Pavel Křivka

Dimension of the sum of several copies of a graph

Czechoslovak Mathematical Journal, Vol. 35 (1985), No. 3, 347–354

Persistent URL: http://dml.cz/dmlcz/102025

Czechoslovak Mathematical Journal, 35 (110) 1985, Praha

DIMENSION OF THE SUM OF SEVERAL COPIES OF A GRAPH

PAVEL KŘIVKA, Pardubice

(Received July 22, 1982)

1. PRELIMINARIES

1.1. Conventions and notation. We shall not repeat the definitions of the basic notions (graph, homomorphism, embedding, spanned subgraph, degree of a vertex — $d_G(x)$, cardinality of $V(G)$ — $|G|$, λ-partite graph, complete graph — K_n, complete λ-partite graph — $K_{r_1 \dots r_\lambda}$, cycles C_m). We also suppose as already known the operations of sum (the sum of k copies of G being denoted as $k \,.\, G$), cartesian product, categorial product. Nevertheless, let us mention some of the less current notation: Let X be a set. Then we write $P(X) = \{Y\colon Y \in X\}$ and $P_{\leqq d}(X) = \{Y \subset X\colon |Y| \leqq d\}$. Let $\Gamma\colon X \to P(X)$. Then $\bar{\Gamma}\colon X \to P(X)$ is defined as follows:

$$y \in \bar{\Gamma}(x) \Leftrightarrow x \in \Gamma(y) .$$

Obviously, Γ corresponds to a graph iff $\Gamma = \bar{\Gamma}$ and $x \notin \Gamma(x)$. Such Γ will be sometimes denoted by Γ_G where G is the associated graph, and will be called a *graph-mapping*.

The upper integral approximation of a real number r will be denoted by $(r)^+$.

The symbol $I(N \times d)$ denotes a matrix $N \times d$, the i-th row of which is the vector $i, \dots, i$.

1.2. Some basic facts about the dimension and encodings. The dimension of G defined as the least natural number n such that G can be embedded into N^n (where the N^n is the n-th categorial power of the complete graph whose vertices are all natural numbers) leads in a natural way to an encoding, i.e. associating the vertices $x \in V(X)$ with distinct vectors $v(x) = v_1(x), \dots, v_n(x)$ in natural numbers so that for $\{x, y\} \in E(G)$ the vectors $v(x)$ and $v(y)$ do not meet and for $\{x, y\} \notin E(G)$ they meet in at least one coordinate.

For a graph G define the *strong matching number* $\mu(G)$ as the maximal cardinality of an $X \subset V(G)$ such that there exists a one-to-one mapping φ of X onto a $Y \subset V(G)$, disjoint from X, for which

$$\{x, y\} \in E(G) , \quad x \in X \quad \text{and} \quad y \in Y \quad \text{iff} \quad y = \varphi(x) .$$

Then $\dim(G) \geqq \log_2 \mu(G)$ whenever $\mu(G) \geqq 3$ [4].

Another graph characteristic is $\bar{\mu}(G)$: Let there be distinct $x^1, \ldots, x^k$ in $V(G)$ such that for some $y^1, \ldots, y^k \in V(G)$,

$$\{x^i, y^i\} \in E(G) \quad \text{and} \quad \{x^i, y^j\} \notin E(G) \quad \text{for} \quad i < j .$$

This k is then denoted by $\bar{\mu}(G)$ and we have $\dim(G) \geqq \log_2 \bar{\mu}(G)$ [4].

For C_{2n} we can put $x^i = i - 1$, $y^i = i$ ($i = 1, \ldots, n - 2$), consequently $\bar{\mu}(C_{2n}) \geqq \geqq 2(n - 1)$ i.e. $\dim C_{2n} \geqq \log_2(n - 1) + 1$. Actually, $\dim C_{2n} = (\log_2(n - 1))^+ + + 1$ for $n > 2$ [4].

We shall also need the following propositions [4]:

$$\dim(k \,.\, K_2) = (\log_2 k)^+ + 1 , \quad \dim(K_1 + K_n) = n .$$

Finally, recall that, by [3], $\dim(K_n + K_n) = n$ (actually, we have proved more – that $\dim(K_n + K_m) = \max(m, n)$ – for further generalization see 3.5.a). Besides, it is clear that for a spanned subgraph H of G, $\dim(H) \leqq \dim(G)$, $\dim K_n = 1$, $\dim K_{r_1 \ldots r_\lambda} = 2$ (in the first coordinate there are the same numbers for the vertices of one colour, in the second there are the numbers $1, \ldots, \sum r_i$). Let us also remark that the homomorphism Theorem 2.1 from [3] holds naturally for any number of copies.

2. GENERALIZED LATIN RECTANGLES

2.1. In the paper [3] we defined the Generalized Latin Rectangle corresponding to some $\Gamma: X \to P_{\leqq d}(X)$ (GLR Γ) as a matrix $X \times d$ with columns $p_1, \ldots, p_d$ (p_i being permutations of X) such that for all $x \in X$,

$$\Gamma(x) \subset \{p_1(x), \ldots, p_d(x)\} = v(x) .$$

In Lemma 2.2 in [3] we have proved that such GLR Γ exists whenever Γ is a graph-mapping. We call two GLR Γ P and Q (corresponding to the same Γ) *independent* iff both $Q^{-1} \circ P$ and $P^{-1} \circ Q$ are again GLR Γ (by composition we mean the composition of the column permutations).

2.2. Lemma. *Let Q and P be two* GLR Γ. *Denote $F(x) = \{y \in X: v(y)$ in Q meets $v(x)$ in $P\}$. Then $Q^{-1} \circ P$ is again* GLR Γ *iff*

$$x \in X \Rightarrow \Gamma(x) \subset F(x) .$$

Proof. We have $F(x) = (Q^{-1} \circ P)(x) : y \in F(x)$ iff there exists a column s such that $p_s(x) = q_s(y) \Leftrightarrow y = (q_s^{-1} \circ p_s)(x) \Leftrightarrow y \in (Q^{-1} \circ P)(x)$. □

2.3. Corollary. a) *If Γ is a graph-mapping then $Q^{-1} \circ P$ is* GLR Γ *iff*

$$P^{-1} \circ Q \quad \textit{is} \quad \text{GLR}\, \Gamma .$$

b) *If Γ is a graph G mapping then P and Q are independent iff*

$$\{x, y\} \in E(G) \Rightarrow v(x) \;\; \text{in } P \text{ meets } v(y) \text{ in } Q .$$

c) *Two latin squares P and Q of the same order are independent iff each row of P meets each row of Q.* □

2.4. Two matrices P and Q of the same dimension, their elements being integers from 1 to n, are called *orthogonal* if for every pair a, b ($1 \leqq a, b \leqq n$) there exists at most one pair (i, j) such that $P_{i,j} = a$ and $Q_{i,j} = b$ [1].

Lemma. *Two* GLR Γ $(N \times m)$ *P and Q are orthogonal if and only if* $Q \circ P^{-1}$ *is* LR.

Proof. Let P, Q be not orthogonal, i.e. there exist $a, b \leqq N$ and two different pairs (i_1, j_1) and (i_2, j_2) such that $P_{i_1,j_1} = P_{i_2,j_2} = a$ and $Q_{i_1,j_1} = Q_{i_2,j_2} = b$. Consequently,

$$p_{j_1}(i_1) = p_{j_2}(i_2) = a \quad \text{and} \quad q_{j_1}(i_1) = q_{j_2}(i_2) = b\,.$$

This means

$$i_1 = (p_{j_1})^{-1}(a)\,, \quad i_2 = (p_{j_2})^{-1}(a)\,.$$

After substituting into the second equation we get

$$(q_{j_1} p_{j_1}^{-1})(a) = (q_{j_2} p_{j_2}^{-1})(a) = b\,.$$

Thus in the a-th row of the matrix $Q \circ P^{-1}$ the same element b is in both columns j_1, j_2. By reversing the reasoning we obtain the other implication. □

2.5. Corollary. *The number of independent* LS *of order n equals the number of orthogonal* LS *of order n.*

Proof. Let $Q_1, \ldots, Q_t$ be independent LS of order n. Thus $Q_i^{-1} \circ Q_j$ for $i \neq j$ is again LS. It is easy to prove that if P is LS then P^{-1} si again LS so that for $Q_1^{-1}, \ldots$ $\ldots, Q_t^{-1}$ we have that $Q_i^{-1} \circ (Q_j^{-1})^{-1} = Q_i^{-1} \circ Q_j$ is again LS and according to the previous lemma $Q_1^{-1}, \ldots, Q_t^{-1}$ are orthogonal LS of order n. If we start with orthogonal LS it is easy to show that their inversion is a system of independent LS. □

We shall denote the number of orthogonal LS of order n by $N(n)$. Let us recall that $N(n) \leqq n - 1$, with equality holding whenever n is a prime power, $N(12) \geqq 5$, $n \neq 2, 6 \Rightarrow N(n) \geqq 2$. If $n = p_1^{r_1} \ldots p_k^{r_k}$ is the prime decomposition of n then $N(n) \geqq$ $\geqq \min (p_i^{r_i} - 1)$ [1].

3. BASIC FACTS

3.1. Theorem. a) *Let G be a graph such that there exist* $t - 1$ *independent* GLR Γ_G $(|G| \times \Delta(G))$. *Then*

$$\dim (t \,.\, G) \leqq \dim G + \Delta(G)\,.$$

b) *For a given k, let m be the minimal number such that there exist* $k - 1$ *independent* GLR Γ_G $(|G| \times m)$. *Then*

$$\dim (k \,.\, G) \leqq \dim G + m\,.$$

Proof is similar to that of Theorem 2.3 in [3]. There we have added the matrix $I(|G| \times \Delta(G))$ to one of the encodings of G and the corresponding GLR Γ_G to the other. Now, we have t (or k) encodings of G, to one of them we add the matrix $I(|G| \times \Delta(G))$ (or $I(|G| \times m)$, respectively) and to each of the rest one of the $t - 1$ independent GLR Γ_G (or one of the $k - 1$ independent GLR Γ_G $(|G| \times m)$). □

3.2. Lemma. *Let G be a graph, k a given integer. There exist $k - 1$ independent* GLR Γ_G $(|G| \times \Delta(G)(\log_t k)^+)$ *where t is the maximal number such that there exist $t - 1$ independent* GLR Γ_G $(|G| \times \Delta(G))$.

Proof. Let $r = (\log_t k)^+$. We construct $t^r - 1 \geqq k - 1$ independent GLR Γ_G: Let us denote $t - 1$ independent GLR Γ_G $(|G| \times \Delta(G))$ by $P_1, \ldots, P_{t-1}$ and let $P_0 = I(|G| \times \Delta(G))$. Let

$$M = \{v: v = (v_1, \ldots, v_r),\ 0 \leqq v_i \leqq t - 1,\ \text{at least one } v_i \neq 0\}$$

$(|M| = t^r - 1)$ and for $v \in M$ let P_v be the matrix $|G| \times \Delta(G) . r$ given by $P_v = P_{v_1} P_{v_2} \ldots P_{v_r}$ (P_{v_i} are just written down one after another).

It is easy to prove that each P_v is GLR Γ_G and that they are pairwise independent. □

3.3. Corollary. *Let G be a graph with $t - 1$ independent* GLR Γ_G $(|G| \times \Delta(G))$, *k an integer. Then*

$$\dim (k . G) \leqq \dim G + \Delta(G) (\log_t k)^+ .$$

In paticular, $\dim (k . G) \leqq \dim G + \Delta(G) (\log_2 k)^+$. □

3.4. Proposition. $\dim (k . K_n) = n$ *iff there exist at least $k - 1$ independent* LS *of order n.*

Proof. $\Leftarrow$: It is clear that every LS is an encoding of K_n and $k - 1$ independent LS form an encoding of the sum $(k - 1) . K_n$. Besides, $I(n \times n)$ is also an encoding of K_n and each row of this matrix meets each row of each LS.

$\Rightarrow$: As each row of one encoding of K_n meets each row of another encoding of K_n, two rows of one encoding never meet, at each place some two of the rows are bound to meet (for each pair of K_n). So we can suppose that each column is a permutation of the numbers $1, \ldots, n$. If we transform each encoding in such a way that the first columns are identical permutations, we can easily check that these encodings are independent LS.

3.5. Corollary. a) *For any integer n,* $\dim ((n + 1) . K_n) > n$.

b) *When n is a prime power then* $\dim (n . K_n) = n$.

c) *If $N(n)$ is the number of* OLS *of order n, then*

$$\dim ((N(n) + 1) . K_n) = n\,, \quad \dim ((N(n) + 2) . K_n) > n\,.$$

d) $\dim (K_{n_1} + \ldots + K_{n_p}) = \max (n_1, \ldots, n_p)$ *iff* $p \leqq N\ (\max (n_1, \ldots, n^p)) + 1$. □

3.6. Corollary. a) *Let* $n = p_1^{r_1} \dots p_k^{r_k}$ *be the prime decomposition of* n, *let* $a = \min(p_i^{r_i} - 1)$. *Then* $\dim(a . K_n) = n$.
b) $\dim(6 . K_{12}) = 12$.
c) *For* $n \neq 2, 6$, $\dim(3 . K_n) = n$.

Proof follows from the properties of OLS. □

3.7. Proposition. a) *Let n be a prime power, k an integer, then*

$$\dim(k . K_n) \leqq 1 + (n - 1)(\log_n k)^+ .$$

b) *For* $n \neq 2, 6$ $\dim(k . K_n) \leqq 1 + (n - 1)(\log_3 k)^+$.
c) $\dim(k . K_n) \leqq 1 + (n - 1)(\log_{N(n)+1} k)^+$.

Proof. If we define $\Gamma_{K_n}(x) = X \setminus \{x\}$ we can complete each GLR Γ_{K_n} is a unique way to an LS and vice versa, i.e. the number of independent GLR Γ_{K_n} equals the number of independent (orthogonal) LS. The rest follows from Corollary 3.3. □

3.8. Lemma. *Let G be a graph, k an integer. Let there exist t independent* GLR Γ_G $(|G| \times m)$. *Then there exist t independent* GLR Γ_H $(k|G| \times m)$ *where* Γ_H *is a graph-mapping corresponding to the sum* $k . G$.

Proof. If p is a permutation of the set $1, \dots, N$ $(N = |G|)$ let $t . p$ be a permutation of the set $1, \dots, t . N$ given by

$$j . N \leqq i \leqq (j + 1) . N \Rightarrow (t . p)(i) - j . N = p(i - j . N) .$$

If $P = \{p_1, \dots, p_m\}$ is a GLR Γ_G then $t . P = \{t . p_1, \dots, t . p_m\}$ is a GLR Γ_H: It suffices to number the vertices of $k . G$ in such a way that $\Gamma_H(i) \subset (t . P)(i)$. One copy is already numbered (P is GLR Γ_G); let us number the others in such a way that if i and j are numbers of the same vertex in different copies of G then i and j are congruent mod N. Now, for $j . N < i \leqq (j + 1) . N$ we have $\Gamma_H(i) = \Gamma_G(i - j . N) + J . N$ and since $(t . P)(i) = P(i - j . N) + j . N$, we get $(t . P)(i) \supset \Gamma_H(i)$. Besides, if P and Q are independent GLR Γ_G then $t . P$ and $t . Q$ are independent GLR Γ_H as $(t . P)^{-1} = t . (p)^{-1}$ and $(t . p)(t . q) = t(p \circ q)$ yields $(t . P)^{-1} \circ (t . Q) = t(P^{-1} \circ Q)$. □

3.9. Proposition. a) *Let* m, n *be integers,* $m > n$, $N(m) > N(n)$. *Then*

$$\dim((N(m) + 1) . K_n) \leqq m .$$

b) *Let* $n, n_1, \dots, n_k$ *be integers,* $n \leqq n_i$, *let* $m = \prod_{i=1}^{k} (N(n_i) + 1)$. *Then* $\dim(m . K_n) \leqq 1 + \sum_{i=1}^{k} (n_i - 1)$.

Proof. a) Follows immediately from Corollary 2.5.c).
b) By induction: $k = 1$ – this is part a). Let now $m' = m(N(n_{k+1}) + 1)$ where $n_{k+1} \geqq n$. As $\dim((N(n_{k+1}) + 1) . K_n) \leqq n$ and as we can suppose that the first column of the encoding is the identity, we get (after removing this first column) $N(n_{k+1})$ independent GLR Γ_{K_n} $(n \times d)$ where $d \leqq n_{k+1} - 1$. According to Lemma

3.8 there exist $N(n_{k+1})$ independent GLR Γ_H $(mn \times d)$ (Γ_H corresponding to the graph $m . K_n$). Now we take $N(n_{k+1}) + 1$ identical encodings of the sum $m . K_n$, to one of them we add the matrix $I(mn \times d)$, to each of the others one of the GLR Γ_H. □

3.10. Corollary. a) *If k is a prime power, $n < k$, then*

$$\dim (k . K_n) \leqq k .$$

b) *Let k, n be integers, $k > n$, $n_1, \ldots, n_k$, $n_i \geqq n$ prime powers, denote $m = n_1^{r_1}, \ldots \ldots, n_t^{r_t}$ $(m \geqq k)$. Then*

$$\dim (k . K_n) \leqq d_m + 1 \quad \textit{where} \quad d_m = \sum_{i=1}^{k} r_i(n_i - 1) .$$

4. SOME APPLICATIONS

4.1. Example. Estimation of the dimension of the sums of K_2, K_3, K_4.

K_2: From 3.7.a) we have $\dim (k . K_2) \leqq 1 + 1(\log_2 k)^+$, i.e. our estimate coincides with the actual dimension (see [4]).

K_3: We get the best estimate if we put $m = 3^{r_1} . 4^{r_2}$, r_1 any integer, $r_2 < 2$ in 3.10. For an upper estimate we have $\dim (k . K_3) \leqq 2(r_1 - 1) + 4 = 2r_1 + 2$ for $3^{r_1} < k \leqq 3^{r_1 - 1} . 4$, $\dim (k . K_3) \leqq 2(r_1 + 1) + 1 = 2r_1 + 3$ for $3^{r_1} . 4 < k \leqq \leqq 3^{r_1 + 1}$.

For a lower estimate we get $\dim (k . K_3) \geqq 4$ from 3.5.a), and, using the strong matching number, $\dim (k . K_3) \geqq \log_2 \mu(k . K_3) = \log_2 k$. The results up to $m = = 729$ are collected in Table 1.

Table 1.

m	4	9	12	27	36	81	108	243	324	729
$d_m + 1$	4	5	6	7	8	9	10	11	12	13
lower esitmate	4	4	4	5	6	7	7	8	9	10

K_4: The best results are for $m = 4^{r_1} . 5^{r_2}$, r_1 any integer, $r_2 < 3$. For an upper estimate we have

for $4^{r_1} < k \leqq 4^{r_1 - 1} . 5$: $\dim (k . K_4) \leqq 3(r_1 - 1) + 5 = 3r_1 + 2$,

for $4^{r_1 - 1} . 5 < k \leqq 4^{r_1 - 2} . 5^2$: $\dim (k . K_4) \leqq 3(r_1 - 2) + 2 . 4 + 1 = = 3r_1 + 3$,

for $4^{r_1 - 2} . 5^2 < k \leqq 4^{r_1 + 1}$: $\dim (k . K_4) \leqq 3(r_1 + 1) + 1 = 3r_1 + 4$.

For a lower estimate we get again $\dim (k . K_4) \geqq \max (5, \log_2 k)$. The results up to 320 are collected in Table 2. We could get quite analogous results foı K_5 $(m =$

$= 5^{r_1} . 7^{r_2} . 8^{r_3}$ where $2r_2 + 3r_3 < 4$, i.e. $r_2 + r_3 \leqq 1$). A different situation would occur for K_6, as $N(6) = 1$ so that the best upper estimate is obtained from the inequality $\dim(k . K_6) \leqq \dim(k . K_7)$ (for K_7 we get the best upper estimate when $m = 7^{r_1} . 8^{r_2} . 9^{r_3} . 11^{r_4}$ where $r_2 + 2r_3 + 4r_4 < 6$). It is possible to give a general

Table 2.

m	5	16	20	25	64	80	100	256	320
decomposition		4^2	$4 . 5$	5^2	4^3	$4^2 . 5$	$4 . 5^2$	4^4	$4^3 . 5$
$d_m + 1$	5	7	8	9	10	11	12	13	14
lower estimate	5	5	5	5	6	7	7	8	8

expression for the optimal m but it is a little bit clumsy (for an interested reader it is available from the author). For small n it can be simplified to

$$\sum_{i=1}^{t} r_i(n_i - n) < n - 1 .$$

Now, we turn to λ-partite graphs and cycles. The following theorem is actually Theorem 2.5 from [3] in a more general form.

4.2. Theorem. *If G is a λ-partite graph then*

$$\dim(k . G) \leqq \dim(k . K_\lambda) - 1 + \dim G .$$

Proof. It is clear that there exists such an encoding (in the proper dimension) of $k . K_\lambda$ that the first column of all copies is the same encoding of K_λ and by means of the homomorphism Theorem 2.1 from [3] we get the rest. □

4.3. Corollary. *Let G be a λ-partite graph. Then*

a) $\dim(k . G) \leqq \dim G + (\lambda - 1)(\log_\lambda k)^+$ *for λ prime power;*
b) $\dim(k . G) \leqq \dim G + (\log_2 k)^+$ *for $\lambda = 2$;*
c) $\dim(k . G) \leqq \dim G + (\lambda - 1)(\log_3 k)^+$ *for $\lambda \neq 2, 6$;*
d) $\dim(k . G) \leqq \dim G + (\lambda - 1)(\log_{N(\lambda)+1} k)^+$;
e) $\dim(k . K_\lambda) \leqq \dim(k . K_{r_1,\ldots,r_\lambda}) \leqq \dim(k . K_\lambda) + 1$,

in particular:

$$(\log_2 k)^+ + 1 \leqq \dim(k . K_{r_1,r_2}) \leqq (\log_2 k)^+ + 2 .$$

Proof follows from Proposition 3.7. □

4.4. Proposition.

$$\log_2 k(n - 1) + 1 \leqq \dim(k . C_{2n}) \leqq (\log_2(n - 1))^+ + 1 + (\log_2 k)^+ \quad \textit{for} \quad n > 2 .$$

Proof. From 4.3.b) we have $\dim(k . C_{2n}) \leqq \dim C_{2n} + (\log_2 k)^+ =$ $= (\log_2(n - 1))^+ + 1 + (\log_2 k)^+$. For a lower estimate we use $\bar{\mu}$. By means of the construction from [4] (mentioned in 1.2) we get $\bar{\mu}(k . C_{2n}) = k . 2 . (n - 1)$, i.e. $\dim (k . C_{2n}) \geqq \log_2 k(n - 1) + 1$. □

4.5. Corollary. *Whenever*

$$n > 2 \text{ and } (\log_2 (n - 1))^+ + (\log_2 k)^+ < 1 + \log_2 k(n - 1)$$

then

$$\dim (k . C_{2n}) = (\log_2 (n - 1))^+ + (\log_2 k)^+ + 1 ,$$

in particular:

$$\dim (2^t . C_{2n}) = (\log_2 (n - 1))^+ + t + 1$$

and if $n = 2^t + 1$ *then*

$$\dim (k . C_{2n}) = t + (\log_2 k)^+ + 1 .$$ □

In the same way we could get estimates for odd cycles. Possible generalizations of Theorem 3.6, 4.1, 4.2 and 4.3 are left to the reader.

Acknowledgement. I am indebted to A. Pultr for useful discussions concerning these topics.

References

[1] *J. Denes, A. D. Keedwell:* Latin squares and their application, Akademiai Kiado, Budapest, 1974.
[2] *P. Křivka:* The dimension of odd cycles and cartesian cubes, Coll. Math. Soc. Janos Bolyai 25. Alg. meth. in graph th., Szeged 1978, 435—443.
[3] *P. Křivka:* Dimension of the sum of two copies of a graph, Czech. Math. J., *31 (106)*, 1981, 514—520.
[4] *L. Lovasz, J. Nešetřil, A. Pultr:* On a product dimension of graphs, J. Comb. Theory, 1981, B37, 5, 231—256.
[5] *J. Nešetřil, A. Pultr:* A Dushnik-Miller type dimension of graphs and its complexity, Proc. of the Conf. on Fundam. of Comp. Theory, Lecture Notes in Comp. Sc. 56, Springer 1977, 482—493.
[6] *J. Nešetřil, A. Pultr:* Product and other representations of graphs and related characteristics, Coll. Math. Soc. Janos Bolyai 25. Alg. meth. in graph th., Szeged 1978.
[7] *J. Nešetřil, V. Rödl:* A simple proof of the Galvin-Ramsey property of graphs and a dimension of a graph, Discrete Math. *23* (1978), 49—55.
[8] *S. Poljak, V. Rödl:* Orthogonal partitions and covering of graphs, Czech. Math. J. *30 (105)*, 1980, 475—485.

Author's address: 532 10 Pardubice, Leninovo n. 565, Czechoslovakia (VŠChT).