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SEVENTIETH ANNIVERSARY OF BIRTHDAY OF ACADEMICIAN ŠTEFAN SCHWARZ

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In May 1984, Academician Štefan Schwarz celebrated the seventieth anniversary of his birthday.

He was born on 18 May 1914 at Nové Mesto nad Váhom. After having completed his studies at Charles University in Prague in 1936, he became Assistant Professor at the Mathematical Institute of the Faculty of Science, Charles University, and later, in 1939, he became member of staff of the newly established Slovak Technical University. In 1946 he was appointed Associate Professor at the Faculty of Science in Bratislava and since 1947 he has been Full Professor at the Slovak Technical University. He was elected corresponding member and ordinary member (Academician) of the Czechoslovak Academy of Sciences in 1952 and 1960, respectively. In 1953 he also became member of the then established Slovak Academy of Sciences. Since 1 January 1966 he has been Head of the Mathematical Institute of this Academy.

The scientific activity of Professor Schwarz has been concentrated on algebra and the theory of numbers. Nonetheless, it covers also other fields of mathematics, as combinatorics, topology and probability theory. So far he has written 94 original scientific papers. It is not easy to give a short outline of his results. In the paper [4] published on the occasion of the 60th anniversary of birthday of Š. Schwarz, the authors of the present paper described some of his results appearing in [A1]–[A80] and attempted to show further prospects of his work. In the present paper we should like to characterize his results achieved during the past decade. The paper concerns partly amendments of his previous results, partly results in new fields. The range of problems studied by Š. Schwarz demonstrates his ability to follow the continual development of various branches of mathematics.

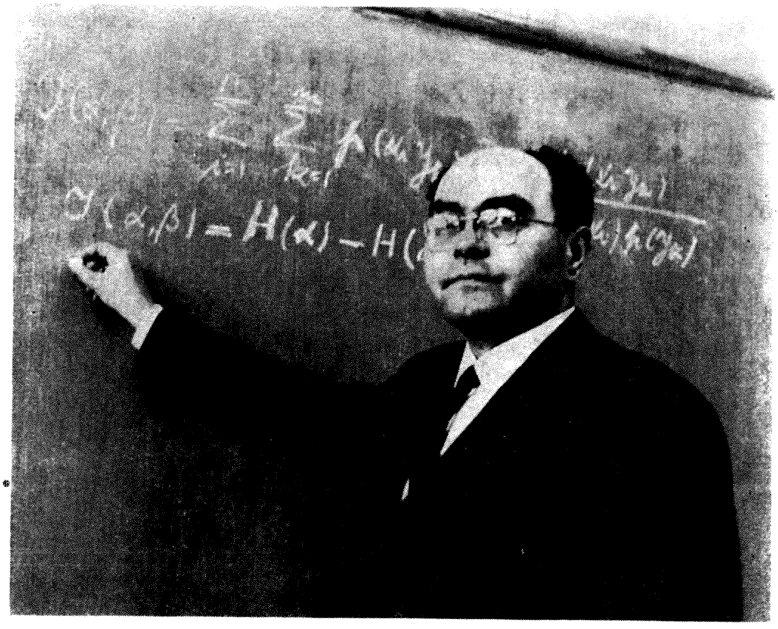
The tendency of Š. Schwarz has always been to test the depth of abstract theories on concrete examples and thus to prove their usefulness. This trend manifests itself also in his scientific activity during the recent years.

The paper [A84] concerns the general theory of semigroups. Let L^* be the intersection of all maximal left ideals of a semigroups S (if they exist). Let R^* and M^* have a similar meaning with respect to the right and two-sided ideals, respectively. (For broad classes of rings, M^* is Jacobson's radical.) In the paper Schwarz investigates e.g. conditions for the existence of L^*, for the identity $L^* = M^*$, etc.

The problems solved in [A84] also play a role in the paper [A81]. It concerns a class

of semigroups obtained in a natural way when studying the small categories, provided the partial semigroup of morphisms is completed by a zero element and the undefined products are put equal to zero. Such semigroups can be defined abstractly. The aim of the paper is to describe the structure of such semigroups and to give their construction.

It is generally extremely difficult to construct all semigroups having certain given properties. The problem considered in [A94] has such a character. Let $\{S_i: i \in J\}$ be a system of disjoint semigroups. Under which conditions does there exist an as-



sociative multiplication in the set $S = \bigcup\{S_i : i \in J\}$, such that S_i are subsemigroups of S and $S_i \,.\, S_j \subset S_j$? The solution of this problem by R. Yosida and M. Petrich from 1965 – 1969 gives no explicit method of effective construction of such a semigroup S. Schwarz [A94] found several "reasonable" sufficient conditions and the corresponding constructions.

Since the beginning of his scientific activity Š. Schwarz has repeatedly resumed the study of various aspects of the matrix theory. (The papers [A62], [A67] and [A68] are often referred to in the literature.) In [A88] he dealt with the existence of infinite products of doubly stochastic matrices. The paper [A93] concerns the multiplicative semigroup of $n \times n$ matrices over the finite field $GF(q)$, $q = p^r$. Let $\lambda(l, q) = p^t \,.\, l \,.\, c \,.\, m \,.\, [q^l - 1, q^{l-1} - 1, \ldots, q - 1]$, where t is the smallest integer for which $p^t \geqq 1$. It is known (essentially due to J. Niwen, 1948) that $A^{\lambda(n,q)+1} = A$

holds for each $n \times n$ regular matrix A. Schwarz proved, by using a typically semigroup theoretical method: If A has a rank $\leqq h$ and $1 \leqq h \leqq n-1$, then $A^{h+1} = A^{h+1+\lambda(h,q)}$ and this result cannot be improved.

In several problems concerning nonnegative $n \times n$ matrices, the sequence of powers $A, A^2, A^3, \ldots$ plays a role. It is only the distribution of zeros and positive elements that is important, while the values of nonzero matrix elements are inessential. Thus we come to the study of Boolean matrices, i.e., matrices whose elements are the elements of the two-element Boolean algebra with naturally defined addition and multiplication. Such matrices can also be interpreted as binary relations on an n-element set. The set of all $n \times n$ Boolean matrices under multiplication forms a semigroup S_n. Schwarz has paid much attention to the study of the semigroup S_n and its subsemigroups (see his papers [A72], [A73], [A74], [A77], [A79]). The reason is that these semigroups are important in combinatorics, automata theory and in several new branches of mathematics. He has investigated in detail the semigroup structure of the so called circulant Boolean matrices ([A82], [A83], [A87]). A long series of Schwarz's results concerning Boolean matrices is included in a recent monograph by K. H. Kim [3]. (Dozens of authors have quoted the paper [A72].)

Let A be a finite acceptor with n states, $L(A)$ the set of words accepted by A. It is known that $L(A)$ is infinite if and only if there is a word of length l, $1 \leqq l \leqq 2n-1$, accepted by A. Using the methods of finite semigroups Schwarz [A86] has found a new lucid proof of this statement and several new results concerning the lengths of the words belonging to $L(A)$.

Another field in which Schwarz used methods of the theory of semigroups was the elementary number theory. In [A89] the results of [A43] and [A51] are extended. Let $m = p_1^{\alpha_1} \ldots p_r^{\alpha_r}$ be the decomposition of the integer $m > 0$ into primes. Let Z_m be the residue class ring mod m. In the paper the multiplicative structure of Z_m is described in detail. Given $a \in Z_m$, there is a single idempotent e in the sequence $a, a^2, a^3, \ldots$. We shall say that a belongs to the idempotent e. Z_m contains 2^r different idempotents forming a Boolean algebra (under naturally defined Boolean operations). The set of all elements of Z_m belonging to the idempotent e forms a multiplicative semigroup $P(e)$, so that $Z_m = \bigcup_e P(e)$. $P(e)$ contains a unique maximal group $G(e)$ – the greatest subgroup of the multiplicative semigroup Z_m having e as the unit element. In the paper Schwarz described the construction of the idempotents, the construction of the groups $G(e)$ and semigroups $P(e)$, the direct decompositions of the semigroups Z_m and $P(e)$. He also determined the number of specific elements of Z_m and settled some other questions. The results of this paper made it possible to prove, from a unique point of view, practically all generalizations of the so called Euler-Fermat's theorem of the congruence theory, which appear in literature again and again.

The systematic use of idempotents (different from 0 and 1) offers the possibility of solving some unconventional problems of the number theory. In [A90] the fol-

lowing problem (among other) is solved. Let $(a, m) = 1$, m odd, and let $\varphi(m)$ be Euler's function. Find a formula for the sum (in Z_m) $\sigma = a + a^2 + \ldots + a^{\varphi(m)}$. It is proved: If $a - 1$ belongs to the idempotent e, then $\sigma = (1 - e)\varphi(m)$. (Note that theorems in which the idempotents of Z_m different from 1 appear, did not explicitly occur in literature before.)

In [A91] the following problem (among other) is solved: Enumerate the products $\prod(x - v: v \in G(e))$ and $\prod(x - v: x \in P(e))$. Bauer (1902) and Vandiver (1917) solved this problem for $e = 1$, obtaining a generalization of Lagrange's identity for prime modules $p: (x - 1)(x - 2)\ldots(x - p + 1) \equiv x^{p-1} - 1 \pmod{p}$. The elegance of the results in [A91] generalizing the former results was achieved by using the idempotents of the semigroup Z_m.

The last paper [A92] we want to speak about is of a combinatoric character. Consider an oriented graph without multiple edges with a set of vertices $V = \{a_1, \ldots, a_n\}$. To this graph a binary relation ϱ on V is assigned in an obvious manner. Denote $a_i\varrho = \{x \in V: (a_i, x) \in \varrho\}$. We say that two vertices a_i, a_j have a common successor of a length $l > 0$ if $a_i\varrho^l \cap a_j\varrho^l \neq \emptyset$ (*). If such an l exists, denote by $L(a_i, a_j)$ the smallest l fulfilling (*). It is not difficult to prove $L(a_i, a_j) \leqq \leqq \frac{1}{2}n(n - 1)$. A more precise estimate was unknown until Schwarz [A92] proved that $L(a_i, a_j) = (n^2/2) - n + \varepsilon_n$, where $\varepsilon_n = 1$ for n even and $\varepsilon_n = \frac{3}{2}$ for n odd, and this result cannot improved. The proof of this statement requires considerable experience in theory of Boolean matrices.

Schwarz's results on the semigroup theory are partly included in the monographs of E. S. Ljapin [2] and A. H. Clifford and G. B. Preston [1], which belong today to classical books on the theory of semigroups. To the authors' knowledge only, his results are referred to in about 400 works of mathematicians from many countries, about 26 of them being monographs or textbooks. His papers on linear algebra have been often cited in monographs and reviews.

Professor Schwarz is an excellent teacher. The main principles of his lecture have been: sufficient motivation of the matter under discussion, proper explication of the importance of results, their formulation in an explicit form with emphasis laid upon constructive processes and algorithms. The same principles may be found in his books [B1]–[B6]. He has observed them in the presentation of his results in papers and reports at conferences as well. Besides lecturing at the Slovak Technical University he worked for several years at the Faculty of Science of Comenius University in Bratislava. He has been a supervisor to a large number of predoctoral students, and many mathematicians are grateful to him for his advide at the beginning of their professional career. The effort to help young mathematicians can be seen in all spheres of his activity.

His pedagogical talent and interest in more general social problems are also reflected in his popular-scientific papers and other contributions in periodicals (see [C1]–[C57]). He took an active part in the organization of our scientific and cultural life and in the field of education especially as Chairman of the Slovak Academy of

Sciences (1965 – 1970) and Vice-Chairman of the Czechoslovak Academy of Sciences (1965 – 1970) as well as in other institutions.

In [4] we have pointed out that Š. Schwarz had never refrained from the social development and never avoided the problems. This is true even today.

His persistent effort is – as far as it is possible during the present explosion of information – to follow the development of mathematics and its applications as a whole. We know from our own experience that he has also followed the discussion on the importance of mathematics for the society in all major countries of the world. The bustle incited by the reforming tendencies in teaching mathematics at all education levels did not leave him cold. He expressed his views on these problems in [C51], [C54], [C55], [C57] as well as in other articles in various periodicals. In his talk [C57] he expressed, tactfully but clearly, his opinion against increasing formalism in teaching mathematics leading to confusion and obscurity, against inadequate abstractness and exaggerated exactness.

Even during the last decade Schwarz's activity was appreciated in many ways. In 1982, on the occasion of the 30th anniversary of the foundation of the Czechoslovak Academy of Sciences his outstanding merits for the development of Czechoslovak science were acknowledged by awarding him for the second time the Order of Labour. In 1980 he was awarded the National Prize of the Slovak Socialist Republic. He was awarded the Medal of J. A. Komenský by the President of the Czechoslovak Socialist Republic for his pedagogic activity. Further distinctions were conferred upon him by both the Czechoslovak and Slovak Academies of Sciences, by Czechoslovak Universities and by the Union of Socialist Youth.

We wish Professor Schwarz that his creative enthusiasm and energy may last for many years, for the benefit and further progress of Czechoslovak science.

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[1]) This list is a continuation of the list in [4]. It is not complete as well as that in [4].