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ON THE INTEGRATION THEOREM FOR LIE GROUPOIDS

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The theorem referred to in the title says that any infinitesimal representation of one Lie groupoid into another extends uniquely to an actual representation (under certain connectivity conditions); and it has some other basic integrability theorems as immediate corollaries. The theorem was first formulated and proved by Quê, [12] p. 35 (and seems to be related also to work of Pradines [11]).

Our formulation of the theorem dispenses with the sheaf of Lie algebras which is the carrier of the classical (Quê-) formulation, and considers instead the "first neighbourhood of the diagonal" of a manifold as a basic combinatorial structure. This notion, although dating back to Grothendieck, Malgrange [9] and others in the late 1950's, obtains its real simplicity only when considered in the context of synthetic differential geometry (SDG, for short; see e.g. [3]). This means in particular that the category of smooth manifolds is embedded fully into a topos $\mathcal{E}$ ("a well adapted topos"), about which we shall talk as if it were the category of sets. According to [1], for each manifold M, there is in $\mathcal{E}$ a sub "set" $M_{(1)} \subset M \times M$, "the first neighbourhood of the diagonal"; we write $x \sim y$ for $(x, y) \in M_{(1)}$. The relation $\sim$ is reflexive and symmetric, but not transitive.

I want to point out that the notions and results we consider make sense for any set equipped with a reflexive symmetric $\sim$, so they should have a combinatorial significance besides the differential-geometric (cf. also [5] for this viewpoint). So in principle, they are more general than the corresponding notions and results of differential geometry out of which they were paraphrased. However, for me, their virtue is not primarily the added generality, but their simplicity, allowing insight into differential geometry at very low cost (no mathematical analysis involved).

The content of the present paper has to a certain extent been announced in [6].

1. GENERAL NOTIONS

In the present paper, the word 'graph' means 'oriented graph with identities and inversion', that is, a set M of *vertices* or *objects*, a set Φ of *edges* or *arrows*, and maps

$$\partial_0, \partial_1 : \Phi \to M \; ; \quad i : M \to \Phi \; ; \quad (\;)^{-1} : \Phi \to \Phi \; ,$$

satisfying

$$\begin{aligned}
\partial_0(i(a)) &= \partial_1(i(a)) = a && \forall a \in M\,, \\
(\alpha^{-1})^{-1} &= \alpha && \forall \alpha \in \Phi\,, \\
\partial_0(\alpha^{-1}) &= \partial_1(\alpha) && \forall \alpha \in \Phi\,, \\
\partial_1(\alpha^{-1}) &= \partial_0(\alpha) && \forall \alpha \in \Phi\,.
\end{aligned}$$

We write (Φ, M) or even just Φ for such a graph, when the rest of the data can be understood from the context.

A *graph map* $(\Phi, M) \to (\Psi, N)$ is a pair $T = (T_1, T_0)$ of maps, $T_1: \Phi \to \Psi$, $T_0: M \to N$, commuting in an evident way with ∂_0, ∂_1, i, and $(\ )^{-1}$. Also, a *subgraph* of (Φ, M) is a pair $\Phi' \subset \Phi$, $M' \subset M$, stable under ∂_0, ∂_1, i, and $(\ )^{-1}$. If $M = N$ and T_0 is the identity map, we say that T is a graph map *over* M. Similarly, a subgraph (Φ', M') as above is a subgraph *over* M if $M' = M$.

Any groupoid (= small category with all arrows invertible) may be considered a graph, by forgetting the composition structure, and a functor between groupoids is in particular a graph map.

For any set M, we have the *codiscrete groupoid* over M, which is $(M \times M, M)$, with the projections to first and second factor as ∂_0 and ∂_1, respectively, and with composition given by

$$(y, z) \circ (x, y) = (x, z)$$

(we compose from right to left).

Any group may be considered a groupoid with the one-element set 1 as object set.

A functor from a codisrete groupoid $(M \times M, M)$ to G is thus a map

$$M \times M \to^{F} G$$

which satisfies

$$F(b, c)\,.\,F(a, b) = F(a, c) \quad \forall a, b, c \in M\,,$$

and (hence) $F(a, a) = e$ (neutral element of G), and $(F(a, b))^{-1} = F(b, a)$.

For any fixed element $i \in M$, there is a 1-1 correspondence between the set of such functors, and the set of all functions $f: M \to G$ with $f(i) = e$, namely to F associate $f = F(i, -)$, and to f associate $F = \mathrm{d}f: M \times M \to G$ given by

(1.1) $$F(a, b) = \mathrm{d}f(a, b) := f(b)\,.\,f(a)^{-1}\,.$$

A subgraph $M_{(1)}$ over M of the codiscrete groupoid $(M \times M, M)$ is the same thing as a reflexive symmetric relation $\sim$ on M:

$$a \sim b \quad \text{iff} \quad (a, b) \in M_{(1)} \subset M \times M\,.$$

We shall consider the question: when can a graph map $r: M_{(1)} \to G$ be extended to a functor $M \times M \to G$? More generally, let (Φ', M) be a subgraph over M of a groupoid (Φ, M); then we shall consider the question: when can a graph map $r: (\Phi', M) \to (\Psi, N)$ into a groupoid (Ψ, N) be extended to a functor $\Phi \to \Psi$? Clearly, a necessary condition for such an extension to exist is that r preserves those composites that happen to exist in Φ', more precisely, r must be curvature free in the following sense:

Definition 1.1. A graph map $r: \Phi' \to \Psi$ (with $\Phi' \subset \Phi$ and Ψ as above) is called *curvature free* if for any composable pair α, β in Φ with α, β, and $\beta \circ \alpha \in \Phi'$, we have

$$r(\beta \circ \alpha) = r(\beta) \circ r(\alpha)$$

in Ψ.

By a 'global integration theorem', we understand presently a theorem, whose conclusion is the converse, that is, a theorem with conclusion that curvature free graph maps uniquely extend to functors. The present paper interrelates some such global integration theorems.

The basic type of global integration theorem is for the case when Φ is a codiscrete groupoid (so Φ' is a reflexive symmetric relation), and Ψ is a group:

. **Definition 1.2.** Let M be a set equipped with a reflexive symmetric relation $\sim$ (or equivalently, with a subgraph $M_{(1)} \subset M \times M$ over M), and let G be a group. We say that the pair $((M, \sim), G)$ *admits integration* if every curvature free graph map $M_{(1)} \to G$ extends uniquely to a functor $M \times M \to G$.

If $i \in M$ is a given element, a functor $F: M \times M \to G$ is, by (2.1) of form $\mathrm{d}f$ for a unique $f: M \to G$ with $f(i) = e$, so that we have

Proposition 1.3. *The pair $((M, \sim), G)$ admits integration if and only if the following conditions hold*: 1) *A graph map $\omega: M_{(1)} \to G$ is curvature free iff there exists $f: M \to G$ with $\mathrm{d}f = \omega$, i.e. with*

$$f(b) \,.\, f(a)^{-1} = \omega(a, b) \quad \forall a \sim b\,;$$

and, 2) *such f is unique if one requires furthermore $f(i) = e$.*

Graph maps $M_{(1)} \to G$ are considered in [4] under the name: G-valued 1-forms on M, and saying that the pair $((M, \sim), G)$ admits integration can thus be expressed verbally by "closed G-valued 1-forms ω are (uniquely) exact", i.e. of form $\mathrm{d}f$ for a unique $f: M \to G$ with $f(i) = e$; f deserves the name: *primitive* of the 1-form ω. In the standard well adpated models for SDG, we have by the main theorem in [4] (in conjunction with [9] – see Theorem 3.5 in [9]) that if M is a connected simply connected manifold, and G a Lie group, then the pair $((M, \sim), G)$ does admit integration.

Proposition 1.4. *If the pair $((M, \sim), G)$ admits integration, and $f: M \to G$ satisfies $f(c) = f(d)$ whenever $c \sim d$, then f is constant.*

Proof. Consider the function $F = \mathrm{d}f$ given by (1.1). By the assumption, the restriction of F to $M_{(1)} \subset M \times M$ has constant value e; this, however, is also the restriction of the map $M \times M \to G$ with constant value e, and since this map is a functor as well, it equals F, by the uniqueness assumption in Definition 1.2. So $F \equiv e$, or $f(b) \,.\, f(a)^{-1} = e \,\forall a, b$, so $f(a) = f(b) \,\forall a, b$.

We present finally a few auxiliary concepts.

A *trivialization* of a graph (Φ, M) is a graph map $Q: M \times M \to \Phi$ over M; so for any pair of elements a, b in M, Q selects an arrow $Q(a, b)$ from a to b. A graph Φ is called *locally trivializable* if it admits a trivilization. (When Φ is a groupoid, we don't assume Q to be a functor; this would give a stronger notion, which we do not consider here).

If (Φ, M) is a graph, and $a, b \in M$, we denote by $\Phi(a, b)$ the set of arrows α with $\partial_0(\alpha) = a$, $\partial_1(\alpha) = b$. If Φ is a groupoid, $\Phi(a, a)$ is a group, called the *vertex group* of Φ at a.

If (Ψ, N) is a groupoid, and $T: M \to N$ a map, we get a groupoid (Ψ_T, M) over M by a standard pull-back procedure: for $a, b \in M$

$$\Psi_T(a, b) := \Psi(T(a), T(b))$$

('the *full image along T*', in standard category theoretic terminology). If Ψ is locally trivializable, then so is Ψ_T.

2. THE INTEGRATION THEOREM

The integration theorem deals with two groupoids (Φ, M) and (Ψ, N); the set Φ of arrows of the first is assumed to be equipped with a reflexive symmetric relation $\sim$, compatible with the groupoid structure in the sense that

(2.1) $$\beta \sim \gamma \Leftrightarrow \beta^{-1} \sim \gamma^{-1},$$

(2.2) $$\beta \sim \gamma \Leftrightarrow \beta \circ \delta \sim \gamma \circ \delta,$$

(2.3) $$\beta \sim \gamma \Leftrightarrow \delta \circ \beta \sim \delta \circ \gamma,$$

for the two last under the assumptions that the composites make sense, (thus for (2.2): $\partial_0(\beta) = \partial_0(\gamma) = \partial_1(\delta)$).

We say that an arrow $\beta \in \Phi$ is a *near-identity* if

$$\beta \sim i(\partial_0(\beta)) \quad \text{and} \quad \beta \sim i(\partial_1(\beta)).$$

(At the cost of some complications of formulations, we could have carried the theory through with only the first of these assumptions, but in the applications in SDG, the first implies the second). Clearly, the set $\Phi' \subset \Phi$ of near-identities is a subgraph over M. Also, if β and γ are arrows with $\partial_0(\beta) = \partial_0(\gamma)$, then $\gamma \circ \beta^{-1}$ makes sense, and is a near-identity iff $\beta \sim \gamma$; similarly with $\gamma^{-1} \circ \beta$ if $\partial_1(\beta) = \partial_1(\gamma)$. This is an easy consequence of (2.1)–(2.3).

We note that, for $a \in M$, the subset $\Phi_a \subset \Phi$ defined by

$$\Phi_a = \{\alpha \in \Phi \mid \partial_0(\alpha) = a\}$$

inherits a reflexive symmetric relation $\sim$ from that of Φ, and can then state the main theorem.

Theorem 2.1. *Let (Φ, M) be a groupoid with a reflexive symmetric relation* $\sim$

on Φ, satisfying (2.1)–(2.3), and let $\Phi' \subset \Phi$ be the set of near-identities. Let (Ψ, N) be a locally trivializable groupoid. Assume that for each $a \in M$ and each vertex group G of Ψ, the pair $((\Phi_a, \sim), G)$ admits integration. Then any curvature-free graph map $r: (\Phi', M) \to (\Psi, N)$ extends uniquely to a functor $R: \Phi \to \Psi$.

Remarks. In the context of SDG, if Φ and Ψ are differentiable groupoids, and $\sim$ the first neighbourhood of the diagonal, such a curvature free graph map $r: \Phi' \to \Psi$ can be proved the same thing as an "infinitesimal representation" of Φ in Ψ in the terminology of [12] (cf. § 3 for some indications in this direction), so that the theorem in this case has for conclusion that any infinitesimal representation is induced by a unique actual representation (= functor).

Proof of the theorem. By replacing (Ψ, N) by the full image along $r_0: M \to N$ (where $r = (r_1, r_0)$), we immediately reduce to the case where $M = N$, and r_0 is the identity map. This reduction will make notation easier, and has no other purpose; it allows us in particular to write r and R instead of r_1 and R_1.

We first prove uniqueness. Let R^1 and R^2 be functors $\Phi \to \Psi$ extending r. Pick some trivialization Q of Ψ. For each $a \in M$, we have maps f_1, and f_2:

$$\Phi_a \to^{f_j} \Psi(a, a) \quad j = 1, 2$$

given by

$$f_j(\beta) = Q(b, a) \circ R^j(\beta) \quad (\text{where } b = \partial_1(\beta)) . \tag{2.4}$$

We clearly have $f_1(i(a)) = f_2(i(a)) = i(a)$. Let $\beta \sim \gamma$ in Φ_a, with $b = \partial_1(\beta)$, $c = \partial_1(\gamma)$. Then, for $j = 1, 2$,

$$\begin{aligned} df_j(\beta, \gamma) &= f_j(\gamma) \circ f_j(\beta)^{-1} \\ &= Q(c, a) \circ R^j(\gamma) \circ R^j(\beta)^{-1} \circ Q(b, a)^{-1} \\ &= Q(c, a) \circ R^j(\gamma \circ \beta^{-1}) \circ Q(b, a)^{-1} \\ &= Q(c, a) \circ r(\gamma \circ \beta^{-1}) \circ Q(b, a)^{-1} , \end{aligned} \tag{2.5}$$

since R^j is a functor, and it agrees with r on the near-identity $\gamma \circ \beta^{-1}$. By the uniqueness assertion of Proposition 1.3 (with $\Psi(a, a)$ for G), we conclude that $f_1 = f_2$. Multiplying (2.4) on the left by $Q(b, a)^{-1}$, we get $R^1(\beta) = R^2(\beta)$. So $R^1 = R^2$.

To prove existence of $R: \Phi \to \Psi$, we again pick some trivialization Q of Ψ. We shall for each $a \in M$ use Q to define a map $R_a: \Phi_a \to \Psi$, and the collection of these R_a's will be our R. Let $\omega = \omega_a: (\Phi_a)_{(1)} \to \Psi(a, a)$ be the graph map given by the right hand side of (2.5), i.e.

$$\begin{aligned} \omega(\beta, \gamma) &= Q(c, a) \circ r(\gamma \circ \beta^{-1}) \circ Q(a, b) \\ &= Q(c)^{-1} \circ r(\gamma \circ \beta^{-1}) \circ Q(b) \end{aligned}$$

(writing $Q(b)$ for $Q(a, b)$, and similarly $Q(c)$ for $Q(a, c) = Q(c, a)^{-1}$). We attempt to use Proposition 1.3 to construct a primitive $f = f_a\colon \Phi_a \to \Psi(a, a)$ for this ω, so we wish to prove that ω is curvature free. Let β, γ, and δ be elements of Φ_a with $\beta \sim \gamma \sim \delta \sim \beta$ and with b, c, and d, respectively, as codomain. Then

$$\begin{aligned}\omega(\gamma, \delta) \circ \omega(\beta, \gamma) &= Q(d)^{-1} \circ r(\delta \circ \gamma^{-1}) \circ Q(c) \circ Q(c)^{-1} \circ r(\gamma \circ \beta^{-1}) \circ Q(b) \\ &= Q(d)^{-1} \circ r(\delta \circ \gamma^{-1}) \circ r(\gamma \circ \beta^{-1}) \circ Q(b) \\ &= Q(d)^{-1} \circ r(\delta \circ \beta^{-1}) \circ Q(b) = \omega(\beta, \delta),\end{aligned}$$

where the penultimate equality follows by the assumption that r is curvature free, so that it preserves those composites of near-identities which are near-identities. So ω is curvature free, and by Proposition 1.3, there exists a unique $f\colon \Phi_a \to \Psi(a, a)$ with $f(i(a)) = i(a)$ and with $\mathrm{d}f = \omega$, i.e. with (for $\beta \sim \gamma$)

$$f(\gamma) \circ f(\beta)^{-1} = \omega(\beta, \gamma) = Q(c)^{-1} \circ r(\gamma \circ \beta^{-1}) \circ Q(b). \tag{2.6}$$

For $\gamma \in \Phi_a$ with codomain c, we then put

$$R_a(\gamma) := Q(c) \circ f(\gamma) \in \Psi(a, c). \tag{2.7}$$

We shall prove that the collection of these R_a's is a functor $\Phi \to \Psi$ extending r; (by the uniqueness already proved, it therefore follows also that the choice of the trivialization Q has no influence on the result of the construction).

First we prove that R does extend r. Let $\gamma \in \Phi(a, c)$ be a near-identity. Then with $f = f_a$ and $\omega = \omega_a$ as above

$$\begin{aligned}R(\gamma) = R_a(\gamma) &= Q(c) \circ f(\gamma) \\ &= Q(c) \circ f(\gamma) \circ f(i(a))^{-1} \\ &= Q(c) \circ \mathrm{d}f(i(a), \gamma) \\ &= Q(c) \circ \omega(i(a), \gamma) \\ &= Q(c) \circ Q(c)^{-1} \circ r(\gamma \circ i(a)^{-1}) \circ Q(a) = r(\gamma),\end{aligned}$$

since $Q(a) = Q(a, a) = i(a)$.

We must next prove that R is a functor, i.e. commutes with composition. Let $\beta\colon a \to b$ and $\delta\colon b \to c$ be arrows; we should prove

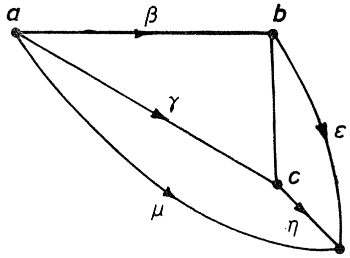
$$R(\delta \circ \beta) = R(\delta) \circ R(\beta). \tag{2.8}$$

We first do the special case where δ is a near-identity, so $R(\delta) = r(\delta)$. Let $\gamma = \delta \circ \beta$, so $\beta \sim \gamma$ in Φ_a. By (2.7) and (2.6), we have

$$\begin{aligned}R(\gamma) \circ R(\beta)^{-1} &= Q(c) \circ f(\gamma) \circ f(\beta)^{-1} \circ Q(b)^{-1} \\ &= Q(c) \circ Q(c)^{-1} \circ r(\gamma \circ \beta^{-1}) \circ Q(b) \circ Q(b)^{-1} \\ &= r(\gamma \circ \beta^{-1}) \\ &= r(\delta) = R(\delta),\end{aligned}$$

and the total equation here is equivalent to (2.8), so the special case is proved.

To prove (2.8) in general, we fix $\beta: a \to b$ in Φ and let $\delta: b \to c$ vary. For $\delta \sim \varepsilon$ in Φ_b, we consider the commutative diagram



where $\gamma = \delta \circ \beta$, $\mu = \varepsilon \circ \beta$, and $\eta = \varepsilon \circ \delta^{-1} = \mu \circ \gamma^{-1}$; η is a near-identity. From the special case already proved (applied to Φ_a and to Φ_b), we get that R transforms the two small triangles involving η into commutative triangles in Ψ. The function $k: \Phi_b \to \Psi(a, a)$, which measures the possible 'lack of functorality', i.e.

$$k(\delta): = R(\delta \circ \beta)^{-1} \circ R(\delta) \circ R(\beta)$$

therefore takes same value on ε as it does on δ, and thus must be constant, by Proposition 1.4. Its value on $\delta = i(b)$ clearly is $i(a)$, so $k(\delta) = i(a)$ for all $\delta \in \Phi_b$, so $R(\delta \circ \beta) = R(\delta) \circ R(\beta)$ $\forall \delta \in \Phi_b$. This proves that R is a functor, giving thus the existence part of the theorem.

3. SOME COROLLARIES

Let M be a set equipped with a reflexive symmetric relation $\sim$, i.e. a subgraph $M_{(1)}$ of $M \times M$. Let Ψ be a groupoid over M. A *connection* on Ψ is a graph map $\nabla: M_{(1)} \to \Psi$ over M. This is a combinatorial paraphrasing of Ehresmann's notion of 'infinitesimal connection in differentiable groupoid', [2], cf. also [6], [7], and [8].

We assume that the set $M \times M$ can be equipped with a reflexive symmetric relation $\approx$ such that

(3.1) $\quad (a, b) \approx (a', b') \Rightarrow (b, a) \approx (b', a')$

(3.2) $\quad (a, b) \approx (a', b') \Rightarrow a \sim a' \quad \text{and} \quad b \sim b'$

(3.3) $\quad a \sim a' \qquad \Rightarrow (a, b) \approx (a', b)$

(3.4) $\quad b \sim b' \qquad \Rightarrow (a, b) \approx (a, b')$.

(Note that in (3.3) and (3.4), we may write $\Leftrightarrow$ instead of $\Rightarrow$, in virtue of (3.2).) One may get one such relation by delcaring $(a, b) \approx (a', b')$ iff: either $a = a'$ and $b \sim b'$, or $b = b'$ and $a \sim a'$. This is clearly the *smallest* relation $\approx$ that satisfies (3.1)–(3.4). One may get another such relation $\approx$ by declaring $(a, b) \approx (a', b')$ iff $a \sim a'$ and $b \sim b'$, which is clearly the *largest* one. In the context of SDG, when $\sim$ is the first neighbourhood of the diagonal, the interesting $\approx$ on $M \times M$ lies strictly between these two extremes, in general.

We then have the following global integration theorem for curvature free connections, first proved in combinatorial form in [8]:

Corollary 3.1. *Assume that the pair* $((M, \sim), \Psi(b, b))$ *admits integration, for all* $b \in M$, *and that* Ψ *is locally trivializable. Then any curvature free connection* $\nabla: M_{(1)} \to \Psi$ *admits a unique integral* (i.e. *an extension to a functor* $M \times M \to \Psi$ *over* M).

Proof. For any reflexive symmetric relation $\approx$ on $M \times M$ satisfying (3.1)–(3.4), the assumptions (2.1)–(2.3) (with $\Phi = M \times M$) are satisfied; and $(a, b) \in \Phi$ is a near-identity iff

$$(a, a) \approx (a, b) \approx (b, b),$$

which by (3.2)–(3.4) is equivalent to $a \sim b$; so $\Phi' = M_{(1)}$. Also, for $a \in M$ fixed, $\Phi_a = \{(a, b) \mid b \in M\} \cong M$, with $\approx$ on Φ_a corresponding to $\sim$ on M, in virtue of (3.2) and (3.4). Thus, the assumptions of the theorem are satisfied, and we get a functor $R: \Phi \to \Psi$ extending $\nabla: \Phi' \to \Psi$. Since $\Phi = M \times M$, this is the desired result.

Of course, the very assumption on $((M, \sim), G)$ admitting integration comes out in turn as a special case of the conclusion of the Corollary, namely by taking $\Psi = M \times G \times M$, which in an evident way is a trivialized groupoid over M, all of whose vertex groups are G.

We finally consider a group G, equipped with a reflexive symmetric relation $\sim$ such that (2.1)–(2.3) are satisfied $\forall \beta, \gamma, \delta \in G$. Let $\mathsf{G} \subset G$ be the set $\{g \in G \mid g \sim e\}$, where e is the neutral element.

Corollary 3.2. *Let* G *and* G *be as above, and assume* H *is a group such that the pair* $((G, \sim), H)$ *admits integration. Then for any map* $r: \mathsf{G} \to H$ *satisfying*

$$r(\beta \,.\, \alpha) = r(\beta) \,.\, r(\alpha) \tag{3.5}$$

whenever α, β, *and* $\beta \,.\, \alpha \in \mathsf{G}$, *there exists a unique group homomorphism* $R: G \to H$ *extending* r.

Proof. This is just the special case of the theorem where $M = N = 1$, the one-point set.

In the context of SDG, this implies one of the fundamental facts of Lie group theory. For, if G and H are Lie groups, and G is connected simply connected, the pair $((G, \sim), H)$ admits integration, by [4] and [9], and a map $R: \mathsf{G} \to H$ with $r(e) = e$ can be proved to be the same thing as a linear map $\varrho: T_eG \to T_eH$ between the respective tangent spaces. Essentially the same calculation as in [4] shows that the condition (3.5) is equivalent to ϱ preserving Lie brackets (T_eG and T_eH being of course Lie algebras); the Corollary thus establishes a bijection between Lie algebra homomorphisms $\varrho: T_eG \to T_eH$, and Lie group homomorphisms $R: G \to H$.

One advantage of the synthetic formulation is that instead of getting this correspondence by some *inducing* process ("R induces ϱ"), we get the correspondence by restriction ("R restricts to r").

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