Zbyněk Kubáček

A generalization of N. Aronszajn's theorem on connectedness of the fixed point set of a compact mapping

*Czechoslovak Mathematical Journal*, Vol. 37 (1987), No. 3, 415–423

Persistent URL: http://dml.cz/dmlcz/102169

Czechoslovak Mathematical Journal, 37 (112) 1987, Praha

# A GENERALIZATION OF N. ARONSZAJN'S THEOREM ON CONNECTEDNESS OF THE FIXED POINT SET OF A COMPACT MAPPING

ZBYNĚK KUBÁČEK, Bratislava

(Received August 23, 1985)

## INTRODUCTION

Š. Belohorec proved in [2] the following theorem (as a generalization of results of N. Aronszajn, see [1]; for other generalizations, see e.g. [4], [6]):

Let $(X, d)$ be a Fréchet space, $M$ a non-empty closed convex subset of $X$, $S$ a continuous mapping such that $S(M) \subset M$ and $\mathrm{cl}\,(S(M))$ is a compact set. Let the following conditions be satisfied:

1. for each $n \in N$ there exists a continuous mapping $S_n$ such that the set $\mathrm{cl}\,(S_n(M))$ is compact and
$$\forall x \in M:\ d(S_n x, Sx) \leqq 1/n\ ;$$

2. the mapping $T_n := I - S_n$ ($I$ is the identity mapping) is an injection and, moreover, there exists a $\delta > 0$ (independent of $n$) such that $T_n(M) \supset \{x \in X;\ d(x, 0) \leqq \delta\}$ holds for each $n \in N$.

Then the set $F$ of fixed points of the mapping $S$ is nonempty, compact and connected.

The aim of this paper is a generalization of that assertion. Under the assumptions mentioned the non-emptiness of the set $F$ is a consequence of the fact that the mapping $I - S$ is 0-closed; this property follows from the compactness of the mapping $S$ (see 2.4) (0-closed mappings are used by Szufla in [5]). Therefore the assertions in this paper are formulated for 0-closed mappings; on the basis of 1.7, 2.4 and 3.6 they can be formulated for compact mappings. The $\cdot$-connectedness is introduced as a generalization of the connectedness in order to get a more objective view of some proofs; by 1.6 and 1.7, the corresponding assertions can be stated for connected sets.

## I. $\cdot$-CONNECTEDNESS

For comparison, before introducing the notion of $\cdot$-connectedness, let us repeat the known definition of connectedness.

**1.1. Definition.** Let $(X, \mathscr{T})$ be a topological space, $A, B$ non-empty subsets of $X$. $A$ and $B$ are called *separated* iff

$$(1) \qquad \exists R \in \mathscr{T} \quad \exists S \in \mathscr{T}: A \subset R \wedge B \subset S \wedge R \cap S \cap (A \cup B) = \emptyset .$$

((1) is equivalent to the condition $\bar{A} \cap B = A \cap \bar{B} = \emptyset$.)

**1.2. Definition.** Let $(X, \mathscr{T})$ be a topological space. A set $C \subset X$ is called *connected* iff it cannot be decomposed into two non-empty separated subsets.

**1.3. Definition.** Let $(X, \mathscr{T})$ be a topological space, $A, B, D$ non-empty subsets of $X$. The sets $A$ and $B$ are called *$D$-separated* iff they both are subsets of $D$ and

$$(2) \qquad \exists R \in \mathscr{T} \quad \exists S \in \mathscr{T}: A \subset R \wedge B \subset S \wedge R \cap S \cap D = \emptyset .$$

$A$ and $B$ are called *disconnected* iff they are $X$-separated.

**1.4. Note.** Comparing (1) and (2) we have: If $A$ and $B$ are $D$-separated and $A \cup B \subset E \subset D$, then $A$ and $B$ are $E$-separated. $A$ and $B$ are separated iff they are $(A \cup B)$-separated.

**1.5. Definition.** Let $(X, \mathscr{T})$ be a topological space, $C \subset D \subset X$. The set $C$ is called *$D$-connected* iff it cannot be decomposed into two non-empty $D$-separated subsets.

**1.6. Note.** 1.4 implies that the set $C$ is connected iff it is $C$-connected. If $C$ is $E$-connected and $E \subset D$, then $C$ is $D$-connected.

**1.7. Lemma.** *Let $(X, \mathscr{T})$ be a Hausdorff topological space and $C$ a compact subset of $X$. Then the following implication holds: If $C$ is $X$-connected, then $C$ is connected.*

Proof. Indirectly. Let $C$ be compact and not connected. The compact set $C$ is closed as $(X, \mathscr{T})$ is a Hausdorff space. $C$ can be written as a union of two closed non-empty disjoint sets $A$ and $B$ as $C$ is closed and not connected. As $A$ and $B$ are closed subsets of the compact set $C$, they are compact. $A$ and $B$ are compact disjoint sets in a Hausdorff topological space, thus they are disconnected ($=X$-separated). As $C = A \cup B$, $C$ is not $X$-connected.

**1.8. Note.** The following assertions can be proved analogously as Lemma 1.7: Each open $X$-connected set in a topological space $(X, \mathscr{T})$ is connected. Each closed $X$-connected set in a normal topological space $(X, \mathscr{T})$ is connected. Each $X$-connected set in a completely normal topological space $(X, \mathscr{T})$ is connected.

**1.9. Lemma.** *Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces, $G: X \to Y$ a continuous mapping, $C \subset D \subset X$ and $C$ a $D$-connected set. Then the set $G(C)$ is $G(D)$-connected.*

Proof. This assertion can be proved by contradiction. If the set $G(C)$ is not $G(D)$-

connected, then there exist non-empty open sets $U \in \mathscr{S}$, $V \in \mathscr{S}$ such that

$$U \cap G(C) \neq \emptyset\,, \quad V \cap G(C) \neq \emptyset\,, \quad U \cap V \cap G(D) = \emptyset\,, \quad G(C) \subset U \cup V\,.$$

As the mapping $G$ is continuous, the sets $G^{-1}(U)$, $G^{-1}(V)$ are open (in the topology $\mathscr{T}$). Moreover, $C \cap G^{-1}(U) \neq \emptyset$, $C \cap G^{-1}(V) \neq \emptyset$, $G^{-1}(U) \cap G^{-1}(V) \cap D = \emptyset$, $C \subset G^{-1}(U) \cup G^{-1}(V)$, i.e., $C$ is not $D$-connected, which is a contradiction.

**1.10. Note.** According to Note 1.6 the set $C$ is connected iff it is $C$-connected. If the set $C$ in Lemma 1.9 is connected, then the set $G(C)$ is $G(C)$-connected, i.e., $G(C)$ is connected.

## II. 0-CLOSED MAPPINGS

**2.1. Definition.** Let $(X, \mathscr{T})$ be a topological space, $M$ a non-empty subset of $X$. A mapping $S$ from $M$ into $X$ is called *compact* iff it is continuous and the set $\operatorname{cl}(S(M))$ is compact.

**2.2. Definition.** Let $(X, \mathscr{T})$ be a topological space, $M$ a non-empty closed subset of $X$, and $a \in X$. A mapping $S$ from $M$ into $X$ is called *a-closed* iff $a \in \operatorname{cl}(S(V)) \Rightarrow$ $\Rightarrow a \in S(V)$ for each closed subset $V$ of $M$.

**2.3. Definition.** An ordered triad $(X, +, \mathscr{T})$ such that $(X, +)$ is a group, $(X, \mathscr{T})$ a topological space and the mappings $+ : X \times X \to X$ $((a, b) \mapsto a + b)$ and $- :$ $X \to X$ $(a \mapsto -a)$ are continuous, is called a *topological group.*

**2.4. Lemma.** *Let $(X, +, \mathscr{T})$ be a topological group, $M$ a non-empty closed subset of $X$, $S \colon M \to X$ a compact mapping. Then the mapping $T := I - S$ ($I$ is the identity mapping) is 0-closed.*

Proof. Let $V$ be a closed subset of $M$, $0 \in \operatorname{cl}(T(V))$. Then there exists a net $\{y_n;\ n \in I\}$ such that $\lim y_n = 0$ and $y_n \in T(V)$ for each $n \in I$. As $T = I - S$, by virtue of the axiom of choice there exists a net $\{x_n;\ n \in I\}$ such that $x_n \in V$ and $y_n =$ $= x_n - Sx_n$ for each $n \in I$. The set $\operatorname{cl}(S(V))$ is compact as a closed subset of the compact set $\operatorname{cl}(S(M))$ ($\operatorname{cl}(S(M))$ is compact because $S$ is a compact mapping). As $\{Sx_n;$ $n \in I\}$ is a net of points of the compact set $\operatorname{cl}(S(V))$, there exists a subnet $\{Sx_m; m \in J\}$ which converges to a point $z \in \operatorname{cl}(S(V))$. As $\{y_m;\ m \in J\}$ (where $y_m = x_m - Sx_m$) is a subnet of the convergent net $\{y_n;\ n \in I\}$, we have $\lim y_m = 0$. As the operation $+$ is continuous, we get

$$\lim x_m = \lim (y_m + Sx_m) = \lim y_m + \lim Sx_m = 0 + z = z\,.$$

As $\lim x_m = z$ and $\{x_m;\ m \in J\}$ is a net of points of $V$, we have $z \in \overline{V}$. As the set $V$ is closed, $z \in V$. The mapping $T := I - S$ is continuous (because $I$ and $S$ are continuous mappings and $(X, +, \mathscr{T})$ is a topological group), thus

$$Tz = T(\lim x_m) = \lim Tx_m = \lim (x_m - Sx_m) = \lim y_m = 0\,,$$

i.e. $0 \in T(V)$.

## III. CONNECTEDNESS AND .-CONNECTEDNESS OF SETS OF FIXED POINTS

**3.1. Lemma.** *Let $(X, \mathscr{T})$ be a topological space, $M$ a nonempty closed subset of $X$, $a \in X$, $T: M \to X$ an $a$-closed mapping, let $\mathscr{B}$ denote a neighbourhood base of the point $a$. Let the system of sets $\{F_i;\ i \in I\}$ satisfy the following conditions:*

(1) $\forall i \in I\colon F_i \neq \emptyset$;

(2) $\forall i \in I\colon F_i \subset M$;

(3) $\forall U \in \mathscr{B}\ \exists i \in I\ \forall x \in F_i\colon Tx \in U$.

*Then the set $F := \{x \in M;\ Tx = a\}$ is non-empty and*

$$(*) \qquad \forall G \in \mathscr{T}\colon F \subset G \quad \exists i \in I\colon F_i \subset G\,.$$

Proof. It follows from (1), (2) and (3) that

$$\forall U \in \mathscr{B} \quad \exists x_U \in M\colon Tx_U \in U\,.$$

Thus $a \in \operatorname{cl}(T(M))$. As the mapping $T$ is $a$-closed, we have $a \in T(M)$, and consequently, the set $F$ is non-empty.

The statement $(*)$ can be proved by contradiction. Let

$$(**) \qquad \exists G \in \mathscr{T}\colon F \subset G \quad \forall i \in I \quad \exists x_i \in F_i\colon x_i \notin G$$

and $S := \{x_i;\ i \in I\}$. It follows from $(**)$ and (3) that $a \in \operatorname{cl}(T(\bar{S}))$. As the mapping $T$ is $a$-closed, we have $a \in T(\bar{S})$. Thus

$$(+) \qquad \bar{S} \cap F \neq \emptyset\,.$$

On the other hand, $S \subset M - G$. As $M$ is a closed and $G$ an open set, $\bar{S} \subset M - G$. By the assumption $F \subset G$, thus $\bar{S} \cap F = \emptyset$, which is a contradiction with $(+)$.

**3.2. Theorem.** *Let $(X, \mathscr{T})$ be a topological space, $a \in X$, $M$ a non-empty closed subset of $X$, $T: M \to X$ an $a$-closed mapping, let $\mathscr{B}$ denote a neighbourhood base of the point $a$, $F := \{x \in M;\ Tx = a\}$. Let there exist a system of sets $\{F_i;\ i \in I\}$ satisfying the following conditions:*

(i) $\forall i \in I\colon F_i \neq \emptyset$;

(ii) $\forall i \in I\colon F_i \subset M$;

(iii) $\forall U \in \mathscr{B}\ \exists i \in I\ \forall x \in F_i\colon Tx \in U$;

(iv) *for each $i \in I$ the set $F_i$ is $M$-connected;*

(v) $\forall i \in I\colon F \subset F_i$.

*Then the set $F$ is non-empty and $M$-connected.*

Proof. By Lemma 3.1 the set $F$ is non-empty. Next we shall proceed by contradiction. Let the set $F$ be not $M$-connected. Then there exist non-empty open sets $G$, $H$ such that

$$F \subset G \cup H\,, \quad F \cap G \neq \emptyset\,, \quad F \cap H \neq \emptyset\,, \quad G \cap H \cap M = \emptyset\,.$$

Let $E := G \cup H$. By Lemma 3.1 there exists $i \in I$ such that $F_i \subset E$. Then

$$F_i \subset E = G \cup H\,, \quad F_i \cap G \supset F \cap G \neq \emptyset\,, \quad F_i \cap H \supset F \cap H \neq \emptyset\,,$$
$$G \cap H \cap M = \emptyset\,,$$

i.e., $F_i$ is not $M$-connected, which is a contradiction with (iv).

**3.3. Note.** Before formulating Theorem 3.4 let us mention the following facts:

Let $(X, +, \mathcal{T})$ be a topological group. Then

(1) there exists a neighbourhood base of the point 0 consisting of symmetric sets;

(2) if $\mathcal{B}$ denotes a neighbourhood base of the point 0, then

$$\forall U \in \mathcal{B} \quad \exists V \in \mathcal{B}\colon V + V \subset U\,.$$

Proof. The first statement is a consequence of the continuity of the mapping $-$. (If $W$ is a neighbourhood of 0, then $-W$ and consequently $W \cap (-W)$ are neighbourhoods of 0 as well. The set $W \cap (-W)$ is symmetric.) The second follows from the continuity of the mapping $+$ at the point $(0, 0)$.

**3.4. Theorem.** *Let $(X, +, \mathcal{T})$ be a topological group, $M$ a non-empty closed subset of $X$, $S\colon M \to X$ a mapping such that the mapping $T := I - S$ ($I$ is the identity mapping) is 0-closed.*

*Let there exist a neighbourhood base $\mathcal{B}$ of the point 0 consisting of symmetric sets and possessing the following property: for each set $U \in \mathcal{B}$ there exists a mapping $T_U\colon M \to X$ such that*

(1) $\forall x \in M\colon Tx - T_U x \in U$;

(2) *the set $E_U := \{x \in M;\ T_U x \in U\}$ is non-empty and $M$-connected.*

*Then the set $F$ of fixed points of $S$ is non-empty and $M$-connected.*

Proof. We shall demonstrate that the system $\{E_U; U \in \mathcal{B}\}$ satisfies the conditions of Theorem 3.2. Let $U \in \mathcal{B}$, $x \in F$ (i.e. $Tx = 0$), then by (1)

$$T_U x = T_U x - 0 = T_U x - Tx = -(Tx - T_U x) \in -U\,.$$

As the base $\mathcal{B}$ consists of symmetric sets, $U = -U$; thus

$$\forall U \in \mathcal{B} \quad \forall x \in F\colon T_U x \in U\,,$$

i.e.

$$\forall U \in \mathcal{B}\colon F \subset E_U\,.$$

This means that the system $\{E_U;\ U \in \mathcal{B}\}$ satisfies condition (v) from Theorem 3.2. Condition (ii) is satisfied evidently; the validity of conditions (i) and (iv) follows from (2). Let $x \in E_V$, then

$$(*) \qquad Tx = (Tx - T_V x) + T_V x \in V + V\,.$$

As $(X, +, \mathcal{T})$ is a topological group, with respect to Note 3.3

$$(**) \qquad \forall U \in \mathcal{B} \quad \exists V \in \mathcal{B}\colon V + V \subset U\,.$$

Taking into account $(*)$ and $(**)$ we get

$$\forall U \in \mathscr{B} \quad \exists V \in \mathscr{B} \quad \forall x \in E_V: Tx \in U,$$

i.e., the system $\{E_V; V \in \mathscr{B}\}$ satisfies condition (iii) from Theorem 3.2. Now Theorem 3.2 yields that $F$ is a non-empty $M$-connected set.

**3.5. Note.** If the base $\mathscr{B}$ in Theorem 3.4 (and in Theorems 3.7, 3.8 and 3.15, too) does not consist of symmetric sets, it is necessary instead of (1) to suppose

(1') $\forall x \in M: Tx - T_U x \in U$,
$\forall x \in M: T_U x - Tx \in U$.

**3.6. Note.** Before formulating Theorem 3.7 it is necessary to realize the following: *Let $(X, \mathscr{T})$ be a topological space, $M$ a non-empty closed subset of $X$, $S: M \to X$ a compact mapping. Then the set $F$ of fixed points of $S$ is compact.*

Proof. The set $F$ is closed in the topology $\mathscr{T}/M$. As $M$ is a closed set, $F$ is closed. As $F \subset S(M) \subset \mathrm{cl}(S(M))$, $F$ is a closed subset of the compact set $\mathrm{cl}(S(M))$, consequently, $F$ is a compact set.

**3.7. Theorem.** *Let $(X, +, \mathscr{T})$ be a Hausdorff topological group, $M$ a non-empty closed subset of $X$, $S: M \to X$ a compact mapping. Let there exist a neighbourhood base $\mathscr{B}$ of the point 0 consisting of symmetric sets possessing the following property: for each set $U \in \mathscr{B}$ there exists a mapping $T_U: M \to X$ such that*
(1) $\forall x \in M: Tx - T_U x \in U$;
(2) *the set $E_U := \{x \in M; T_U x \in U\}$ is non-empty and $M$-connected.*
*Then the set $F$ of fixed points of $S$ is non-empty, compact and connected.*

Proof. As $S$ is a compact mapping, the mapping $T := I - S$ is 0-closed in accordance with Lemma 2.4. By Theorem 3.4 the set $F$ is non-empty and $M$-connected. The set $F$ is compact by Note 3.6. As $(X, \mathscr{T})$ is a Hausdorff topological space and $F$ is a compact $M$-connected set, by Lemma 1.7 $F$ is connected. Consequently, $F$ is a non-empty compact connected set.

**3.8. Theorem.** *Let $(X, +, \mathscr{T})$ be a topological group, $M$ a non-empty closed subset of $X$, $S: M \to X$a mapping such that the mapping $T := I - S$ is 0-closed. Let there exist a neighbourhood base $\mathscr{B}$ of the point 0 consisting of symmetric sets possessing the following property: for each $U \in \mathscr{B}$ there exists a mapping $T_U: M \to X$ such that*
(1) $\forall x \in M: Tx - T_U x \in U$;
(2) *there exists a set $Z_U \subset X$ such that $U \subset Z_U$, $U$ is $Z_U$-connected, the equation $T_U y = x$ has a unique solution $y \in M$ for a given $x \in Z_U$ and the mapping $V_U: Z_U \to M$ assigning the point $y$ such that $T_U y = x$ to the point $x$ is continuous. Then the set $F$ of fixed points of the mapping $S$ is non-empty and $M$-connected.*

Proof. As $U$ is $Z_U$-connected by (2), the set $V_U(U)$ is $V_U(Z_U)$-connected in virtue of

Lemma 1.9. As $V_U(Z_U) \subset M$, the set $V_U(U)$ is $M$-connected by Note 1.6. Consequently, the system of sets $\{V_U(U); U \in \mathscr{B}\}$ satisfies condition (iv) from Theorem 3.2. Further, let $U \in \mathscr{B}$, $x \in F$ (i.e. $Tx = 0$), then (taking into account the symmetry of the set $U$; see Note 3.5, too)

$$T_U x = -(Tx - T_U x) \in -U = U .$$

On the set $U$ the mapping $V_U$ is defined; as $T_U x \in U$, the point $V_U(T_U x)$ exists. In accordance with the definition of the mapping $V_U$ we have $x = V_U(T_U x)$, thus $x \in V_U(U)$. In this way it is proved that

$$\forall U \in \mathscr{B}: F \subset V_U(U) ,$$

i.e., the system $\{V_U(U); U \in \mathscr{B}\}$ satisfies condition (v) from Theorem 3.2. With respect to (2), condition (i) is satisfied, the validity of (ii) is evident. The proof that condition (iii) is satisfied is analogous as in Theorem 3.4.

**3.9. Lemma.** *Let $(X, +, \mathscr{T})$ be a Hausdorff topological group, $M$ a non-empty subset of $X$, $S: M \to X$ a compact mapping. Let $T := I - S$. Then the image $T(N)$ of a closed set $N \subset M$ is a closed set.*

Proof. See [3, p. 100]; it is necessary to note that in that proof it suffices to consider a set $V$ such that $V + V \subset U$ instead of the set $(1/2)\, U$; the local convexity (it is an assumption in the quoted theorem) is not used in the proof. Consequently, it suffices to assume that $X$ is a Hausdorff topological group.

**3.10. Lemma.** *Let $(X, +, \mathscr{T})$ be a Hausdorff topological group, $M$ a non-empty closed subset of $X$, $S: M \to X$ a compact mapping, $U$ a non-empty subset of $X$, $P := I - S$. Let there exist a unique solution $y \in M$ of the equation $Py = x$ for each $x \in U$. Then the mapping $V: U \to M$ assigning the point $y$ such that $Py = x$ to the point $x$ is continuous.*

Proof. It is necessary to demonstrate that for each set $A \subset M$ closed in the topology $\mathscr{T}/M$ the set $V^{-1}(A)$ is closed in the topology $\mathscr{T}$. Let $A \subset M$ be a set closed in the topology $\mathscr{T}/M$. As $M$ is a closed set the set $A$ is closed, too. Further, $V^{-1}(A) = P(A)$. By Lemma 3.9 the set $P(A)$ is closed.

**3.11. Definition.** A topological space $(X, \mathscr{T})$ is said to be *locally connected* if every point has a neighbourhood base consisting of connected sets.

**3.12. Note.** Evidently, in a topological group the existence of a neighbourhood base of the point 0 consisting of connected sets is sufficient for the local connectedness.

**3.13. Note.** As in a topological vector space there exists a neighbourhood base of the point 0 consisting of balanced sets and every balanced set is connected, every topological vector space is locally connected.

**3.14. Note.** It is not difficult to prove the following assertion: Let $(X, +, \mathscr{T})$ be a locally connected topological group. Then there exists a neighbourhood base of the point 0 consisting of symmetric connected sets.

**3.15. Theorem.** *Let $(X, +, \mathcal{T})$ be a Hausdorff locally connected topological group, $M$ a non-empty closed subset of $X$, $S$: $M \to X$ a compact mapping, and let $\mathcal{B}$ denote the neighbourhood base of the point 0 consisting of symmetric connected sets. Let the following conditions be satisfied:*

(1) *for each set $U \in \mathcal{B}$ there exists a compact mapping $S_U$: $M \to X$ such that*

$$\forall x \in M: Sx - S_U x \in U$$

*(see also Note 3.5);*

(2) *the equation $T_U y = x$ has a unique solution $y \in M$ for each $x \in U$ (where $T_U := I - S_U$).*

*Then the set $F$ of fixed points of the mapping $S$ is non-empty, connected and compact.*

Proof. The mapping $T := I - S$ is 0-closed by Lemma 2.4. The mappings $V_U$: $U \to M$ assigning the point $y$ such that $T_U y = x$ to the point $x$ are continuous for each $U \in \mathcal{B}$ by Lemma 3.10. The set $U$ is connected, i.e., it is $U$-connected by Note 1.6. It suffices to define $Z_U := U$ to satisfy all the conditions of Theorem 3.8, by which the set $F$ is non-empty and $M$-connected. The set $F$ is compact by Note 3.6. As $F$ is a $X$-connected compact set in a Hausdorff topological space, the set $F$ is connected by Lemma 1.7. Consequently, $F$ is a non-empty compact connected set.

**3.16. Note.** As in every topological vector space there exists a neighbourhood base of the point 0 consisting of balanced sets and every balanced set is connected and symmetric, the following consequence of Theorem 3.16 is true:

*Let $(X, +, \cdot, \mathcal{T})$ be a Hausdorff topological vector space, $M$ a non-empty closed subset of $X$, $S$: $M \to X$ a compact mapping, $T$ a mapping defined as $T := I - S$, let $\mathcal{B}$ denote the neighbourhood base of the point 0 consisting of balanced sets. Let the following conditions be satisfied:*

(1) *for each set $U \in \mathcal{B}$ there exists a compact mapping $S_U$: $M \to X$ such that*

$$\forall x \in M: Sx - S_U x \in U ;$$

(2) *the equation $T_U y = x$ (where $T_U := I - S_U$) has a unique solution $y \in M$ for each $x \in U$.*

*Then the set $F$ of fixed points of the mapping $S$ is non-empty, compact and connected.*

**Acknowledgement.** The autor thanks Professor V. Šeda for his generous consultations and aid in preparing this paper.

### *References*

[1] *Aronszajn, N.:* Le correspondant topologique de l'unicité dans la théorie des équations différentielles. Annals of Mathematics, *43*, 1942, 730—738.

[2] *Belohorec, Š.:* Generalization of a certain theorem of N. Aronszajn and its application in the theory of functional differential equations (in Slovak). Manuscript.

[3] *Deimling, K.:* Nichtlineare Gleichungen und Abbildungsgrade. Berlin, Springer-Verlag 1974.

[4] *Dubois, J.* and *Morales, P.:* On the Hukuhara-Kneser property for some Cauchy problems in locally convex topological vector spaces. In: Ordinary and Partial Differential Equations, Proceedings, Dundee, Scotland 1982. Berlin, Springer-Verlag 1982, p. 162—170.

[5] *Szufla, S.:* Sets of fixed points of nonlinear mappings in function spaces. Funkcialaj Ekvacioj, *22*, 1979, 121—126.

[6] *Vidossich, G.:* On the structure of the set of solutions of nonlinear equations. Journal of Mathematical Analysis and Applications, *34*, 1971, 602—617.

*Author's address:* 842 15 Bratislava, Mlynská dolina (Matematicko-fyzikálna fakulta UKo) Czechoslovakia.