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Czechoslovak Mathematical Journal, Vol. 39 (1989), No. 3, 522–527

Persistent URL: `http://dml.cz/dmlcz/102324`

Czechoslovak Mathematical Journal, 39 (114) 1989, Praha

ON α-IDEALS AND GENERALIZED α-IDEALS IN SEMIGROUPS

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(Received February 23, 1988)

Let S be a semigroup. A non-empty subset A of S is called a *generalized* (m, n)-*ideal of* S if the inclusion

$$A^m S A^n \subseteqq A$$

holds, where m, n are arbitrary non-negative integers. Here, as usual, $A^0 S A^n = SA^n$, $A^m S A^0 = A^m S$ and $A^0 S A^0 = S$ (see [4]). A generalized (m, n)-ideal A of S is said to be *an* (m, n)-*ideal of* S if A is a subsemigroup of S. It is easy to see that one-side (left or right) ideals are particular cases of (m, n)-ideals. S. Lajos, in [3], [5], [6] and [7], characterized certain classes of semigroups through the generalized $(1, 1)$-ideals.

In this paper we shall generalize some results on (m, n)-ideals and generalized (m, n)-ideals in semigroups. In section 1 we introduce the α-ideals and the generalized α-ideals in semigroups, where α is a finite sequence of zeros and units containing at least one zero.

In Section 2 we characterize the semigroups for which any generalized α-ideal is an α-ideal. Moreover, we prove among other things that if every generalized $(3, 3)$-ideal is a $(3, 3)$-ideal then every generalized α-ideal is an α-ideal.

In Section 3 we investigate the semigroup by all generalized α-ideals of a semigroup S. In particular, we prove that there is an isomorphism between the semigroup which consists of all 101-ideals of a regular semigroup S and the semigroup which consists of all 101-ideals of the semigroup S/μ, where μ is the maximal idempotent separating congruence on S. Moreover, we answer a question by S. Lajos on the semigroup which consists of all left ideals of a semigroup S.

The reader is referred to [2] for basic notions and terminology of algebraic semigroups theory.

1.

By X^* we denote the free monoid over an alphabet X. Let S be a semigroup. By $\mathscr{P}(S)$ we denote the semigroup of all subsets of S under set product with the unity $\emptyset$. For $\alpha \in \{0, 1\}^*$ we shall define $f_\alpha^S \colon \mathscr{P}(S) \to \mathscr{P}(S)$ as follows: $f_\alpha^S(A) = \emptyset$ if α is the empty word and

$$f_\alpha^S(A) = A_1 A_2 \dots A_n$$

if $\alpha = \alpha_1\alpha_2 \dots \alpha_n$, $\alpha_i \in \{0, 1\}$, where

$$A_i = \begin{cases} A & \text{for} \quad \alpha_i = 1\,, \\ S & \text{for} \quad \alpha_i = 0\,. \end{cases}$$

1.1. Lemma. *$f_\alpha^S(A) \subseteqq f_\alpha^S(B)$, whenever $A \subseteqq B \subseteqq S$.*

1.2. Lemma. *$f_{\alpha\beta}^S(A) = f_\alpha^S(A)\, f_\beta^S(A)$ for $A \subseteqq S$.*

Let us put $\Lambda = \{0, 1\}^* \setminus \{1\}^*$.

1.3. Lemma. *If $A \in \mathscr{P}(S)$ and $\alpha \in \Lambda$, then $Af_\alpha^S(A) \subseteqq f_\alpha^S(A)$.*

Proof. We can suppose that $\alpha = \alpha_1\alpha_2 \dots \alpha_n$, $\alpha_i \in \{0, 1\}$, and $\alpha_j = 0$ for some j with $\alpha_i = 1$ for $i < j$. If $j = 1$, then $Af_\alpha^S(A) = ASA_2 \dots A_n \subseteqq SA_2 \dots A_n = f_\alpha^S(A)$. If $j > 1$, then $Af_\alpha^S(A) = A^jSA_{j+1} \dots A_n \subseteqq A^{j-1}SA_{j+1} \dots A_n = f_\alpha^S(A)$.

1.4. Lemma. *If $A \in \mathscr{P}(S)$ and $\alpha \in \Lambda$, then $f_\alpha^S(A)\, f_\alpha^S(A) \subseteqq f_\alpha^S(A)$.*

Proof. We have $f_\alpha^S(A) = A_1A_2 \dots A_n$ and $A_j = S$ for some j. Then $f_\alpha^S(A)\, f_\alpha^S(A) = A_1A_2 \dots A_{j-1}(A_j \dots A_nA_1 \dots A_j)\, A_{j+1} \dots A_n \subseteqq A_1A_2 \dots A_{j-1}SA_{j+1} \dots A_n = f_\alpha^S(A)$.

1.5. Lemma. *If $A \in \mathscr{P}(S)$ and $\alpha \in \Lambda$, then $f_\alpha^S(A \cup f_\alpha^S(A)) \subseteqq f_\alpha^S(A)$.*

Proof. Let $A \in \mathscr{P}(S)$. First we note that for every positive integer n we have

$$f_\alpha^S(A \cup f_\alpha^S(A)) = S^n = f_\alpha^S(A)\,, \tag{1}$$

where $\alpha = 0^n$.

Let $\alpha \in \Lambda$. We prove our statement by induction on the length n of α. It follows from (1) that the result is true for $n = 1$. Assume now that $n \geqq 2$ and the result holds for $n - 1$.

Case 1: $\alpha = 1\beta$. Then $\beta \in \Lambda$ and $f_\beta^S(A \cup f_\beta^S(A)) \subseteqq f_\beta^S(A)$. Using Lemmas 1.1, 1.2, 1.3 and 1.4 we obtain

$$\begin{aligned} f_\alpha^S(A \cup f_\alpha^S(A)) &= (A \cup Af_\beta^S(A))\, f_\beta^S(A \cup Af_\beta^S(A)) \subseteqq \\ &\subseteqq (A \cup Af_\beta^S(A))\, f_\beta^S(A \cup f_\beta^S(A)) \subseteqq (A \cup Af_\beta^S(A))\, f_\beta^S(A) \subseteqq \\ &\subseteqq Af_\beta^S(A) \cup Af_\beta^S(A)\, f_\beta^S(A) = Af_\beta^S(A) = f_\alpha^S(A)\,. \end{aligned}$$

Case 2: $\alpha = \beta 1$. This is dual to Case 1.

Case 3: $\alpha = 0\beta 0$, where $\beta \in \{0, 1\}^*$. According to (1) we can suppose that $\beta \notin \{0\}^*$. Then $n \geqq 3$, $\beta = \alpha_2 \dots \alpha_{n-1}$ and $I \neq \emptyset$, where $i \in I$ if and only if $i \in \{2, \dots, n - 1\}$ and $\alpha_i = 1$. Therefore $f_\alpha^S(A \cup f_\alpha^S(A)) = SA_2 \dots A_{n-1}S$, where $A_i = A \cup f_\alpha^S(A)$ for $i \in I$ and $A_i = S$ for $i \in \bar{I} = \{2, \dots, n - 1\} \setminus I$.

By Z we denote the set of all words of $\{0, 1\}^*$ having the length $n - 2$. Let us put $B_{i0} = A$, $B_{i1} = f_\alpha^S(A)$ if $i \in I$, $B_{i0} = S = B_{i1}$ if $i \in \bar{I}$ and $B_\gamma = SB_{2\gamma_2} \dots B_{n-1,\gamma_{n-1}}S$ if $\gamma = \gamma_2 \dots \gamma_{n-1} \in Z$. It is easy to show that $B_\gamma = f_\alpha^S(A)$ if $\gamma = 0^{n-2}$ and $B_\gamma \subseteqq Sf_\alpha^S(A)\, S \subseteqq f_\alpha^S(A)$ if $\gamma \neq 0^{n-2}$. Hence we have $f_\alpha^S(A \cup f_\alpha^S(A)) = \bigcup_{\gamma \in Z} B_\gamma \subseteqq f_\alpha^S(A)$.

1.6. Definition. Let $\alpha \in \Lambda$. A non-empty subset M of a semigroup S is called a *generalized α-ideal* of S if $f_\alpha^S(M) \subseteqq M$. A generalized α-ideal M of S is said to be an *α-ideal* of S if M is a subsemigroup of S.

1.7. Theorem. *Let A be a non-empty subset of a semigroup S. Then $A \cup f_\alpha^S(A)$ is a generalized α-ideal of S for every $\alpha \in \Lambda$.*

The proof follows from Lemma 1.5.

2.

S. Lajos, in [3], gave an example of a semigroup for which certain generalized (m, n)-ideal are not (m, n)-ideals. F. Catino, in [1], characterized the semigroups for which any generalized $(1, 1)$-ideal is a $(1, 1)$-ideal.

2.1. Theorem. *Let S be a semigroup and $\alpha \in \Lambda$. Then every generalized α-ideal of S is an α-ideal of S if and only if $ab \in f_\alpha^S(\{a, b\})$ for all $a, b \in S$.*

Proof. Suppose that $M^2 \subseteqq M$ for every generalized α-ideal M of S. Let $a, b \in S$ and put $A = \{a, b\}$ and $M = A \cup f_\alpha^S(A)$. According to Theorem 1.7, M is a generalized α-ideal of S and so $ab \in M^2 \subseteqq M$. If $ab \in A$, then $ab = a$ or $ab = b$. In both cases we have $ab \in f_\alpha^S(A)$.

Assume that $ab \in f_\alpha^S(\{a, b\})$ for all $a, b \in S$. Let M be a generalized α-ideal of S. If $x \in M^2$, then $x = ab$, where $a, b \in M$ and so, by Lemma 1.1, we have $ab \in f_\alpha^S(\{a, b\}) \subseteqq f_\alpha^S(M) \subseteqq M$. Therefore $M^2 \subseteqq M$ and the proof is complete.

Let us put $W(a, b) = \{a^2, b^2, ba^2, ab^2, aba\}$. Recall that an element a of a semigroup S is said to be *left regular* if $a \in a^2S$. Dually, a right regular element of S.

2.2. Theorem. *Let S be a semigroup and $\beta \in \{0, 1\}^*$. Then the following statements are equivalent:*

1. *For any $\alpha \in \{0, 1\}^*$, every generalized $\alpha 0 \beta$-ideal of S is an $\alpha 0 \beta$-ideal of S.*
2. *Every generalized $1^3 0 \beta$-ideal of S is a $1^3 0 \beta$-ideal of S.*
3. *For all $a, b \in S$ we have $ab \in W(a, b)\, S f_\beta^S(\{a, b\})$ and moreover a^2 is a left regular element of S.*

Proof. $1 \Rightarrow 2$. It is clear.

$2 \Rightarrow 3$. Let $a, b \in S$. Put $A = \{a, b\}$. By Theorem 2.1 and Lemma 1.2 we have $ab \in f_{1^3 0 \beta}^S(A) = A^3 S f_\beta^S(A)$. This implies $ab \in W(a, b)\, S f_\beta^S(A)$ or $ab \in babSf_\beta^S(A)$. In the second case we obtain $ab \in b^2 abS f_\beta^S(A) \subseteqq b^2 S f_\beta^S(A) \subseteqq W(a, b)\, S f_\beta^S(A)$.

Moreover, for $a = b$ we have $a^2 \in a^3 S$ and so $a^2 \in a^4 S$.

$3 \Rightarrow 1$. Let $a, b \in S$. Then $ab \in W(a, b)\, S f_\beta^S(A)$, where $A = \{a, b\}$, and $a^2 \in a^m S$, $b^2 \in b^m S$ for all integers $m \geqq 2$. It is easy to show that $ab \in A^m S f_\beta^S(A)$ for all integers $m \geqq 2$, whenever $ab \in \{a^2, b^2, ba^2, ab^2\}\, S f_\beta^S(A)$. Suppose that $ab \in abaS f_\beta^S(A)$. We shall distinguish two cases.

Case 1: $ba \in \{b^2, a^2, ab^2, ba^2\}\, S f_\beta^S(A)$. Then $ba \in A^m S f_\beta^S(A)$ for all integers $m \geqq 2$ and so $ab \in A^m S f_\beta^S(A)$ for all integers $m \geqq 2$.

Case 2: $ba \in babSf_\beta^S(A)$. Then $ab \in (ab)^2\, Sf_\beta^S(A)$ and so $ab \in (ab)^m\, Sf_\beta^S(A) \subseteqq$ $\subseteqq A^m S f_\beta^S(A)$ for all integers $m \geqq 2$.

Therefore we have $ab \in A^m S f_\beta^S(A)$ for all integers $m \geqq 2$ and so $ab \in f_\alpha^S(A)$ . . $Sf_\beta^S(A) = f_{\alpha 0\beta}^S(A)$ for all $\alpha \in \{0, 1\}^*$. It follows from Theorem 2.1 that every generalized $\alpha 0\beta$-ideal of S is an $\alpha 0\beta$-ideal of S and the proof is complete.

We recall that an element a of a semigroup S is called *completely regular* if there exists an element x of S such that $a = axa$, $ax = xa$. It is well known that an element of S is completely regular if it is left regular and right regular.

Using the same method of proof as in Theorem 2.2, we obtain:

2.3. Theorem. *Let S be a semigroup. Then the following statements are equivalent*:

1. *For all $\alpha, \beta \in \{0, 1\}^*$, every generalized $\alpha 0\beta$-ideal of S is an $\alpha 0\beta$-ideal of S.*
2. *Every generalized $1^3 01^3$-ideal of S is a $1^3 01^3$-ideal of S.*
3. *For all $a, b \in S$ we have $ab \in W(a, b)\, SW(b, a)$ and moreover a^2 is a completely regular element of S.*

3.

Let ϱ be a congruence on a semigroup S. Put $T = S/\varrho$ and define $\psi\colon \mathscr{P}(T) \to \mathscr{P}(S)$ as follows:

$$\psi(M) = \bigcup_{z \in M} z$$

for any $M \subseteqq T$.

3.1. Lemma. *Let $P, Q \in \mathscr{P}(T)$. Then $\psi(P) \subseteqq \psi(Q)$ if and only if $P \subseteqq Q$.*

3.2. Lemma. *Let $P, Q \in \mathscr{P}(T)$. Then $\psi(P)\, \psi(Q) \subseteqq \psi(PQ)$.*

3.3. Lemma. *Let $M \in \mathscr{P}(T)$ and $P_i \in \mathscr{P}(T)$ for $i = 1, 2, \ldots, n$. Then $\psi(P_1)\, \psi(P_2) \ldots$ $\ldots \psi(P_n) \subseteqq \psi(M)$ if and only if $P_1 P_2 \ldots P_n \subseteqq M$.*

Proof. Suppose that $\psi(P_1)\, \psi(P_2) \ldots \psi(P_n) \subseteqq \psi(M)$. If $y \in P_1 P_2 \ldots P_n$, then there exists $z_i \in P_i$ for $i = 1, 2, \ldots, n$ such that $z_1 z_2 \ldots z_n \subseteqq y$. For $i = 1, 2, \ldots, n$ we have $z_i \subseteqq \psi(P_i)$ and so $z_1 z_2 \ldots z_n \subseteqq \psi(M)$. Therefore $y \cap \psi(M) \neq \emptyset$, hence $y \subseteqq$ $\subseteqq \psi(M)$. Then $y \in M$. Consequently $P_1 P_2 \ldots P_n \subseteqq M$.

Conversely, assume now that $P_1 P_2 \ldots P_n \subseteqq M$. Using Lemma 3.1 and Lemma 3.2 we obtain $\psi(P_1)\, \psi(P_2) \ldots \psi(P_n) \subseteqq \psi(P_1 P_2 \ldots P_n) \subseteqq \psi(M)$.

3.4. Lemma. *Let $M \in \mathscr{P}(T)$. Then $f_\alpha^S\, \psi(M) \subseteqq \psi(M)$ if and only if $f_\alpha^T(M) \subseteqq M$.*

3.5. Lemma. *Let M be a non-empty subset of T. Then M is an α-ideal [a generalized α-ideal] of T if and only if $\psi(M)$ is an α-ideal [a generalized α-ideal] of S.*

3.6. Lemma. *Let $P_i \in \mathscr{P}(T)$ for $i = 1, 2, \ldots, n$ and $\psi(P_1)\, \psi(P_2) \ldots \psi(P_n) \in \psi(\mathscr{P}(T))$. Then $\psi(P_1 P_2 \ldots P_n) = \psi(P_1)\, \psi(P_2) \ldots \psi(P_n)$.*

Proof. Suppose that $\psi(P_1)\, \psi(P_2) \ldots \psi(P_n) = \psi(M)$ for some $M \in \mathscr{P}(T)$. It follows from Lemma 3.3 that $P_1 P_2 \ldots P_n \subseteqq M$. According to Theorem 3.2 we have $\psi(M) \subseteqq$

$\subseteqq \psi(P_1P_2 \dots P_n)$. Lemma 3.1 implies $M \subseteqq P_1P_2 \dots P_n$ and this completes the proof.

Let $\emptyset \neq \mathscr{A} \subseteqq \mathscr{P}(T)$. By $[\mathscr{A}]$, $[\psi(\mathscr{A})]$, respectively, we denote the subsemigroup of $\mathscr{P}(T)$ generated by $\mathscr{A}$, the subsemigroup of $\mathscr{P}(S)$ generated by $\psi(\mathscr{A})$.

3.7. Lemma. *Let $\emptyset \neq \mathscr{A} \subseteqq \mathscr{P}(T)$ such that $[\psi(\mathscr{A})] \subseteqq \psi(\mathscr{P}(T))$. Then $\psi/[\mathscr{A}]$ is an isomorphism of $[\mathscr{A}]$ onto $[\psi(\mathscr{A})]$.*

Proof. Assume that $M \in [\mathscr{A}]$, then $M = P_1P_2 \dots P_n$, where $P_i \in \mathscr{A}$ $(i = 1, 2, \dots, n)$. Hence we have $\psi(P_1)\,\psi(P_2) \dots \psi(P_n) \in [\psi(\mathscr{A})] \subseteqq \psi(\mathscr{P}(T))$. According to Lemma 3.6, we obtain $\psi(M) \in [\psi(\mathscr{A})]$. Thus $\psi([\mathscr{A}]) \subseteqq [\psi(\mathscr{A})]$.

Let $A \in [\psi(\mathscr{A})]$. Then $A = \psi(P_1)\,\psi(P_2) \dots \psi(P_n) \in \psi(\mathscr{P}(T))$, where $P_i \in \mathscr{A}$ $(i = 1, 2, \dots, n)$. It follows from Lemma 3.6 that $A = \psi(P_1P_2 \dots P_n) \in \psi([\mathscr{A}])$. Therefore $\psi([\mathscr{A}]) = [\psi(\mathscr{A})]$. By Lemma 3.6 and Lemma 3.1 we obtain that $\psi/[\mathscr{A}]$ is an isomorphism of $[\mathscr{A}]$ onto $[\psi(\mathscr{A})]$. The proof is complete.

Let $\alpha \in A$. By $\mathscr{F}_\alpha^S[{}^-\mathscr{F}_\alpha^S]$ we denote the subsemigroup of $\mathscr{P}(S)$ generated by all α-ideals [generated α-ideals] of S. An equivalence relation $\sigma(\mathscr{F}_\alpha^S)$ $[\sigma({}^-\mathscr{F}_\alpha^S)]$ on S is defined by the rule that

$$(a, b) \in \sigma(\mathscr{F}_\alpha^S) \quad \text{iff} \quad \forall H \in \mathscr{F}_\alpha^S\colon \quad a \in H \Leftrightarrow b \in H$$

$$[(a, b) \in \sigma({}^-\mathscr{F}_\alpha^S) \quad \text{iff} \quad \forall H \in {}^-\mathscr{F}_\alpha^S\colon\ a \in H \Leftrightarrow b \in H].$$

3.8. Theorem. *Let S be a semigroup and $\alpha \in A$. If ϱ is a congruence on S such that $\varrho \subseteqq \sigma(\mathscr{F}_\alpha^S)$ $[\varrho \subseteqq \sigma({}^-\mathscr{F}_\alpha^S)]$ then the semigroups $\mathscr{F}_\alpha^S$ and $\mathscr{F}_\alpha^{S/\varrho}[{}^-\mathscr{F}_\alpha^S$ and ${}^-\mathscr{F}_\alpha^{S/\varrho}]$ are isomorphic.*

Proof. It is easy to show that according to Lemma 3.5, $\varrho \subseteqq \sigma(\mathscr{F}_\alpha^S)$ implies $\mathscr{F}_\alpha^S \subseteqq \psi(\mathscr{P}(T))$, where $T = S/\varrho$. By $\mathscr{A}$ we denote the set of all α-ideals of T. Lemma 3.5 implies that $\psi(\mathscr{A})$ is the set of all α- ideals of S and so $\mathscr{F}_\alpha^S = [\psi(\mathscr{A})]$ and $\mathscr{F}_\alpha^T = [\mathscr{A}]$. Analogously for generalized α-ideals. The rest of the proof follows from Lemma 3.7.

If S is a regular semigroup and $\alpha = 101$, by Proposition 4.1 of [11], $\sigma(\mathscr{F}_{101}^S) = \mathscr{H}$.

3.9. Corollary. *Let S be a regular semigroup and let ϱ be a congruence relation on S such that $\varrho \subseteqq \mathscr{H}$. Then the semigroups $\mathscr{F}_{101}^S$ and $\mathscr{F}_{101}^{S/\varrho}$ are isomorphic.*

Theorem 2 in [9] is a consequence of the last corollary.

This Corollary gives more information on the semigroup $\mathscr{F}_{101}^S$: For instance, if S is a ω-regular bisimple semigroup it follows from Corollaty 4 [10] that $\mathscr{F}_{101}^S$ is a rectangular band. Now, since $\mathscr{H}$ is a congruence relation and since $S/\mathscr{H}$ is isomorphic to the bicyclic semigroup $\mathscr{C}(p, q)$ (this latter has already been studied in [8]). Moreover from the description of $\mathscr{F}_{101}^{\mathscr{C}(p,q)}$ given in [8] one easily derives a description of $\mathscr{F}_{101}^S$.

The analogue of Corollary 3.9 does not hold if $\mathscr{H}$ is replaced by Green's relations $\mathscr{L}$ and $\mathscr{R}$. Indeed, if S is a left zero semigroup and $|S| > 1$, then $\mathscr{L} = S \times S$, $|\mathscr{F}_{101}^{S/\mathscr{L}}| = 1$ and $|\mathscr{F}_{101}^S| \neq 1$, hence $\mathscr{F}_{101}^S$ and $\mathscr{F}_{101}^{S/\mathscr{L}}$ are not isomorphic.

The following, however, is true, solving problem by S. Lajos.

3.10. Corollary. *Let S be a semigroup. If ϱ is a congruence contained in $\mathscr{L}$, then $\mathscr{F}_{01}^{S}$ and $\mathscr{F}_{01}^{S/\varrho}$ are isomorphic.*

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