mathbf{H}$.

2. We construct the submatrices $\mathbf{E}_I$, $\mathbf{I}_{vIII}$ and $\mathbf{Y}$. Using Eq. (14) we calculate the submatrix $\tilde{\mathbf{I}}_{vII}$ and by aid of Eq. (10) we obtain the matrix $\tilde{\mathbf{Y}}$ which we divide into submatrices $\tilde{\mathbf{Y}}_{ij}$ $(i, j = 1, 2)$.

3. We calculate the inverse $\tilde{\mathbf{Y}}_{22}^{-1}$ of the submatrix $\tilde{\mathbf{Y}}_{22}$.

4. From Eq. (17) we calculate the matrix $\mathbf{U}_{II}$; then we form the matrix $\tilde{\mathbf{U}}$ according to Eq. (12).

5. From Eq. (11) we calculate the matrix $\mathbf{U}$.

6. Matrix $\mathbf{I}$ is determined as follows:

(a) From Eq. (18) we calculate the submatrix of currents in the passive link branches $\mathbf{I}_{II}$.

(b) We introduce a well oriented complete basic set of loops of the network [7]. We number the loops in the same order as the corresponding link branches. We set up the incidence matrix $\mathbf{C}'(m; n)$ whose rows and columns correspond to tree branches and independent loops, respectively.

(c) The submatrix of tree-branch currents $\mathbf{I}_I(m; 1)$ is then determined according to the relation

$$\mathbf{I}_I(m; 1) = \mathbf{C}' \begin{array}{|c|} \hline \mathbf{I}_{II} \\ \hline \mathbf{I}_{III} \\ \hline \end{array}$$

To illustrate the method just described we trace the execution of the analysis of two networks.

I. Let us solve the network of Fig. 3. It was already oriented, a tree containing a voltage branch was chosen (in Fig. 3 this tree is indicated by heavy lines) and a set

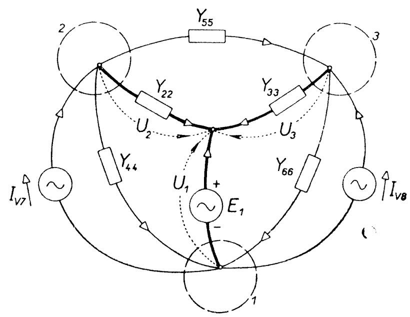

Fig. 3. Electric network without magnetic couplings, analysed by the cut-set method ($l = 8$, $m = 3$, $n = 5$, $p = 1$, $q = 2$, $r = 5$).

of well oriented basic cut-sets was introduced. The topological structure of the network is characterized by the third incidence matrix

$$\mathbf{H} = \begin{array}{c|c|c|c|} & 1 & 2 & 3 \\ \hline 1 & 1 & & \\ \hline 2 & & 1 & \\ \hline 3 & & & 1 \\ \hline\hline 4 & -1 & 1 & \\ \hline 5 & & 1 & -1 \\ \hline 6 & -1 & & 1 \\ \hline 7 & 1 & -1 & \\ \hline 8 & 1 & & -1 \\ \hline \end{array}$$

Its physical structure is characterized by the matrices:

$$\mathbf{Y} = \operatorname{diag}\left[Y_{11}, \ldots, Y_{66}, 0\right] \tag{20}$$

where

$$\mathbf{I}_{vIII} = \begin{array}{|c|}\hline I_{v7} \\ \hline I_{v8} \\ \hline\end{array} \tag{21}$$

and

$$\mathbf{E}_I = \begin{array}{|c|}\hline E_1 \\ \hline\end{array} \tag{22}$$

We perform a transformation according to Eqs. (9) and (10) and divide the calculated matrices into submatrices according to Eqs. (14) and (15) and obtain

$$\tilde{\mathbf{I}}_{vII} = \begin{array}{|c|}\hline -I_{v7} \\ \hline -I_{v8} \\ \hline\end{array} \tag{23}$$

$$\tilde{\mathbf{Y}}_{22} = \begin{array}{|c|c|}\hline Y_{22} + Y_{44} + Y_{55} & -Y_{55} \\ \hline -Y_{55} & Y_{33} + Y_{55} + Y_{66} \\ \hline\end{array} \qquad \tilde{\mathbf{Y}}_{21} = \begin{array}{|c|}\hline -Y_{44} \\ \hline -Y_{66} \\ \hline\end{array} \tag{24}$$

Substituting into Eq. (17) we find

$$\mathbf{U}_{II} = \begin{array}{|c|}\hline U_2 \\ \hline U_3 \\ \hline\end{array} \tag{25}$$

The matrix

$$\mathbf{U}_I = -\mathbf{E}_I = \begin{array}{|c|}\hline -E_1 \\ \hline\end{array}$$

is known. Using Eqs. (4), (11) and (12) we find the voltages across the remaining branches of the network:

$$\mathbf{U}_{III} = \mathbf{H}_{II} \begin{array}{|c|}\hline -\mathbf{E}_I \\ \hline \mathbf{U}_{II} \\ \hline\end{array} \tag{26}$$

The given network is very simple, so that each of the three cut-sets introduced separates from it the part containing only one node. Thus the cut-set method actually turns into the node voltage method. The solution of a more complex network will be shown in the following example.

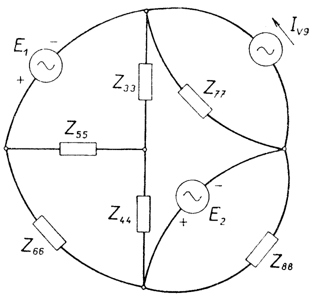

Fig. 4. Electric network without magnetic couplings, analysed by the cut-set method ($l = 9$, $m = 4$, $n = 5$, $p = 2$, $q = 1$, $r = 6$).

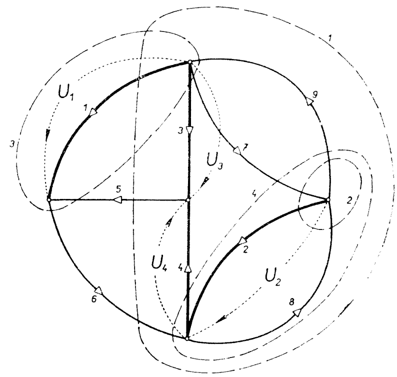

Fig. 5. Oriented graph of the network shown in Fig. 4 into which a well oriented set of basic cut-sets has been introduced.

II. Let us consider the network of Fig. 4. Fig. 5 shows its oriented graph with the indicated tree (by heavy lines) and with the introduced set of well oriented basic cut-sets. The third incidence matrix is

$$
\mathbf{H} = \begin{array}{c|c|c|c|c|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & & & \\
2 & & 1 & & \\
3 & & & 1 & \\
4 & & & & 1 \\
\hline\hline
5 & 1 & & -1 & \\
6 & -1 & & 1 & -1 \\
7 & & -1 & 1 & -1 \\
8 & & -1 & & \\
9 & & 1 & -1 & 1 \\
\hline
\end{array}
= \begin{array}{|c|}
\hline
\mathbf{J}(m) \\
\hline
\mathbf{H}_{II}(n; m) \\
\hline
\end{array} . \tag{27}
$$

Now we write the matrices

$$\mathbf{Y}(9) = \operatorname{diag}\left[Y_{11}, Y_{22}, \ldots, Y_{88}, 0\right] \tag{28}$$

where $Y_{11} = Y_{22} \rightarrow +\infty$

$$\mathbf{I}_{vIII}(q; 1) = \begin{array}{|c|}\hline I_{v9} \\ \hline\end{array} \quad \text{and} \quad \mathbf{E}_{I}(p; 1) = \begin{array}{|c|}\hline E_1 \\ \hline E_2 \\ \hline\end{array} \tag{29}$$

perform their transformation and divide them into submatrices. We obtain

$$^{\sim}\mathbf{I}_{vI}(p; 1) = \begin{array}{|c|}\hline 0 \\ \hline I_{v9} \\ \hline\end{array} \qquad ^{\sim}\mathbf{I}_{vII}(m - p; 1) = \begin{array}{|c|}\hline -I_{v9} \\ \hline I_{v9} \\ \hline\end{array} \tag{30}$$

$$^{\sim}\mathbf{Y}_{22}(m - p) = \begin{array}{|c|c|}\hline Y_{33} + Y_{55} + \\ + Y_{66} + Y_{77} & -Y_{66} - Y_{77} \\ \hline -Y_{66} - Y_{77} & Y_{44} + Y_{66} + \\ + Y_{77} \\ \hline\end{array}$$

$$^{\sim}\mathbf{Y}_{21}(m - p; p) = \begin{array}{|c|c|}\hline -Y_{55} - Y_{66} & -Y_{77} \\ \hline Y_{66} & Y_{77} \\ \hline\end{array}$$

By substituting these matrices into Eq. (17) we calculate $\mathbf{U}_{II}$. The matrix of the cut-set voltages (i.e. the matrix of the voltages across the tree branches) is then

$$^{\sim}\mathbf{U}(m; 1) = \begin{array}{|c|}\hline -\mathbf{E}_I \\ \hline \mathbf{U}_{II} \\ \hline\end{array} = \begin{array}{|c|}\hline -E_1 \\ \hline -E_2 \\ \hline U_3 \\ \hline U_4 \\ \hline\end{array} \tag{31}$$

From the equation $^{\sim}\mathbf{U}_{III}(n; 1) = \mathbf{H}_{II}{}^{\sim}\mathbf{U}$ we then determine the voltages of the link branches.

The branch voltage matrix of the given network is

$$\mathbf{U}(l; 1) = \begin{array}{|c|} \hline ^{\sim}\mathbf{U} \\ \hline \mathbf{U}_{III} \\ \hline \end{array} \tag{32}$$

4. GENERALIZATION OF THE CUT-SET MATRIX METHOD TO THE ANALYSIS OF NETWORKS WITH MAGNETIC COUPLINGS

If the network to be analysed contains magnetic couplings the method described above cannot be applied directly. The reasons are the same as those concerning the nodal method [1]. We shall show how the cut-set matrix method may be generalized to be applicable to networks with mutual inductances. Thus we arrive at a method having an entirely general validity in the field of linear networks.

The method is based on the validity of the superposition theorem. According to this theorem we can split the network solved into two partial networks, formulate for each of them the basic equations and then pass from these equations to the equation of the given network. By finding the solution of this equation we have substantially brought the analysis to its end.

First we shall describe in detail how to establish the two partial networks.

The first partial network is obtained by omitting all the branches containing inductances with magnetic couplings. However, formally we shall continue to take account of these omitted branches, their admittances and magnetic couplings being, of course, equal to zero. The second partial network is constituted by the removed branches of the network (i.e. those containing magnetically coupled inductances); the remaining branches of the network (i.e. those without magnetically coupled inductances) will be formally considered, their impedances will, of course, be infinitely large.

Now we shall proceed to the mathematical application of the suggested ideas. Let the network contain several groups of magnetically coupled branches. We select a tree in such a way that all the voltage branches be in the tree branches and all the current branches in the link branches. Then we number the branches of the network. The numbering may be arbitrary but in order to maintain reference to the third chapter of this article and to facilitate numerical calculations we shall accept the convention adopted in the third chapter completing it by the order of numbering the branches with mutual inductances. The branches are numbered in the following sequence:

- voltage tree branches (their number is p)
- passive tree branches without magnetic couplings (their number is r_1)
- passive tree branches with magnetic couplings (their number is r_2)
- passive link branches with magnetic couplings (their number is r_3)
- passive link branches without magnetic couplings (their number is r_4) and
- current link branches (their number is q).

Now we split the network into the first and second partial networks. The quantities belonging to the first partial network are denoted by the index $^{(1)}$ and those of the second partial network by the index $^{(2)}$.

If we apply the cut-set method to the first partial network then, with regard to Eq. (16), the cut-set matrix equation holds in the form

$$\tilde{\mathbf{I}}_{vII}^{(1)} - \tilde{\mathbf{Y}}_{21}^{(1)}\mathbf{E}^{(1)} + \tilde{\mathbf{Y}}_{22}^{(1)}\mathbf{U}_{II}^{(1)} = \tilde{\mathbf{I}}^{(1)} \tag{33}$$

This equation is a matrix representation of the generalized Kirchhoff's current law for $m - p$ independent cut-sets belonging to the passive tree branches. On the left hand side of Eq. (33) we obtain (after multiplication) column matrices whose elements in the i-th rows are the sums of the currents in the current branches, the currents in the voltage branches and the currents in the passive branches, incident to the i-th cut-set ($i = p + 1, \ldots, m$). The submatrices $\tilde{\mathbf{Y}}_{21}^{(1)}$ and $\tilde{\mathbf{Y}}_{22}^{(1)}$ are determined from matrix $\tilde{\mathbf{Y}}^{(1)}$ obtained by transforming the matrix of branch admittances

$$\mathbf{Y}^{(1)}(l) = \operatorname{diag}\,[Y_{11}, \ldots, Y_{p+r_1,p+r_1}, \underbrace{0, \ldots, 0}_{r_2+r_3}, Y_{l-q-r_4,l-q-r_4}, \ldots, Y_{l-q,l-q}, \underbrace{0, \ldots, 0}_{q}] \tag{34}$$

where $Y_{ii} \to +\infty$ ($i = 1, \ldots, p$). With regard to Eq. (10) the matrix $\tilde{\mathbf{Y}}^{(1)}$ is determined from the transformation relation

$$\tilde{\mathbf{Y}}^{(1)} = {}^t\mathbf{H}\mathbf{Y}^{(1)}\mathbf{H} \tag{35}$$

For the matrix of branch voltages of the second partial network $\mathbf{U}^{(2)}(l)$, the equation

$$\mathbf{U}^{(2)} = \mathbf{Z}^{(2)}\mathbf{I}^{(2)} \tag{36}$$

holds where $\mathbf{Z}^{(2)}(l)$ denotes the symmetric matrix of branch impedances and $\mathbf{I}^{(2)}(l; 1)$ the matrix of branch currents. With regard to the adopted method of numbering branches, matrix $\mathbf{Z}^{(2)}$ may be expressed as a quasidiagonal one in the form

$$\mathbf{Z}^{(2)}(l) = \operatorname{diag}\,[\mathbf{Z}_{11}(p + r_1), \mathbf{Z}_{22}(r_2 + r_3), \mathbf{Z}_{33}(r_4 + q)] \tag{37}$$

where $\mathbf{Z}_{11}$ and $\mathbf{Z}_{33}$ are diagonal submatrices and their elements are the values of branch impedances of the first partial network; these impedances assume infinitely large values. The submatrix $\mathbf{Z}_{22}$ is symmetrical and its elements Z_{ij} are the impedance values of the self-inductances (for $i = j$) and the mutual inductances (for $i \neq j$) of the second partial network.

From Eq. (36) we express

$$\mathbf{I}^{(2)} = \mathbf{Y}^{(2)}\mathbf{U}^{(2)} \tag{38}$$

where $\mathbf{Y}^{(2)} = \mathbf{Z}^{(2)-1}$ is the matrix of branch admittances. With regard to Eq. (37) matrix $\mathbf{Y}^{(2)}$ must be quasidiagonal too:

$$\mathbf{Y}^{(2)}(l) = \operatorname{diag}\left[\mathbf{Y}_{11}(p + r_1),\ \mathbf{Y}_{22}(r_2 + r_3),\ \mathbf{Y}_{33}(r_4 + q)\right] \tag{39}$$

The submatrices $\mathbf{Y}_{11} = \mathbf{Z}_{11}^{-1}$ and $\mathbf{Y}_{33} = \mathbf{Z}_{33}^{-1}$ are both equal to zero – this can be proved as in the paper [1] – and the matrix $\mathbf{Y}_{22} = \mathbf{Z}_{22}^{-1}$ is symmetrical. Therefore

$$\mathbf{Y}^{(2)} = \operatorname{diag}\left[\mathbf{0}(p + r_1),\ \mathbf{Z}_{22}^{-1}(r_2 + r_3),\ \mathbf{0}(r_4 + q)\right] \tag{40}$$

For easier numerical inversion of the symmetric submatrix $\mathbf{Z}_{22}$ (i.e. the calculation of $\mathbf{Y}_{22} = \mathbf{Z}_{22}^{-1}$), that is to say, for an easier calculation of the matrix $\mathbf{Y}^{(2)}$, it is of advantage if the non-zero elements of matrix $\mathbf{Z}_{22}$ are as close to the principal diagonal as possible. (Matrix $\mathbf{Z}_{22}$ can then be expressed, by dividing it into further submatrices, as a quasidiagonal matrix; the inversion of this matrix is relatively easy.) This form of the matrix $\mathbf{Z}_{22}$ can be attained if, when numbering the branches, we indicate the branches of each magnetically coupled branch group by successive numbers. By this we achieve that the order of the diagonal submatrices of the quasidiagonal matrix $\mathbf{Z}_{22}$ will be minimum; it is equal to the number of branches of the releveant group. If the network considered includes only groups of magnetically coupled branch pairs, then this technique enables us to achieve that all the diagonal submatrices of the matrix $\mathbf{Z}_{22}$ are only of the second order and thus their inversion is very easy.

By using Eqs. (9), (38) and (11) we obtain the cut-set matrix equation expressing the application of Kirchhoff's current law to m independent cut-sets of the second partial network:

$$\tilde{\mathbf{I}}^{(2)} = {}^t\mathbf{H}\mathbf{I}^{(2)} = {}^t\mathbf{H}\mathbf{Y}^{(2)}\mathbf{U}^{(2)} = {}^t\mathbf{H}\mathbf{Y}^{(2)}\mathbf{H}\,\tilde{\mathbf{U}}^{(2)} \tag{41}$$

Denoting

$$\tilde{\mathbf{Y}}^{(2)} = {}^t\mathbf{H}\mathbf{Y}^{(2)}\mathbf{H} \tag{42}$$

we obtain the cut-set matrix equation for the second partial network:

$$\tilde{\mathbf{I}}^{(2)} = \tilde{\mathbf{Y}}^{(2)}\,\tilde{\mathbf{U}}^{(2)} \tag{43}$$

The matrix $\tilde{\mathbf{I}}^{(2)}$ is a column matrix and the value of the element located in its i-th row is given by the currents in the branches with magnetic couplings, incident to the

i-th cut-set ($i = 1, \dots, m$). We are, however, not interested in all of the m independent cut-sets, but only in $m - p$ of them belonging to the passive tree branches. Therefore, we express Eq. (43) in terms of partitioned matrices:

$$(44) \qquad {}^{\sim}\mathbf{I}^{(2)} = \left[\begin{array}{|c|} \hline {}^{\sim}\mathbf{I}_I^{(2)} \\ \hline {}^{\sim}\mathbf{I}_{II}^{(2)} \\ \hline \end{array}\right] \begin{array}{l} \}\,p \\ \}\,m-p \end{array} = \left[\begin{array}{|c|c|} \hline {}^{\sim}\mathbf{Y}_{11}^{(2)} & {}^{\sim}\mathbf{Y}_{12}^{(2)} \\ \hline {}^{\sim}\mathbf{Y}_{21}^{(2)} & {}^{\sim}\mathbf{Y}_{22}^{(2)} \\ \hline \end{array}\right] \begin{array}{l} \}\,p \\ \}\,m-p \end{array} \cdot \left[\begin{array}{|c|} \hline -\mathbf{E}_I^{(2)} \\ \hline {}^{\sim}\mathbf{U}_{II}^{(2)} \\ \hline \end{array}\right] \begin{array}{l} \}\,p \\ \}\,m-p \end{array}$$

The voltage branches of the second partial network belong to the removed branches and we consider them only formally. Nevertheless we must take into account the voltages of these branches (the nodes of these branches may in fact be identical with those of the actual branches of the second partial network – and then we have to deal with the voltages between these nodes); let these voltages be the elements of a matrix denoted by $-\mathbf{E}_I^{(2)}$. Thus, for $m - p$ independent cut-sets of the second partial network the cut-set equation

$$(45) \qquad -{}^{\sim}\mathbf{Y}_{21}^{(2)}\mathbf{E}_I^{(2)} + {}^{\sim}\mathbf{Y}_{22}^{(2)}\,{}^{\sim}\mathbf{U}_{II}^{(2)} = {}^{\sim}\mathbf{I}_{II}^{(2)}$$

holds. Now we pass from the two partial networks to the given network. Let the voltages of the voltage and passive tree branches be equal in both partial networks. Then it holds:

$$(46) \qquad \mathbf{E}_I^{(1)} = \mathbf{E}_I^{(2)} = \mathbf{E}_I \quad \text{and} \quad \mathbf{U}_{II}^{(1)} = \mathbf{U}_{II}^{(2)} = \mathbf{U}_{II}$$

Therefore, we can interconnect the nodes of all mutually corresponding tree branches of both partial networks. Mathematically, this interconnection corresponds to summing the cut-set equations (33) and (45):

Using Eq. (46) we obtain

$${}^{\sim}\mathbf{I}^{(1)} + {}^{\sim}\mathbf{I}^{(2)} = \mathbf{0}$$

$$(47) \qquad {}^{\sim}\mathbf{I}_{vII}^{(1)} - {}^{\sim}\mathbf{Y}_{21}^{(1)}\mathbf{E}_I + {}^{\sim}\mathbf{Y}_{22}^{(1)}\mathbf{U}_{II} - {}^{\sim}\mathbf{Y}_{21}^{(2)} + \mathbf{E}_I + {}^{\sim}\mathbf{Y}_{22}^{(2)}\mathbf{U}_{II} = \mathbf{0}$$

Thus we have arrived at the cut-set equation of the given network. On its left hand side there are (after multiplication) column matrices whose elements located in the i-th row are the currents in the current branches, the currents in the voltage branches, the currents in the passive branches without magnetic couplings and finally the currents in the passive branches with magnetic couplings incident to the i-th cut-set ($i = p + 1, \dots, m$). Rearranging and solving this cut-set equation we obtain the passive-branch voltage matrix of the given network:

$$(48) \qquad \mathbf{U}_{II} = ({}^{\sim}\mathbf{Y}_{22}^{(1)} + {}^{\sim}\mathbf{Y}_{22}^{(2)})^{-1} \cdot [-{}^{\sim}\mathbf{I}_{vII}^{(1)} + ({}^{\sim}\mathbf{Y}_{21}^{(1)} + {}^{\sim}\mathbf{Y}_{21}^{(2)})\,\mathbf{E}_I]$$

Now it will be easy to determine the voltages across the remaining branches of the network: since we know the voltages of the voltage branches, which are elements of the matrix $-\mathbf{E}_r$, we also know, by Eq. (12), the matrix $^{\sim}\mathbf{U}$ and so we determine from Eq. (11) the branch voltage matrix $\mathbf{U}$ of the given network.

The currents in the link branches and then also the currents in the tree branches are determined in the same manner as for a network without magnetic couplings as described in the conclusion of Chapter 2.

The procedure for the analysis of a network with magnetic couplings by the generalized cut-set method may again be divided into six steps. In comparison with the procedure shown at the end of the preceding chapter for networks without magnetic couplings the individual steps will be changed or supplemented as follows:

1. In numbering the branches we obey a somewhat modified rule supplemented by an instruction for numbering branches with magnetic couplings. We see to it that successive numbers be assigned to every magnetically coupled branch group. (Thus we guarantee that the non-zero elements of matrix $\mathbf{Z}^{(2)}$ will be as close to the principal diagonal as possible.)

2. (a) We split the given network into two partial networks. We set up a diagonal matrix $\mathbf{Y}^{(1)}$ for the first partial network and a quasidiagonal matrix $\mathbf{Z}^{(2)}$ for the second partial network and perform the inversion $\mathbf{Z}^{(2)-1} = \mathbf{Y}^{(2)}$.

(b) Using Eqs. (35) and (42) we calculate the matrices $^{\sim}\mathbf{Y}^{(1)}$ and $^{\sim}\mathbf{Y}^{(2)}$ and divide them into the submatrices $^{\sim}\mathbf{Y}^{(1)}_{21}$, $^{\sim}\mathbf{Y}^{(1)}_{22}$, $^{\sim}\mathbf{Y}^{(2)}_{21}$, $\mathbf{Y}^{(2)}_{22}$.

3. We perform the inversion $(^{\sim}\mathbf{Y}^{(1)}_{22} + {}^{\sim}\mathbf{Y}^{(2)}_{22})^{-1}$.

4.–6. These three steps are the same as those for the networks without magnetic couplings except that the submatrix is calculated according to Eq. (48).

5. EXAMPLES OF AN APPLICATION OF THE GENERALIZED CUT-SET MATRIX METHOD

To illustrate the generalized cut-set matrix method we shall describe the procedure to be followed in the analysis of two networks differing from those solved in Chap. 3 in that there are inductive couplings between paris of some of their branches.

I. Let us consider the network of Fig. 6. The orientation of the network, the choice of the tree and the introduction of a well oriented basic set of cut-sets is the same as in the preceding example so that the third incidence matrix $\mathbf{H}$ too, is expressed by Eq. (19).

The network considered splits into two partial networks shown in Figs. 7 and 8.

For the first partial network (Fig. 7) we find the branch admittance matrices by using Eq. (34):

$$\mathbf{Y}^{(1)} = \operatorname{diag}\left[Y_{11}, Y_{12}, 0, 0, Y_{55}, Y_{66}, 0, 0\right] \tag{49}$$

where $Y_{11} \to +\infty$. The matrix $\mathbf{I}_{vIII}$ and the transformed matrix $^{\sim}\mathbf{I}_{vII}$ are the same as in the preceding example – see Eqs. (21) and (23) and the matrix $\mathbf{E}_I^{(1)}$ is analogous to Eq. (22):

(50)
$$\mathbf{E}_I^{(1)} = \begin{array}{|c|} \hline E_1 \\ \hline \end{array}$$

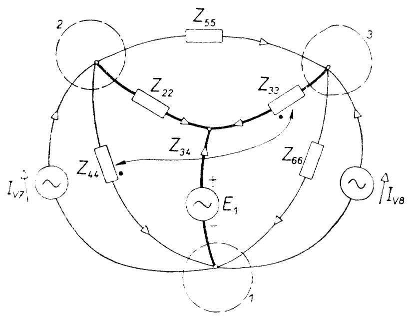

Fig. 6. Electric network with magnetic couplings analysed by the cut-set method ($l = 8$, $m = 3$, $n = 5$, $p = 1$, $r_1 = 1$, $r_2 = 1$, $r_3 = 1$, $r_4 = 2$).

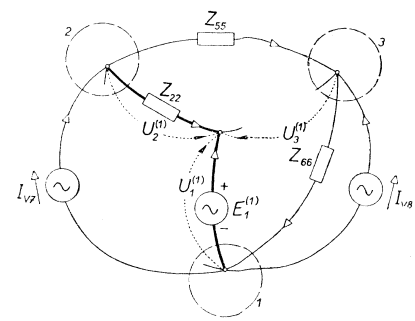

Fig. 7. First partial network of the network shown in Fig. 6.

We transform the matrix $\mathbf{Y}^{(1)}$ of Eq. (49) using the relation (10), and after dividing it into submatrices we find

(51)
$$^{\sim}\mathbf{Y}_{22}^{(1)} = \begin{array}{|c|c|} \hline Y_{22} + Y_{55} & -Y_{55} \\ \hline -Y_{55} & Y_{55} + Y_{66} \\ \hline \end{array} \qquad ^{\sim}\mathbf{Y}_{22}^{(1)} = \begin{array}{|c|} \hline 0 \\ \hline -Y_{66} \\ \hline \end{array}$$

For the second partial network (Fig. 8) we find the branch admittance matrix by using Eq. (40):

(52)
$$\mathbf{Y}^{(2)} = \operatorname{diag}\left[\mathbf{0}(2), \mathbf{Y}_{22}(2), \mathbf{0}(4)\right]$$

where

(53)
$$\mathbf{Y}_{22}^{(2)} = \begin{array}{|c|c|} \hline Y_{33} & Y_{34} \\ \hline Y_{43} & Y_{44} \\ \hline \end{array} = \mathbf{Z}_{22}^{(2)-1} = \frac{1}{Z_{33}Z_{44} - Z_{34}^2} \begin{array}{|c|c|} \hline Z_{44} & -Z_{34} \\ \hline -Z_{43} & Z_{33} \\ \hline \end{array}$$

We transform the matrix $\mathbf{Y}^{(2)}$ of Eq. (52) using the relation (10) and after dividing it into submatrices we find

$$(54) \qquad {}^{\sim}\mathbf{Y}_{22}^{(2)} = \begin{array}{|c|c|} \hline Y_{44} & Y_{43} \\ \hline Y_{34} & Y_{33} \\ \hline \end{array} \qquad {}^{\sim}\mathbf{Y}_{21}^{(2)} = \begin{array}{|c|} \hline -Y_{44} \\ \hline -Y_{34} \\ \hline \end{array}$$

We substitute the matrices expressed by Eqs. (23), (50) (where we put $\mathbf{E}_I^{(1)} = \mathbf{E}_I$), (51) and (54) into Eq. (48) and thus calculate the matrix of voltages across the passive branches $\mathbf{U}_{II}$ of the network considered.

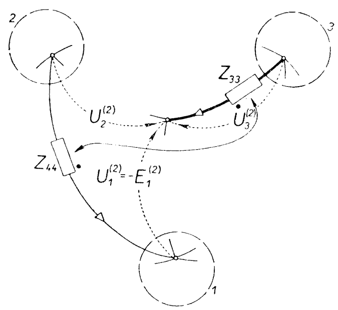

Fig. 8. Second partial network of the network shown in Fig. 6.

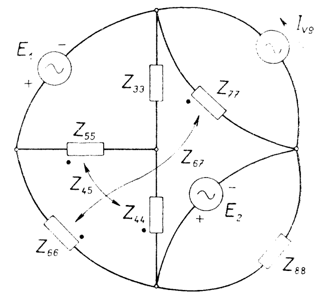

Fig. 9. Electric network with magnetic couplings analysed by the cut-set method ($l = 9$, $m = 4$, $n = 5$, $p = 2$, $r_1 = 1$, $r_2 = 1$, $r_3 = 3$, $r_4 = 1$, $q = 1$).

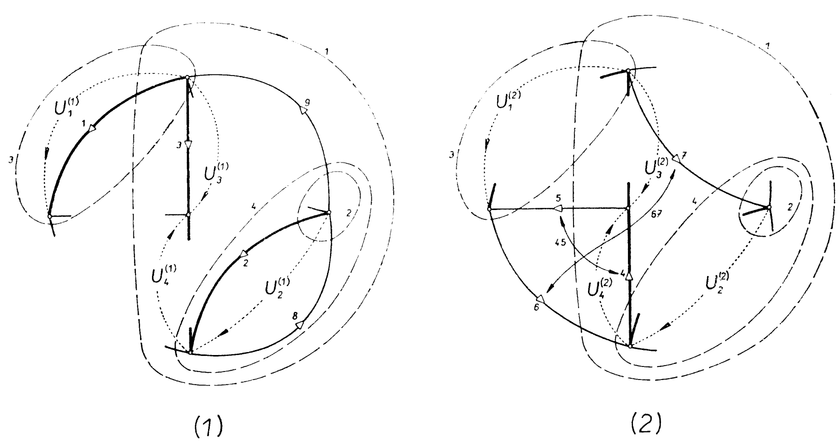

Fig. 10. Oriented graphs of both partial networks belonging to the network of Fig. 9; into each graph a well oriented set of basic cut-sets has been introduced.

II. Let us consider the network of Fig. 9. Its third incidence matrix $\mathbf{H}$ is given by the equation (27). We divide the network into two partial networks whose oriented graphs with indicated tree and with introduced complete set of well oriented basic cut-sets are shown in Fig. 10.

For the first partial network we have

$$\mathbf{Y}^{(1)} = \operatorname{diag}\left[Y_{11}, Y_{22}, Y_{33}, 0, 0, 0, 0, Y_{88}, 0\right] \tag{55}$$

where $Y_{11} \to +\infty$, $Y_{22} \to +\infty$. In this case the submatrices $\mathbf{I}^{(1)}_{vIII}$, $\mathbf{E}^{(1)}_{I}$, ${}^{\sim}\mathbf{I}^{(1)}_{v}$, ${}^{\sim}\mathbf{I}^{(1)}_{vII}$. $\mathbf{U}^{(1)}_{II}$, ${}^{\sim}\mathbf{U}^{(1)}$, $\mathbf{U}^{(1)}$, ${}^{\sim}\mathbf{Y}^{(1)}_{22}$ and ${}^{\sim}\mathbf{Y}^{(1)}_{21}$ are the same as those found in example II of Chap, 3 if we substitute into them $Y_{44} = Y_{55} = Y_{66} = Y_{77} = 0$.

For the second partial network we have

$$\mathbf{Y}^{(2)} = \operatorname{diag}\left[0, 0, 0, \mathbf{Y}^{(2)}_{22}, 0, 0\right] \tag{56}$$

where

$$\mathbf{Y}^{(2)}_{22} = \mathbf{Z}^{(2)-1}_{22} = \begin{bmatrix} \mathbf{Z}'^{-1}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}''^{-1}_{22} \end{bmatrix}$$

and

$$\mathbf{Z}'^{-1}_{22} = \frac{1}{Z_{44}Z_{55} - Z_{45}^2} \begin{bmatrix} Z_{55} & -Z_{45} \\ -Z_{45} & Z_{44} \end{bmatrix} \qquad \mathbf{Z}''^{-1}_{22} = \frac{1}{Z_{66}Z_{77} - Z_{67}^2} \begin{bmatrix} Z_{77} & -Z_{67} \\ -Z_{67} & Z_{66} \end{bmatrix}$$

Using Eq. (42) we calculate the matrix ${}^{\sim}\mathbf{Y}^{(2)}$ and by its division into submatrices we find ${}^{\sim}\mathbf{Y}^{(2)}_{22}$ and ${}^{\sim}\mathbf{Y}^{(2)}_{21}$. It holds further:

$$\mathbf{E}^{(2)}_{I} = \mathbf{E}^{(1)}_{I} = \mathbf{E}_{I} \tag{57}$$

We substitute these values into Eq. (48) and calculate from it the matrix $\mathbf{U}_{II}$. With regard to Eq. (12) we therefore know the matrix ${}^{\sim}\mathbf{U}$:

$${}^{\sim}\mathbf{U} = \begin{bmatrix} -\mathbf{E}_{I} \\ \mathbf{U}_{II} \end{bmatrix}$$

From Eq. (11) we then calculate the matrix $\mathbf{U}$.

6. CONCLUSION

The submitted article formulates a generalized cut-set method. The technique described is applicable to the analysis of linear networks which may contain ideal current sources together with ideal voltage sources and may also include magnetic couplings between their branches. In comparison with other methods of network analysis the generalized cut-set method is very advantageous. In numerical calculations connected with the analysis of a network in steady state we always meet the requirement to perform a matrix inversion; this is the most difficult computing operation whose complexity grows rapidly with the order of the matrix being inverted. Therefore the order of the matrix to be inverted may be considered as a rough criterion of the usefulness of the whole method. From this viewpoint it is therefore an important indication that this method requires to invert a matrix of the $(m - p)$-th order and if the given network includes magnetic couplings one must invert a further matrix of the $(r_2 + r_3)$-th order; for a simpler arrangement of the magnetic couplings this matrix may be quasidiagonal which greatly facilitates its inversion.

The formulation of the generalized cut-set method has been worked out in such a manner that it may be directly used as a basis of an algorithm suitable for writing a program for analysing the network on a digital computer.

In this article we have confined ourselves to the formulation of a generalized cut-set method for the analysis of networks in steady state. However, it is apparent that this method may be used as a basis for the formulation of equations describing transient responses of networks.

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Souhrn

ANALÝZA ELEKTRICKÝCH OBVODŮ ZOBECNĚNOU MATICOVOU METODOU ŘEZŮ

DANIEL MAYER

V práci je zobecněna metoda řezů[1]) tak, aby ji bylo možno aplikovat na analýzu lineárních obvodů, jež obsahují ideální zdroje napětí a současně ideální zdroje proudu, přičemž mezi větvemi analyzovaného obvodu mohou být induktivní vazby. Metoda řezů vychází ze zobecněného prvního Kirchhoffova zákona. Při její formulaci se v některých podrobnostech citují zejména výsledky prací [1] a [2], v nichž autor provedl obdobné zobecnění metody uzlových napětí.

Nejprve je provedeno zobecnění metody řezů pro případ, že analyzovaný obvod obsahuje oba typy zdrojů, ale mezi jeho větvemi nejsou induktivní vazby. Postupuje se tak, že se zvolí takový úplný strom, který obsahuje všechny napěťové větve, libovolné pasivní větve a žádné proudové větve řešeného obvodu. Vhodnou orientací a očíslováním větví obvodu docílíme, aby třetí incidenční matice $\mathbf{H}$, jež popisuje topologickou strukturu obvodu, měla tvar vyjádřený rovnicí (1). Matice, které vystupují v základních rovnicích obvodu vyjádříme jako matice rozdělené. Matici větvových napětí $\mathbf{U}$ určíme takto: podle rov. (17) vypočítáme její submatici $\mathbf{U}_{II}$, jejíž prvky jsou napětími na pasivních větvích stromu. Submatici $\mathbf{U}_I$, jejíž prvky jsou napětí ideálních napěťových zdrojů, vzhledem k rov. (5) známe a submatici $\mathbf{U}_{III}$, jejíž prvky jsou napětí na nezávislých větvích, pak již snadno určíme podle rovnic (11) a (12).

Matici větvových proudů $\mathbf{I}$ určíme takto: Submatici $\mathbf{I}_{II}$, jejíž prvky vyjadřují proudy v pasivních nezávislých větvích, vypočítáme z rov. (18). Prvky submatice $\mathbf{I}_{III}$ jsou hodnoty ideálních proudových zdrojů, tedy tato submatice je dána. Ze submatic $\mathbf{I}_{II}$ a $\mathbf{I}_{III}$ pak již snadno nalezneme submatici $\mathbf{I}_I$, jejímiž prvky jsou proudy ve větvích stromu.

Zobecnění popisované metody na obvody se vzájemnými indukčnostmi je založeno na principu superpozice. Analyzovaný obvod se rozloží na dva dílčí obvody. Prvý dílčí obvod získáme tak, že z řešeného obvodu vypustíme všechny větve, které obsahují indukčnosti s induktivními vazbami. Druhý dílčí obvod sestává právě z těchto větví. Pro prvý dílčí obvod lze na základě výše popsané metody snadno formulovat základní rovnici — tzv. maticovou rovnici řezů, rov. (33). Pro druhý dílčí obvod se vychází z Ohmova zákona, rov. (36), a výpočtem $\mathbf{Y}^{(2)} = \mathbf{Z}^{(2)-1}$ se přechází na rov. (38), jež je analogická k rov. (33). Vhodným očíslováním větví lze docílit, že matice $\mathbf{Z}^{(2)}$ je kvazidiagonální a tedy její inverze je snadná. S užitím principu superpozice přecházíme z rovnic obou dílčích obvodů na rovnici analyzovaného obvodu — rov.

[1]) Místo označení „metoda řezů" (angl. „Cut-Set Method") se v české literatuře (např. [7]) používá též termín „metoda J křivek".

(47). Řešením této rovnice dostáváme submatici napětí pasivních větví stromu $\mathbf{U}_{II}$, rov. (48). Ostatní submatice větvových napětí a proudů určíme pak již snadno.

Aplikace odvozené zobecněné metody řezů je ilustrována čtyřmi příklady. V prvých dvou příkladech je naznačeno provedení analýzy obvodu bez vzájemných indukčností a v dalších dvou příkladech je naznačeno řešení obvodu s induktivně vázanými dvojicemi větví.

Ukazuje se, že v porovnání s jinými metodami analýzy obvodů je popisovaná zobecněná metoda řezů velmi výhodná. V přeložené práci je tato metoda formulována pro analýzu obvodů v ustáleném stavu, lze ji však též použít při formulaci rovnic popisujících přechodné jevy v obvodech.

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