Hana Kamasová

An algorithm for the inversion of partitioned matrices

AN ALGORITHM FOR THE INVERSION OF PARTITIONED MATRICES

Hana Kamasová

1. INTRODUCTION

In paper [1] the following results were proved:

Let A be a square matrix of order $n \geq r$ over a field of characteristic zero, partitioned into blocks $\alpha_{i,k}$ of the type $(n_i \times n_k)$,

$$\sum_{i=1}^{r} n_i = \sum_{k=1}^{r} n_k = n .$$

Let further

$$(1,1) \qquad A_1 = \alpha_{1,1}$$

$$A_2 = \begin{bmatrix} \alpha_{1,1}, & \alpha_{1,2} \\ \alpha_{2,1}, & \alpha_{2,2} \end{bmatrix}$$

$$\vdots$$

$$A_{r-1} = \begin{bmatrix} \alpha_{1,1} & , \ldots, \alpha_{1,r-1} \\ \vdots & \\ \alpha_{r-1,1}, \ldots, \alpha_{r-1,r-1} \end{bmatrix}$$

$$A_r = A .$$

Let us define the matrices $Z_{i,k}^{(p)}$ for $i, k, p = 1, 2, \ldots, r$ in the following way:

1. $Z_{i,k}^{(1)} = \alpha_{i,k}$
2. $Z_{i,k}^{(p)} = Z_{i,k}^{(p-1)} - Z_{i,p-1}^{(p-1)} Z_{p-1}^{-1} Z_{p-1,k}^{(p-1)}$

   for $p = 2, \ldots, r$ where $Z_{p,p}^{(p)} = Z_p$.

For matrices Z_p we have

Theorem 1,1. *The matrices A_p are regular iff Z_p are regular for $p = 1, 2, \ldots, r$.*

For the proof see [1].

For $i \leqq p$, $k \leqq p$ let us introduce $V_{i,k}^{(p)}$ as the set of subsequences of the sequence $(i, i + 1, \ldots, p - 1, p, p - 1, \ldots, k + 1, k)$ which have the following properties:

1° The first element is i, the last is k.

2° Each two neighbouring elements of the subsequences are different.

Theorem 1,2. *Let the matrices* $(1,1)$ *be regular, let* $\mathbf{A}^{-1} = [\beta_{i,k}]$ *be a partitioned matrix conformal to* $\mathbf{A}$. *Then we have*

$$(1,2) \qquad \beta_{i,k} = \sum (-1)^{1 + m(j_1, \ldots, j_s)} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1, j_2}^{(q_1)} \mathbf{Z}_{j_2}^{-1} \ldots \mathbf{Z}_{j_s - 1, j_s}^{(q_s - 1)} \mathbf{Z}_{j_s}^{-1},$$

$$(i = j_1, \ldots, j_s = k) \in V_{i,k}^{(r)}$$

$$g_t = \min (j_t, j_{t+1}), \quad t = 1, \ldots, s - 1$$

where $m(j_1, \ldots, j_s)$ *is the number of the elements of the sequence* $(i = j_1, \ldots, j_s = k)$.

For the proof see [1].

In this paper an algorithm for calculating the blocks of the inverse matrix of $\mathbf{A}$ is presented. It is simultaneously an algorithm for the matrices $\mathbf{Z}_{i,k}^{(p)}$ for $i \geqq p$; $k \geqq p$; $i, k, p = 2, \ldots, r$.

First, let us introduce the following sets:

For $i \leqq p \leqq k$, let us denote by $U_{i,k}^{(p)}$ the set of the subsequences of the sequence $(i, i + 1, \ldots, p - 1, p, k)$ which have the properties 1°, 2°. Let $(U_{i,k}^{(p)})_p$ denote the set of those elements from $U_{i,k}^{(p)}$ which contain the term p.

Further, for $i \geqq p \geqq k$ let $W_{i,k}^{(p)}$ denote the set of the subsequences of the sequence $(i, p, p - 1, \ldots, k + 1, k)$ which have the properties 1°, 2°.

Let $(W_{i,k}^{(p)})_p$ denote the set of those elements from $W_{i,k}^{(p)}$ which contain the term p.

For $i \leqq p$; $k \leqq p$ let $(V_{i,k}^{(p)})_p$ denote the set of those elements from $V_{i,k}^{(p)}$ which contain the term p.

It is obvious that:

$(1,3)$ For $i \leqq k$, $\qquad V_{i,k}^{(k)} = U_{i,k}^{(k)}$.

$(1,4)$ For $i \geqq k$, $\qquad V_{i,k}^{(i)} = W_{i,k}^{(i)}$.

$(1,5)$ For $i \leqq p$, $\qquad U_{i,p+1}^{(p)} = V_{i,p+1}^{(p+1)}$.

$(1,6)$ For $k \leqq p$, $\qquad W_{p+1,k}^{(p)} = V_{p+1,k}^{(p+1)}$.

$(1,7)$ For $i \leqq p$; $k \leqq p$, $V_{i,k}^{(p)} \cup (V_{i,k}^{(p+1)})_{p+1} = V_{i,k}^{(p+1)}$, $V_{i,k}^{(p)} \cap (V_{i,k}^{(p+1)})_{p+1} = \Phi$.

$(1,8)$ For $i \leqq p < k$, $U_{i,k}^{(p)} \cup (U_{i,k}^{(p+1)})_{p+1} = U_{i,k}^{(p+1)}$, $U_{i,k}^{(p)} \cap (U_{i,k}^{(p+1)})_{p+1} = \Phi$.

$(1,9)$ For $i > p \geqq k$, $W_{i,k}^{(p)} \cup (W_{i,k}^{(p+1)})_{p+1} = W_{i,k}^{(p+1)}$, $W_{i,k}^{(p)} \cap (W_{i,k}^{(p+1)})_{p+1} = \Phi$.

2. RESULTS

Theorem 2,1. *Let the matrices* $(1,1)$ *be regular. For* $i, k, p = 1, 2, ..., r$ *let us define the following transformation:*

(2,1) $\qquad \beta_{i,k}^{(0)} = \alpha_{i,k}$

(2,2) $\qquad \beta_{p,p}^{(p)} = \left(\beta_{p,p}^{(p-1)}\right)^{-1}$

(2,3) $\qquad \beta_{p,k}^{(p)} = -\left(\beta_{p,p}^{(p-1)}\right)^{-1} \beta_{p,k}^{(p-1)}$ $\qquad\qquad$ *for* $k \neq p$.

(2,4) $\qquad \beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)}\left(\beta_{p,p}^{(p-1)}\right)^{-1}$ $\qquad\qquad$ *for* $i \neq p$.

(2,5) $\qquad \beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)}\left(\beta_{p,p}^{(p-1)}\right)^{-1} \beta_{p,k}^{(p-1)}$ $\quad$ *for* $i \neq p,\, k \neq p$.

Then for $i \leq p;\; k \leq p;$

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,...,j_s=k)\in V^{(p)}{}_{i,k}} (-1)^{m+1}\, \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1,j_2}^{(q_1)} \mathbf{Z}_{j_2}^{-1} \,...\, \mathbf{Z}_{j_{s-1},j_s}^{(q_{s-1})} \mathbf{Z}_{j_s}^{-1}$$

(2,6) $\quad$ *where* $q_t = \min\left(j_t, j_{t+1}\right)$ *for* $t = 1, ..., s-1$,

$\qquad m$ *is the number of terms of the sequence* $(i = j_1, ..., j_s = k) \in V_{i,k}^{(p)}$,

(2,7) $\quad$ *for* $i > p;\, k > p$,

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)},$$

(2,8) $\quad$ *for* $i \leq p;\, k > p$,

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,...,j_s=k)\in U^{(p)}{}_{i,k}} (-1)^{u+1}\, \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1,j_2}^{(j_1)} \mathbf{Z}_{j_2}^{-1} \mathbf{Z}_{j_2,j_3}^{(j_2)} \,...\, \mathbf{Z}_{j_{s-1}}^{-1} \mathbf{Z}_{j_{s-1},j_s}^{(j_{s-1})}$$

$\qquad$ *where* u *is the number of terms of the sequnce*

$\qquad (i = j_1, ..., j_s = k) \in U_{i,k}^{(p)}.$

(2,9) $\quad$ *For* $i > p;\, k \leq p,$

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1,...,j_s=k)\in W^{(p)}{}_{i,k}} (-1)^{w}\, \mathbf{Z}_{j_1,j_2}^{(j_2)} \mathbf{Z}_{j_2}^{-1} \mathbf{Z}_{j_2,j_3}^{(j_3)} \mathbf{Z}_{j_3}^{-1} \,...\, \mathbf{Z}_{j_{s-1},j_s}^{(j_s)} \mathbf{Z}_{j_s}^{-1}$$

$\qquad$ *where* w *is the number of terms of the sequence*

$\qquad (i = j_1, ..., j_s = k) \in W_{i,k}^{(p)}.$

Particularly, for $p = r$ *the partitioned matrix* $\left[\beta_{i,k}^{(r)}\right] = \mathbf{A}^{-1}.$

Proof by induction.

1. For $p = 1$, $\beta_{1,1}^{(0)} = \mathbf{Z}_1$ is regular and we have by $(2,1)-(2,5)$

$$\beta_{i,k}^{(1)} = \begin{bmatrix} \mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} \\ \mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2, & \mathbf{Z}_{23}^{(2)} & & \mathbf{Z}_{2r}^{(2)} \\ \vdots & \vdots & & & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} & \mathbf{Z}_{r2}^{(2)}, & \dots\dots\dots\dots\dots\dots\dots & & \mathbf{Z}_{rr}^{(2)} \end{bmatrix}.$$

For $p = 2$, $\beta_{2,2}^{(1)} = \mathbf{Z}_2$ is regular by Theorem 1,1 and we obtain

$$\beta_{i,k}^{(2)} = \begin{bmatrix} \mathbf{Z}_1^{-1} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}, \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2^{-1}, \\ \mathbf{Z}_{31}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}, \\ \vdots & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}, \end{bmatrix}$$

$$\begin{bmatrix} -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} + \dot{\mathbf{Z}}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots\dots\dots\dots\dots\dots & -\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ \mathbf{Z}_3, & \dots\dots\dots\dots\dots\dots & \mathbf{Z}_{3r}^{(3)} \\ \\ \mathbf{Z}_{r3}^{(3)}, & \dots\dots\dots\dots\dots\dots & \mathbf{Z}_{rr}^{(3)} \end{bmatrix}.$$

It is obvious that Theorem 2,1 is true for $p = 1, 2$.

2. Let us suppose that our theorem is true for p; then $\beta_{p+1,p+1}^{(p)} = \mathbf{Z}_{p+1}$, therefore by Theorem 1,1 $\beta_{p+1,p+1}^{(p)}$ is regular.

Proof of 2,6:

a) Let $i < p + 1$; $k < p + 1$; then by $(2,5)$

$$\beta_{i,k}^{(p+1)} = \beta_{i,k}^{(p)} - \beta_{i,p+1}^{(p)}\big(\beta_{p+1,p+1}^{(p)}\big)^{-1}\beta_{p+1,k}^{(p)} = \sum_{V^{(p)}{}_{i,k}} (-1)^{m_1+1}\, \mathbf{Z}_i^{-1} \dots \mathbf{Z}_k^{-1} -$$

$$- \Big(\sum_{U^{(p)}{}_{i,p+1}} (-1)^{u+1}\, \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1},p+1}^{(j_{s-1})} \Big) \mathbf{Z}_{p+1}^{-1} \Big(\sum_{W^{(p)}{}_{p+1,k}} (-1)^w\, \mathbf{Z}_{p+1,j_2}^{(j_2)} \dots \mathbf{Z}_k^{-1} \Big) = I + II\,.$$

After performing the multiplication in II we obtain obviously a sum where we summarize over all sequences of the set $\big(V_{i,k}^{(p+1)}\big)_{p+1}$. The sign of every summand in II is $(-1)^{u+w}$. Since the term $p + 1$ is included both in number u and in number w, it is clear that $(-1)^{u+w} = (-1)^{v+1}$ where v is the number of terms of the sequence from $\big(V_{i,k}^{(p+1)}\big)_{p+1}$.

Since $U_{i,p+1}^{(p)}$ contains only increasing sequences and $W_{p+1,k}^{(p)}$ contains only decreasing

sequences, we obtain for every $\mathbf{Z}_{j_t,j_{t+1}}^{(q_t)}$ in II clearly $q_t = \min(j_t, j_{t+1})$. By relation (1,7) we can therefore write

$$\beta_{i,k}^{(p+1)} = \sum_{(i=j_1,\ldots,j_s=k)\in V_{i,k}(p+1)} (-1)^{m+1}\, \mathbf{Z}_{j_1}^{-1}\mathbf{Z}_{j_1,j_2}^{(q_1)}\ldots\mathbf{Z}_{j_s}^{-1}\,.$$

$$q_t = \min(j_t, j_{t+1})$$

b) Let $i = p + 1;\ k = p + 1;$ then

$$\beta_{p+1,p+1}^{(p+1)} = (\beta_{p+1,p+1}^{(p)})^{-1} = \mathbf{Z}_{p+1}^{-1}\,.$$

c) Let $i = p + 1;\ k < p + 1;$ then

$$\beta_{p+1,k}^{(p+1)} = -(\beta_{p+1,p+1}^{(p)})^{-1}\,\beta_{p+1,k}^{(p)} = -\mathbf{Z}_{p+1}^{-1}\Big(\sum_{W_{p+1,k}(p)}(-1)^w\,\mathbf{Z}_{p+1,j_1}^{(j_1)}\ldots\mathbf{Z}_k^{-1}\Big),$$

therefore by (1,6)

$$\beta_{p+1,k}^{(p+1)} = \sum_{V_{p+1,k}(p+1)}(-1)^{m+1}\,\mathbf{Z}_{p+1}^{-1}\ldots\mathbf{Z}_k^{-1}\,.$$

d) Let $i < p + 1;\ k = p + 1;$ then

$$\beta_{i,p+1}^{(p+1)} = \Big(\sum_{U_{i,p+1}(p)}(-1)^{u+1}\,\mathbf{Z}_i^{-1}\ldots\mathbf{Z}_{j_{s-1},p+1}^{(j_{s-1})}\Big)\mathbf{Z}_{p+1}^{-1}$$

and by (1,5) we get

$$\beta_{i,p+1}^{(p+1)} = \sum_{V_{i,p+1}(p+1)}(-1)^{m+1}\,\mathbf{Z}_i^{-1}\ldots\mathbf{Z}_{p+1}^{-1}\,.$$

Proof of (2,7):

Let $i > p + 1;\ k > p + 1;$ then

$$\beta_{i,k}^{(p+1)} = \beta_{i,k}^{(p)} - \beta_{i,p+1}^{(p)}(\beta_{p+1,p+1}^{(p)})^{-1}\beta_{p+1,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)} - \mathbf{Z}_{i,p+1}^{(p+1)}\mathbf{Z}_{p+1}^{-1}\mathbf{Z}_{p+1,k}^{(p+1)} = \mathbf{Z}_{i,k}^{(p+2)}\,.$$

Proof of (2,8):

a) Let $i < p + 1 < k;$ then

$$\beta_{i,k}^{(p+1)} = \sum_{U_{i,k}(p)}(-1)^{u_1+1}\,\mathbf{Z}_i^{-1}\mathbf{Z}_{i,j_2}^{(i)}\ldots\mathbf{Z}_{j_{s-1}}^{-1}\mathbf{Z}_{j_{s-1},k}^{(j_{s-1})} -$$

$$- \Big(\sum_{U_{i,p+1}(p)}(-1)^{u_2+1}\,\mathbf{Z}_i^{-1}\ldots\mathbf{Z}_{j_{s-1},p+1}^{(j_{s-1})}\Big)\mathbf{Z}_{p+1}^{-1}\mathbf{Z}_{p+1,k}^{(p+1)} =$$

$$= \sum_{U_{i,k}(p)}(-1)^{u_1+1}\,\mathbf{Z}_i^{-1}\mathbf{Z}_{i,j_2}^{(i)}\ldots\mathbf{Z}_{j_{s-1},k}^{(j_{s-1})} + \sum_{(U_{i,k}(p+1))_{p+1}}(-1)^{u_3+1}\,\mathbf{Z}_i^{-1}\ldots\mathbf{Z}_{p+1}^{-1}\mathbf{Z}_{p+1,k}^{(p+1)}$$

where $u_3 = u_2 + 1$ is the number of terms of the sequence $(i = j_1,\ldots,j_s = k)\in$ $\in (U_{i,k}^{(p+1)})_{p+1}$.

Using relation (1,8) we obtain

$$\beta_{i,k}^{(p+1)} = \sum_{U_{i,k}(p+1)}(-1)^{u+1}\,\mathbf{Z}_i^{-1}\ldots\mathbf{Z}_{j_{s-1},k}^{(j_{s-1})}\,.$$

b) Let $i = p + 1 < k$; then

$$\beta_{p+1,k}^{(p+1)} = -\mathbf{Z}_{p+1}^{-1}\mathbf{Z}_{p+1,k}^{(p+1)} = \sum_{U_{p+1,k}^{(p+1)}} (-1)^{u+1} \, \mathbf{Z}_{p+1}^{-1} \ldots \mathbf{Z}_{p+1,k}^{(p+1)} \,,$$

for $U_{p+1,k}^{(p+1)}$ contains only one element — the sequence $(p + 1, k)$.

Proof of $(2,9)$:

a) Let $i > p + 1 > k$; then

$$\beta_{i,k}^{(p+1)} = \sum_{W_{i,k}^{(p)}} (-1)^{w_1} \, \mathbf{Z}_{i,j_2}^{(j_2)}\mathbf{Z}_{j_2}^{-1} \ldots \mathbf{Z}_{j_{s-1},k}^{(k)}\mathbf{Z}_{k}^{-1} \; -$$

$$- \, \mathbf{Z}_{i,p+1}^{(p+1)} \sum_{W_{p+1,k}^{(p)}} (-1)^{w_2} \, \mathbf{Z}_{p+1,j_2}^{(j_2)} \ldots \mathbf{Z}_{j_{s-1},k}^{(k)}\mathbf{Z}_{k}^{-1} \; =$$

$$= \sum_{W_{i,k}^{(p)}} (-1)^{w_1} \, \mathbf{Z}_{i,j_2}^{(j_2)} \ldots \mathbf{Z}_{k}^{-1} \; + \sum_{(W_{i,k}^{(p+1)})_{p+1}} (-1)^{w_3} \, \mathbf{Z}_{i,p+1}^{(p+1)} \ldots \mathbf{Z}_{k}^{-1}$$

where $w_3 = w_2 + 1$ is the number of terms of the sequence $(i = j_1, \ldots, j_s = k) \in$
$\in \big(W_{i,k}^{(p+1)}\big)_{p+1}$.

By $(1,9)$ we obtain

$$\beta_{i,k}^{(p+1)} = \sum_{W_{i,k}^{(p+1)}} (-1)^{w} \, \mathbf{Z}_{i,j_2}^{(j_2)} \ldots \mathbf{Z}_{k}^{-1} \,.$$

b) Let $i > p + 1 = k$; then

$$\beta_{i,p+1}^{(p+1)} = \mathbf{Z}_{i,p+1}^{(p+1)}\mathbf{Z}_{p+1}^{-1} \,.$$

For $p = r$, formulas $(2,6)$ for $\beta_{i,k}^{(r)}$ and $(1,2)$ for $\beta_{i,k}$ are identical, thus the partitioned matrix $\big[\beta_{i,k}^{(r)}\big] = \mathbf{A}^{-1}$.

3. EXAMPLE

An algorithm for the calculation of the inverse matrix of a quasi-triangular matrix.

Theorem 3,1. *Let the matrices* $(1,1)$ *be regular. Let* $\alpha_{i,k} = 0$ *for* $i > k$. *Let* $\mathbf{A}^{-1} =$
$= [\beta_{i,k}]$ *be a partitioned matrix conformal to* $\mathbf{A}$. *Then*

$$(3,1) \qquad \mathbf{Z}_{i,k}^{(p)} = \alpha_{i,k} \quad for \quad i \geqq p; \; k \geqq p; \; p \geqq 2 \,,$$

$$(3,2) \quad \beta_{i,k} = \sum_{(i=j_1,\ldots,j_s=k)\in V_{i,k}^{(k)}} (-1)^{m+1} \alpha_{j_1,j_1}^{-1}\alpha_{j_1,j_2}\alpha_{j_2,j_2}^{-1} \ldots \alpha_{j_s,j_s}^{-1} \quad for \quad i \leqq k$$

where m *is the number of terms of the sequence* $(i = j_1, \ldots, j_s = k) \in V_{i,k}^{(k)}$,

$$(3,3) \qquad \beta_{i,k} = 0 \quad for \quad i > k \,.$$

For the proof see [2].

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Lemma. *Let the matrices* $(1,1)$ *be regular, let* $\alpha_{i,k} = 0$ *for* $i > k$. *Then* $\beta_{i,k}^{(p)} = 0$ *for* $i > k$; $p = 1, 2, \ldots, r$.

Proof. For $\beta_{i,k}^{(p)}$ formulas $(2,6)-(2,9)$ are true for $p = 1, 2, \ldots, r$. In the case of a quasi-triangular matrix, these formulas have by $(3,1)-(3,3)$ the following form:

$(2,6)'$ For $i \leq k \leq p$,

$$\beta_{i,k}^{(p)} = \sum_{V_{i,k}^{(k)}} (-1)^{m+1} \alpha_{i,i}^{-1} \ldots \alpha_{k,k}^{-1} ;$$

for $k < i \leq p$, $\beta_{i,k}^{(p)} = 0$.

$(2,7)'$ For $i > p$; $k > p$,

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)} = \alpha_{i,k} .$$

$(2,8)'$ For $i \leq p$; $k > p$,

$$\beta_{i,k}^{(p)} = \sum_{U_{i,k}^{(p)}} (-1)^{u+1} \alpha_{i,i}^{-1} \ldots \alpha_{j_{s-1},k} .$$

$(2,9)'$ For $i > p$; $k \leq p$,

$$\beta_{i,k}^{(p)} = \sum_{W_{i,k}^{(p)}} (-1)^{w} \alpha_{i,j_2} \alpha_{j_2,j_2}^{-1} \ldots \alpha_{k,k}^{-1} .$$

Since $W_{i,k}^{(p)}$ contains only decreasing sequences, it is $j_t > j_{t+1}$ for all $t = 1, \ldots, s - 1$; thus $\alpha_{j_t,j_{t+1}} = 0$. Therefore $\beta_{i,k}^{(p)} = 0$ for $i > k$.

The transformation formulas:

$$\beta_{i,k}^{(0)} = \alpha_{i,k} ,$$

$$\beta_{p,p}^{(p)} = \left(\beta_{p,p}^{(p-1)}\right)^{-1} ,$$

$$\beta_{p,k}^{(p)} = -\left(\beta_{p,p}^{(p-1)}\right)^{-1} \beta_{p,k}^{(p-1)} \quad \text{for} \quad k > p ,$$

$$\beta_{p,k}^{(p)} = 0 \quad \text{for} \quad k < p ,$$

$$\beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)}\left(\beta_{p,p}^{(p-1)}\right)^{-1} \quad \text{for} \quad i < p ,$$

$$\beta_{i,p}^{(p)} = 0 \quad \text{for} \quad i > p ,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)}\left(\beta_{p,p}^{(p-1)}\right)^{-1} \beta_{p,k}^{(p-1)} \quad \text{for} \quad i < p < k ,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} \quad \text{for} \quad i > p; k > p \quad \text{and for} \quad i < p; k < p .$$

References

[1] *H. Kamasová, A. Šimek:* Metoda inverze matice rozdělené na bloky, Aplikace matematiky, 2, 14 (1969), 105—114.

[2] *H. Kamasová, A. Šimek:* Inversion of quasi-triangular matrices, Aplikace matematiky, 2, 15 (1970), 146—148.

Souhrn

ALGORITMUS PRO INVERZI BLOKOVÝCH MATIC

Hana Kamasová

V článku je uveden algoritmus pro výpočet inverzní matice k blokové matici rozdělené na $r \times r$ bloků.

Věta. *Nechť matice* $(1,1)$ *jsou regulární. Pro i, k, p = 1, 2, ..., r definujme transformace* $(2,1)-(2,5)$. *Potom platí* $(2,6)-(2,9)$. *Specielně pro p = r bloková matice* $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}$.

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