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SOME REMARKS ABOUT BIJECTIONS ONTO METRIC SPACES

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Under a *bijection* $f : X \to Y$ we understand a one-to-one continuous mapping of a space X onto a space Y. We are interested in the problem of investigating those topological spaces which can be bijected (usually with some additional conditions) onto metrizable ones.

Compare the class of bijections $\mathscr{F}$ and the class of perfect mappings $\mathscr{P}$ in order to see the origins of this problem. (Recall that a continuous mapping $g : X \to Y$ is called *perfect* if it is closed, i.e. the image of every closed set $A \subset X$ is closed in Y, and proper, i.e. the preimage of every compact set $C \subset Y$ is compact). Both the classes may be in a way considered dual, or mutually complementary: e.g. the intersection $\mathscr{F} \cap \mathscr{P}$ is obviously a class of all homeomorphisms; if $f \in \mathscr{F}$ and $g \in \mathscr{P}$ then the diagonal mapping $f \Delta g$ is just an embedding (see e.g. [1]). Many nice properties of perfect mappings are known, such as the invariance of both metrizability and paracompactness under taking perfect images ([14 p. 355, p. 385]) and the invariance of paracompactness under taking perfect preimages [14 p. 386]. Moreover, the class of p-paracompact spaces, one of the most "popular" classes of topological spaces admits a characterization as the class of all perfect preimages of metrizable spaces.

Unfortunately, bijections do not possess so nice general properties. It is not difficult to construct a Hausdorff non-regular space, regular but non-normal, normal but non-paracompact spaces which admit bijections onto metrizable ones. On the other hand a bijective image of a metric space may be very "bad" and have nothing in common with metrizability. Therefore, there are, at any rate, three possibilities how to obtain more or less profound results about spaces which admit bijections onto metrizable spaces (our principal interest) or which are bijective images of metrizable spaces. The first is to consider not bijections but "bijectivelike" mappings. This approach was used by J. Nagata [17] who obtained interesting characterizations of some classes of spaces (e.g. stratifiable, semistratifiable) as "bijectivelike" images of metrizable spaces. The second is to consider not arbitrary bijections but only bijections satisfying some additional requirements. We can mention such examples as: the characterization of paracompact σ-spaces obtained by A. V. Archangeľskii [2] and the characterizations of stratifiable and M_1-spaces established quite recently by Ju. Bregman [10].

Now, the third possibility is to consider not a single bijection of a given space X onto a metrizable one, but a series of such bijections. Furthermore, it is natural to try to "connect the set of bijections $\{f_\alpha : X \to M_\alpha\}$ into an inverse system" in such way that X be the limit of this system. As a result one can get a characterization of a class of spaces as limits of special inverse systems of metrizable spaces in which all projections are bijective. This approach is the one presented in the paper.

In the first section we characterize paracompacta with a G_δ-diagonal and perfectly normal paracompacta with a G_δ-diagonal as limits of special inverse systems of metrizable spaces (theorems (1.1) and (1.2)).

In the second section we show when a stratifiable space X is a limit of an inverse system $\{M_\lambda, p_{\lambda\lambda'}, \Lambda\}$ of metric absolute neighbourhood retracts, every projection $p_\lambda : X \to M_\lambda$, $p_{\lambda\lambda'} : M_\lambda \to M_{\lambda'}$ where is a weak embedding, i.e. it factorizes as a composition $f \circ i$ an embedding i and a bijection f which is at the same time a homotopy equivalence. (Theorem (2.3)) In order to prove this result it is essential for us to use the technique developed by R. Cauty in [12].

In what follows $\mathscr{M}$ will denote the class of metrizable spaces and $\mathscr{S}$ the class of stratifiable spaces. $A(N)\,R(\mathscr{C})$ means an absolute (neighborhood) retract in a class $\mathscr{C}$ of spaces and $A(N)\,E(\mathscr{C})$ an absolute (neighborhood) extensor for a class $\mathscr{C}$. As usual $\mathbb{R}$ denotes the real line and $\mathbb{N}$ the set of natural numbers.

Regular, normal and paracompact spaces are always assumed to be Hasudorff. In the other cases no separation axiom is assumed unless explicitly stated. All mappings, in particular the bijections, under consideration are continuous.

For a subset A of a topological space X, $\bar{A}$ denotes its closure and A^0 its interior.

1. PARACOMPACTA WITH G_δ-DIAGONALS AS LIMITS OF INVERSE SYSTEMS OF METRIZABLE SPACES

The aim of this section is to prove the following two results:

(1.1) **Theorem.** *A T_1-space is a paracompacta with a G_δ-diagonal iff X is the limit of an inverse system $\{M_\alpha, p_{\alpha\alpha'}, A\}$ in which all projections $p_{\alpha\alpha'}$ are bijective, $M_\alpha \in \mathscr{M}$ and for every open cover $\mathscr{V}$ of X there exists $\alpha \in A$ such that*

$$\bigcup\{(p_\alpha U)^0 : U \in \mathscr{V}\} = \bigcup\{p_\alpha U : U \in \mathscr{V}\} \quad (= M_\alpha)$$

(1.2) **Theorem.** *A T_1-space is a perfectly normal paracompacta with a G_δ-diagonal iff X is a limit of an inverse system $\{M_\alpha, p_{\alpha\alpha'}, A\}$ in which all projections $p_{\alpha\alpha'}$ are bijective, $M_\alpha \in \mathscr{M}$ and for every family $\mathscr{V}$ of open subsets of X there exists $\alpha \in A$ such that $\bigcup\{(p_\alpha U)^0 : U \in \mathscr{V}\} = \bigcup\{p_\alpha U : U \in \mathscr{V}\}$.*

In order to establish these theorems we first need some auxiliary propositions. It is well-known that a paracompact space with a G_δ-diagonal admits a bijection onto a metrizable one (see e.g. [19], [5].) The next lemma expresses of this fact.

(1.3) **Lemma.** *Let X be a paracompacta with a G_δ-diagonal and $\mathscr{V}$ an open cover of X. Then there exists a bijection $f : X \to M$, $M \in \mathscr{M}$ such that $\bigcup\{(f(U))^0 : U \in \in \mathscr{V}\} = M$.*

Proof. Let Δ be a diagonal in $X \times X = X^2$, i.e. $\Delta = \{(x, x) : x \in X\}$ and let $O_1 \supset O_2 \supset \ldots$ be a decreasing sequence of open neighborhoods of Δ in X^2 such that $\bigcap O_n = \Delta$. We shall construct a sequence $\{\mathscr{U}^n : n \in \mathbb{N}\}$ of open covers of X by induction. Let $\mathscr{U}^1$ be a σ-discrete refinement of $\mathscr{V}$ satisfying $U \times U \subset O_1$ for every $U \in \in \mathscr{U}^1$. Assume that $\mathscr{U}^1, \ldots, \mathscr{U}^n$ are already constructed and define $\mathscr{U}^{n+1}$ as a σ-discrete cover of X such that $U \times U \subset O^{n+1}$ holds for every $U \in \mathscr{U}^{n+1}$ and there exists $V \in \in \mathscr{U}^n$ satisfying $\overline{U} \subset V$.

Now let $\mathscr{B} = \bigcup\{\mathscr{U}^n : n \in \mathbb{N}\}$. It is obvious that $\mathscr{B} = \{U_\alpha : \alpha \in A\}$ is a σ-discrete family of open sets, i.e. $\mathscr{B} = \bigcup \mathscr{B}_m$ where every $\mathscr{B}_m$ is discrete. For every pair $(m, n) \in \in \mathbb{N} \times \mathbb{N}$ and every $U_\alpha \in \mathscr{B}_m$ define a set $V_\alpha = \bigcup\{U : U \in \mathscr{B}_n, \overline{U} \subset U_\alpha\}$. Since $\mathscr{B}_n$ is discrete, $\overline{V}_\alpha \subset U_\alpha$ for every U_α. Let $\mathscr{C}_n^m$ denote the family of all V_α constructed for given m and n in this way and let $\mathscr{W} = \bigcup\{\mathscr{C}_m^n : m, n \in \mathbb{N}\}$. It is clear that $\mathscr{W}$ covers X.

Take $V_\alpha \in \mathscr{C}_m^n$ and consider a mapping $f_\alpha : X \to [0, 1]_\alpha = I_\alpha$ such that $f_\alpha(V_\alpha) = 1$ and $f_\alpha(X \setminus U_\alpha) = 0$ (if some V_α is empty we may take $f_\alpha \equiv 0$). Let $J_\tau = \bigcup\{I_\alpha : V_\alpha \in \in \mathscr{C}_m^n\}$ be a metric hedgehog with τ hedges where τ is not less than the cardinality of $\mathscr{C}_m^n$ [14] and define a mapping $f_m^n : X \to J_\tau$ by the formula $f_m^n|_{U_\alpha} = f_\alpha|_{U_\alpha} : U_\alpha \to I_\alpha \subset J_\tau$ if $U_\alpha \in \mathscr{B}_m$ and $f_m^n(x) = 0$ if $x \notin \bigcup\{U_\alpha : U_\alpha \in \mathscr{B}_m\}$. Since $\mathscr{B}_m$ is discrete, the mapping f_m^n is continuous. Construct such mappings f_m^n for all $m, n \in \mathbb{N}$. Taking sufficiently large τ one can assume that the space J_τ is the same for all mappings f_m^n. It is clear from the construction that the family $\{f_m^n : m, n \in \mathbb{N}\}$ distinguishes the points of X and therefore the diagonal mapping [14] $f = \Delta\{f_m^n : m, n \in \mathbb{N}\} : X \to \Pi\{J_m^n : m, n \in \mathbb{N}\}$ with $J_m^n = J_\tau$ for all $n, m \in \mathbb{N}$ is injective. Setting $f(X) = M$ we obtain a bijection $f : X \to \to M$.

It is easy to notice that for every $U_\alpha \in \mathscr{B}_m^n$ we have $f(U_\alpha) \supset N_\alpha \supset f(V_\alpha)$ where $N_\alpha = \Pi\{J_m^n(m, n) \neq (m_0, n_0)\} \times (0, 1]_\alpha \cap M$ and hence $(fU_\alpha)^0 \supset N_\alpha \supset f(V_\alpha)$. Recalling that $\mathscr{W}$ is a cover of X we obtain from here that $\{(f(U_\alpha))^0 : U_\alpha \in \mathscr{B}\}$ is a cover of M and hence $\{(f(U))^0 : U \in \mathscr{V}\}$ is a cover of M as well.

(1.4) **Lemma.** *Let X be a perfectly normal paracompacta with a G_δ-diagonal and $\mathscr{V} = \{U^\alpha : \alpha \in A\}$ a discrete family of open sets in X. Then there exists a bijection $f : X \to M$, $M \in \mathscr{M}$ such that $f(U^\alpha)$ is open for every $\alpha \in A$.*

Proof. Take $\alpha \in A$ and consider a sequence of open subsets U_n^α such that $\overline{U}_n^\alpha \subset \subset U_{n+1}^\alpha$ for every n and $\bigcup_n U_n^\alpha = U^\alpha$.

Let $U_n = \bigcup\{U_n^\alpha : \alpha \in A\}$ and consider an open cover $\mathscr{V}_n = \{U^\alpha, X \setminus \overline{U}_n : \alpha \in A\}$. By Lemma (1.3) there exists a bijection $f_n : X \to M_n$, $M_n \in \mathscr{M}$ such that $\{(f_n(U^\alpha))^0, (f_n(X \setminus \overline{U}_n))^0 : \alpha \in A\}$ is a cover of the space M_n. Let $f = \Delta\{f_n : n \in \mathbb{N}\} : X \to \to \Pi\{M_n : n \in \mathbb{N}\}$ and $M = f(X)$. It is obvious that the mapping $f : X \to M$ is a bijection. We only have to prove that $f(U^\alpha)$ is open in M for every $U^\alpha \in \mathscr{V}$.

For every point $x \in U^\alpha$ we find a number m such that $x \notin U_m^\alpha$, $\overline{U}_m^\alpha \subset \overline{U}_m$. This obviously implies that $f_m(x) \in (f_m(U^\alpha))^0 \equiv V_m^\alpha$ and therefore $f(x) \in (V_m^\alpha \times \Pi\{M_n : : n \neq m\}) \cap M \equiv U_{f(x)}$. Moreover, it is clear that $U_{f(x)} \subset f(U^\alpha)$. Thus, for every $f(x) \in f(U^\alpha)$ there exists an open neighbourhood $U_{f(x)}$ of $f(x)$ which is contained in $f(U^\alpha)$ and therefore $f(U^\alpha)$ is open.

(1.5) **Lemma.** *Let X be a perfectly normal paracompacta with a G_δ-diagonal and $\mathscr{V}$ a family of open subsets of X. Then there exists a bijection $f : X \to M$, $M \in \mathscr{M}$ such that* $\bigcup\{(f(V))^0 : V \in \mathscr{V}\} = \bigcup\{f(V) : V \in \mathscr{V}\}$ $(= f(\bigcup\{V : V \in \mathscr{V}\}))$.

Proof. Since X is a perfectly normal paracompact space, it is hereditarily paracompact and therefore there exists a σ-discrete open refinement $\mathscr{U} = \bigcup \mathscr{U}_n$ of the family $\mathscr{V}$ (i.e. $\bigcup\{U : U \in \mathscr{U}\} = \bigcup\{V : V \in \mathscr{V}\}$ and every $\mathscr{U}_n$ is discrete). By Lemma (1.4) for every n there exists a metrizable space M_n and a bijection $f_n : X \to M_n$ such that $f_n(U)$ is an open subset of M_n for every $U \in \mathscr{U}_n$. Consider a diagonal mapping $f = \Delta\{f_n : n \in \mathbb{N}\} : X \to \Pi\{M_n : n \in \mathbb{N}\}$ and let $M = f(X)$. It is easy to notice, that the equality $f(A) = M \cap (f_m(A) \times \Pi\{M_n : n \neq m\})$ holds for every $A \subset X$ and every $m \in \mathbb{N}$. In particular, this implies that $f(U)$ is an open subset of M for every $U \in \bigcup \mathscr{U}_n$. To conclude the proof it suffices to observe that $\bigcup\{(f(V))^0 : V \in \mathscr{V}\} \supset \bigcup\{(f(U))^0 : : U \in \mathscr{U}\} = \bigcup\{f(U) : U \in \mathscr{U}\} = f(\bigcup\{U : U \in \mathscr{U}\}) = f(\bigcup\{V : V \in \mathscr{V}\}) = \bigcup\{f(V) : : V \in \mathscr{V}\}$.

(1.6) **Lemma.** *If for every open cover $\mathscr{V}$ of a T_1-space X there exists a bijection f of X onto a regular space M such that $\bigcup\{(f(V))^0 : V \in \mathscr{V}\} = M$ then X is regular as well.*

Proof. Let F be a closed subset of X and $a \notin F$. Consider an open cover $\{V_1, V_2\}$ of X where $V_1 = X \setminus \{a\}$, $V_2 = X \setminus F$ and take a bijection $f : X \to M$ such that $\{(f(V_1))^0, (f(V_2))^0\}$ is a cover of a regular space M.

It is obvious that $f(a) \notin f(V_1)$ and hence $f(a) \in (f(V_2))^0$. On the other hand $f(F) = M \setminus f(X \setminus F) = M \setminus f(V_2) \subset M \setminus (f(V_2))^0 \equiv G$. Let O_1 and O_2 be disjoint neighborhoods of $f(a)$ and G respectively in M. Then $f^{-1}(O_1)$ and $f^{-1}(O_2)$ are disjoint neighborhoods of a and F in X and thus X is regular.

(1.7) **Lemma.** *If X is a T_1-space and for every open cover $\mathscr{V}$ of X there exists a bijection f of X onto a paracompact space M such that $\bigcup\{(f(V))^0 : V \in \mathscr{V}\} = M$ then X is paracompact as well.*

Proof. Let $\mathscr{V}$ be an open cover of X and f a bijection of X onto a paracompact space M such that $\{(f(V))^0 : V \in \mathscr{V}\} = \mathscr{W}$ is an open cover of M. Take an open locally-finite refinement $\mathscr{W}'$ of $\mathscr{W}$. It is obvious that $\{f^{-1}(W) : W \in \mathscr{W}'\}$ is a locally-finite refinement of $\mathscr{V}$. Applying the previous Lemma we get now that X is a paracompact space.

(1.8) **Lemma.** *If there exists a bijection $f : X \to M$ and the space M has a G_δ-diagonal, then the space X has a G_δ-diagonal as well.*

Proof is obvious.

(1.9) **Lemma.** *If for every open subset U of a T_1-space X there exists a metrizable space M and a bijection $f : X \to M$ such that $f(U)$ is open in M then X is perfectly normal.*

Proof. Let F and G be closed disjoint subsets of X and consider bijections $f_1 : X \to M_1, f_2 : X \to M_2$ where M_1, M_2 are metrizable spaces, $f_1(F)$ is closed in M_1 and $f_2(G)$ is closed in M_2. Let $M = \{(f_1(x), f_2(x)) : x \in X\} \subset M_1 \times M_2$ and $f = f_1 \Delta f_2 : X \to M$. Since, obviously $f(F) = M \cap (f_1(F) \times M_2)$, $f(G) = M \cap (M_1 \times f_2(G))$ the sets $f(F)$ and $f(G)$ are closed and disjoint in M. Let us separate them by disjoint open neighborhoods $U(\supset f(F))$ and $V(\supset f(G))$. Then $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open neighborhoods of F and G respectively in X and hence X is normal.

In order to prove that a closed set F is a G_δ-set in X it suffices to consider a bijection $f : X \to M$, $M \in \mathcal{M}$ for which $f(F)$ is closed in M.

(1.10) **Proposition.** A *T_1-space X is a paracompacta with a G_δ-diagonal iff for every open cover $\mathscr{V}$ of X there exists a metrizable space M and a bijection $f : X \to M$ such that $\{(f(V))^0 : V \in \mathscr{V}\}$ is a cover of M.*

Proof follows immediately from Lemmas (1.3), (1.7) and (1.8).

(1.11) **Proposition.** *A T_1-space X is a paracompacta with a G_δ-diagonal iff for every family $\mathscr{V}$ of open subsets of X there exists a metrizable space M and a bijection $f : X \to M$ such that $\bigcup\{(f(V))^0 : V \in \mathscr{V}\} = \bigcup\{f(V) : V \in \mathscr{V}\}$.*

Proof follows immediately from Lemma (1.5), Proposition (1.10) and Lemma (1.9).

(1.12) **Remark.** Proposition (1.10) and Lemma (1.9) imply also that.

A paracompact space X with a G_δ-diagonal is perfectly normal iff for every open $U \subset X$ there exists a bijection $f : X \to M$, $M \in \mathcal{M}$ such that $f(U)$ is open in M.

Proof of theorem (1.1). The part "if" is a corollary of Proposition (1.10).

Suppose that X is a paracompacta with a G_δ-diagonal and consider for every open cover $\mathscr{V}$ of X a metrizable space $M_\mathscr{V}$ and bijection $f_\mathscr{V} : X \to M_\mathscr{V}$ such that $\bigcup\{(f_\mathscr{V}(V))^0 : V \in \mathscr{V}\} = M_\mathscr{V}$ (see (1.10)). Let A be the set of all open covers of X and $\mathscr{F} = \{f_\mathscr{V} : \mathscr{V} \in A\}$.

The family $\mathscr{F}$ distinguishes points and closed sets in X. (One can show this quite similarly as in the proof of Lemma (1.6)). Therefore the diagonal mapping $h = \Delta\{f_\mathscr{V} : \mathscr{V} \in A\} : X \to \Pi\{M_\mathscr{V} : \mathscr{V} \in A\}$ is an embedding [14] and $h : X \to Y$, where $Y = h(X)$, is a homeomorphism.

Let Λ be the set of all finite subsets of A. Define a partial order on Λ putting $\lambda' \geqq \lambda''$ for $\lambda', \lambda'' \in \Lambda$ iff $\lambda'' \subset \lambda'$. For every $\lambda = \{\mathscr{V}_1, \ldots, \mathscr{V}_n\}$ let $p_\lambda = \Delta\{f_{\mathscr{V}_k} : \mathscr{V}_k \in \lambda\} : X \to \Pi\{M_{\mathscr{V}_k} : \mathscr{V}_k \in \lambda\}$ and $X_\lambda = p_\lambda(X)$. If $\lambda' < \lambda$ let $p_{\lambda\lambda'} : X_\lambda \to X_{\lambda'}$ be just the restriction to X_λ of a natural projection $\pi_{\lambda\lambda'} : \Pi\{M_{\mathscr{V}_k} : \mathscr{V}_k \in \lambda\} \to \Pi\{M_{\mathscr{V}_k} : \mathscr{V}_k \in \lambda'\}$. In such a way we obtain an inverse system of metrizable spaces $\{X_\lambda, p_{\lambda\lambda'}, \Lambda\}$. A trivial checking shows that X is the inverse limit of this system.

Proof of theorem (1.2) is quite similar. The only difference is that we must apply Proposition (1.11) instead of (1.10).

2. STRATIFIABLE SPACES AND INVERSE SYSTEMS OF METRIC ANR'S

Applying the Kuratowski-Wojdislawski theorem [8] which states that for every metric space M there is a normed vector space L and a convex set $C \subset L$ such that M embeds as a closed subset in C, and the Tietze-Dugundji theorem according to which every convex set C of a normed vector space is an $AR(\mathscr{M})$ [8], one can easily notice that

(2.1) for every metric space M there exists an inverse system $\{M_\alpha, p_{\alpha\alpha'}, A\}$ in which $M_\alpha \in ANR(\mathscr{M})$ for all $\alpha \in A$, all the projections $p_{\alpha\alpha'}$ are just embeddings and $M = \varprojlim M_\alpha$. This fact was repeatedly used for various purposes. Among other R. Fox applied it to develop a "geometric" shape theory for metric spaces [15].

In the last 20 years many authors have studied stratifiable spaces which seem to be one of the best generalizations of metric spaces at least from the geometric point of view, and which have many nice properties (see e.g. [13], [3], [5], [6], [7], [11]). Stratifiable spaces are perfectly normal paracompacta; the class of stratifiable spaces is hereditary and invariant under taking countable products. On the other hand stratifiable spaces are invariant under domination and under taking adjunction spaces [5] and these are very useful properties for the needs of the algebraic topology. Moreover, every convex set C of a topological vector space L is an absolute extensor for stratifiable spaces [5] and for every stratifiable space X there exists $S = S(X) \in AR(\mathscr{S})$ which contains X as a closed subset ([20] see also [11]; by $\mathscr{S}$ we denote the class of stratifiable spaces).

Since metrizability is a hereditary property there cannot be the same result as (2.1) for $M \in \mathscr{S}$. But nevertheless as in many other aspects stratifiable spaces behave themselves "very like" the metric ones also from this point of view, too. To formulate this precisely (it is also the main result of the section) we introduce first the following notions.

(2.2) **Definition.** A mapping $h : X \to Z$ is called a *weak embedding* if there exists a space Y, an embedding $f : X \to Y$ and a bijection which is also a homotopy equivalence $g : Y \to Z$ such that $h = g \circ f$. In case that all spaces under consideration are stratifiable we call h a weak embedding in the class of stratifiable spaces.

The main result of this section may be stated now as follows

(2.3) **Theorem.** *For every stratifiable space X there exists an inverse system $\{M_\beta, p_{\beta\beta'}, B\}$ where $M \in ANR(\mathcal{M})$ for all β, $X = \varprojlim M_\beta$ and all projections $p_{\beta\beta'} : M_\beta \to M_{\beta'}$, $p_\beta : X \to M_\beta$ are weak embeddings in the class of stratifiable spaces.*

Before proving this theorem, first we recall some familiar definitions and facts about $AR(\mathcal{M})$'s and $AR(\mathcal{S})$'s.

Let P_n denote an n-dimensional simplex. For a set A define a mapping $\delta_i : A^n \to \to A^{n-1}$ $(1 \leqq i \leqq n)$ by the formula $\delta_i(a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n) = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots a_n)$.

(2.4) **Definition.** (Borges [6]). A topological space X is called *hyperconnected* if there exist mappings

$$\lambda^m : X^m \times P_{m-1} \to X\,, \quad m \in \mathbb{N}$$

with the following properties:

(2.4.a) if $x \in X^m$, $t \in P_{m-1}$ and $t_i = 0$ then $\lambda^m(x, t) = \lambda^{m-1}(\delta_i x, \delta_i t)$;

(2.4.b) for every $x \in X^m$ the mapping $\varphi : P_{m-1} \to X$ defined by the identity $\varphi(t) = = \lambda^m(x, t)$ is continuous;

(2.4.c) for every point $x \in X$ and every neighborhood V of x there exists a neighborhood W of x such that $W \subset V$ and $\lambda^m(W^m \times P_{m-1}) \subset V$ for every $m \in \mathbb{N}$.

(2.5) **Theorem.** (Borges [6, Th. 5.1], [7, Th. 4.4]). *A metric space X is $AR(\mathcal{M})$ iff X is hyperconnected.*

(2.6) **Theorem.** (Cauty [12, Th. 1.8]). *A stratifiable space X is $AR(\mathcal{S})$ iff X is hyperconnected.*

The proof of theorem (2.3) is based on the Theorem (2.12) and the later in its turn is proved by using of the next proposition, (2.7). We want to emphasize that the just mentioned statement (2.7) has much in common with Cauty's statement [12 th. 2.1]. In its proof we also use the technique developed in [12]. However, special requirements in the statement (2.7) bring about additional problems which are to be overcome in the proof.

(2.7) **Proposition.** *Let X be an $AR(\mathcal{S})$ and Y its open subset. Then there exists a continuous metric d on X such that X_d is an $AR(\mathcal{M})$, Y_d its open subset and the natural bijection $f : X \to X_d$ as well as its restriction $f' : Y \to Y_d$ are homotopy equivalences. Moreover, if α is an open cover of X then the metric d can be chosen in such a way that $\bigcup\{(fU)^0 : U \in \alpha\} = X_d$.*

(X_d denotes the set X topologized by the metric d and $Y_d = f(Y) \subset X_d$.)

Proof. Since X is stratifiable the diagonal Δ is a G_δ-set in X^2. Let $G_1 \supset G_2 \supset \ldots$ be a sequence of its neighborhoods in X^2 satisfying $\bigcap G_n = \Delta$. Choose a sequence of open covers $\{\mathcal{O}_n\}$ of X such that for every n $\bigcup\{O \times O : O \in \mathcal{O}_n\} \subset G_n$ and $\mathcal{O}_{n+1}$

refines $\mathcal{O}_n$. Moreover, if an open cover α of X is given, then let $\mathcal{O}_1$ be also a refinement of α.

Since Y is an open subset of a perfectly normal space X there exists a sequence of open subsets $Y_n \subset Y$ such that $\overline{Y}_n \subset Y_{n+1}$ for every n and $\bigcup Y_n = Y$. Moreover, $Y \in ANR(\mathcal{S})$ as an open subset of $X \in AR(\mathcal{S})$ and therefore there exists an open cover $\mathcal{H}$ of Y such that any two $\mathcal{H}$-close mappings $f, g : Z \to Y$ of an arbitrary topological space Z are homotopic in Y [11, th. 1.4]. Let now $\lambda^m : X^m \times P_m \to X$, $m \in \mathbb{N}$ be mappings satisfying the conditions of Definition (2.4).

We shall construct now by induction two sequences of open covers $\{\mathcal{V}_n\}$ and $\{\mathcal{W}_n\}$ of the space X. Let $\mathcal{W}_0 = \mathcal{V}_0 = \mathcal{H} \bigcup \{X \setminus \overline{Y}_0\}$. Assume that for all $k \leqq n$ we have constructed open covers $\mathcal{W}_k$ and $\mathcal{V}_k$. To construct $\mathcal{W}_{n+1}$ fix for every $x \in X$ sets $V \in \mathcal{V}_n$ and $O \in \mathcal{O}_n$ containing x and let $W_x = W$ be a neighborhood of x for which $\lambda^m(W^m \times P_{m-1}) \subset O \cap V$, $m \in \mathbb{N}$ and $\mathcal{W}_{n+1} = \{W_x : x \in X\}$.

Since by (2.4) $\lambda^1(W \times P_0) = W \subset O \cap V$ for $W \in \mathcal{W}_{n+1}$, the cover $\mathcal{W}_{n+1}$ refines both $\mathcal{V}_n$ and $\mathcal{O}_n$.

To construct $\mathcal{V}_{n+1}$ we introduce first some notations. If $\mathcal{C}$ is a cover of X and $A \subset X$ let $\mathrm{st}(A, \mathcal{C}) = \bigcup\{V : V \in \mathcal{C}, V \cap A \neq \emptyset\}$ and $\mathcal{C}^A = \{C : C \in \mathcal{C}, C \cap A \neq \emptyset\}$. Let $\mathcal{C}^\Delta$ denote a point-star cover obtained from $\mathcal{C}$, i.e. $\mathcal{C}^\Delta = \{\mathrm{st}(\{x\}, \mathcal{C}) : x \in X\}$. Define now $\mathcal{V}_{n+1}$ as an open cover of X which satisfies the following three properties:

(1V) $\mathcal{V}^\Delta_{n+1}$ refines both $\mathcal{W}_{n+1}$ and $\mathcal{O}_{n+1}$,

(2V) $\mathrm{St}(\overline{Y}_n, \mathcal{V}_{n+1}) \subset Y_{n+1}$, $\mathrm{St}(X \setminus Y, \mathcal{V}_{n+1}) \subset X \setminus \overline{Y}_{n+1}$;

(3V) $\mathcal{V}_{n+1}$ refines $\mathcal{H} \cup \{X \setminus \overline{Y}_n\}$.

(It is easy to check that a cover with such properties really exists).

Since the cover $\mathcal{V}^\Delta_{n+1}$ is a refinement of $\mathcal{V}_n$, according to [18, Th. 2.16] there exists a pseudometric d on X such that the family $\mathcal{B}_n = \{B_n(x, d) : x \in X\}$ (where $B_n(x, d) = \{x' : x' \in X, d(x, x') < 2^{-n}\}$) refines $\mathcal{V}_n$ but $\mathcal{V}^\Delta_{n+3}$ refines $\mathcal{B}_n$. Since $\mathcal{V}^\Delta_n$ refines $\mathcal{O}_n$, the pseudometric d separates the points of X and therefore it is a metric.

To prove that $X_d \in AR(\mathcal{M})$ it suffices to refer to (2.5) and to check that the mappings $\lambda^m : X^m_d \times P_{m-1} \to X_d$ satisfy the conditions of (2.4). The conditions (2.4.a) and (2.4.b) are trivially true. To check (2.6.c) take a point $x \in X$ and a neighborhood $B(x, 2^{-n})$. From the definition of d it follows that there exists $V \in \mathcal{V}_{n+3}$ such that $x \in V \subset B_n(x, d)$ while the construction of $\{\mathcal{W}_n\}$ allows us to find $W \in \mathcal{W}_{n+4}$ satisfying $\lambda^m(W^m \times P_{m-1}) \subset V$ for all $m \in \mathbb{N}$. Since $\mathcal{B}_{n+4}$ refines $\mathcal{V}_{n+4}$ and therefore also $\mathcal{W}_{n+4}$, $\lambda^m(B^m_{n+4}(x, d) \times P_{m-1}) \subset B_n(x, d)$ holds for all $m \in \mathbb{N}$, and thus the property (2.6.c) is also true.

Let $f : X \to X_d$ be a natural bijection. It is obvious that f is continuous. Moreover, since $X \in AR(\mathcal{S})$, $X_d \in AR(\mathcal{M})$, f is a homotopy equivalence. The properties (1V) and (2V) imply that the set $Y_d = f(Y)$ is open in X_d. Hence to complete the proof we only have to show that the restriction $f' : Y \to Y_d$ of the bijection f is a homotopy equivalence.

To construct for f' a homotopy inverse $\varphi : Y_d \to Y$ consider for every $k \in \mathbb{N}$ the family $\mathscr{C}_k = \bigcup_{n=0}^{\infty} \mathscr{B}^{Y_n}_{k+n}$. The property (2V) and the definition of d implies that every $\mathscr{C}_k$ is a cover of Y. By (1V) $\mathscr{C}^{\Delta}_{k+4}$ refines $\mathscr{C}_k$ and the property (3V) allows to conclude that for $k \geqq 3$ $\mathscr{C}_k$ refines $\mathscr{H}$. Take an open cover $\mathscr{H}_1$ of Y_d such that $\mathscr{H}^{\Delta}_1$ refines $\mathscr{C}_3$. Recalling that Y_d is an open subset of X_d and hence an $ANR(\mathscr{M})$ we can find an open cover $\mathscr{H}_2$ of Y_d such that for every simplicial complex P and every subcomplex Q containing all vertices of P every $\mathscr{H}_2$ – small mapping $\varphi : Q \to Y$ has an $\mathscr{H}_1$ – small extension $\tilde{\varphi} : P - Y$ (see [16, ch. 4, Th. 4.1]). (A mapping $\varphi : Q \to Y$ is called $\mathscr{U}$ – small where $\mathscr{U}$ is a cover of X if for every simplex $\sigma \subset P$ there exists $U \in \mathscr{U}$ containing $f(\sigma \cap Q)$). Consider an open cover $\mathscr{H}_3$ of Y such that $\mathscr{H}^{\Delta}_3$ refines $\mathscr{H}_2$. Let $\mathscr{K} = \{K_\beta : \beta \in B\}$ be an open locally fine refinement of $\mathscr{H}_3$ and let $i : Y_d \to \mathscr{N}(\mathscr{K})$ be the canonical mapping of Y_d in the nerve $\mathscr{N}(\mathscr{K})$ of the cover $\mathscr{K}$. We shall define a mapping $j : \mathscr{N}(\mathscr{K}) \to Y$ as follows. For every vertex $\beta \in \mathscr{N}(\mathscr{K})$ take a point $y_\beta \in K_\beta$. If $\sigma = (\beta_0, \ldots, \beta_n)$ is a simplex in $\mathscr{N}(\mathscr{K})$ then the set $\{y_{\beta_0}, \ldots, y_{\beta_n}\}$ contained in $K_{\beta_0} \cup \ldots \cup K_{\beta_n} \subset \mathrm{st}\,(y_{\beta_0}, \mathscr{K}) \subset \mathrm{st}\,(y_{\beta_0}, \mathscr{H}_3)$ and therefore there exists $H \in \mathscr{H}_2$ which contains $\{y_{\beta_0}, \ldots, y_{\beta_n}\}$. It is easy to notice that there exists an $\mathscr{H}_1$ – small mapping $j : \mathscr{N}(\mathscr{K}) \to Y$ such that $j(\beta) = y_\beta$ for every vertex $\beta \in \mathscr{N}(\mathscr{K})$. We define now $\varphi : Y_d \to Y$ by the identity $\varphi = j \circ i$.

To show that φ is a homotopy inverse for $f' : Y \to Y_d$ take $y \in Y$ and let $\sigma = (\beta_0, \ldots, \beta_n)$ be a simplex of $\mathscr{N}(\mathscr{K})$ the interior of which contains $i(y)$. It means that $y \in K_{\beta_0} \cap \ldots \cap K_{\beta_n}$ and hence there exists $H \in \mathscr{H}_1$ which contains both $\varphi(y) = j \circ i(y)$ and y_{β_0}. Since $\mathscr{H}^{\Delta}_1$ refines $\mathscr{C}_3$ there exists $U \in \mathscr{C}_3$ which contains both y and $j \circ i(y)$ and hence $\varphi \circ f : Y \to Y$ and $\mathrm{id} : Y \to Y$ are $\mathscr{C}_3$ – close. Now, since $\mathscr{C}_3$ refines $\mathscr{H}$ there exists a homotopy $h : Y \times I \to Y$ such that $h(y, 0) = y$ and $h(y, 1) = \varphi \circ f(y)$. On the other hand reasoning similarly as in the proof of [12, Th. 2.2] one can show that any two $\mathscr{C}_3$-close mappings g_1 and g_2 of a metrizable space M into Y_d are homotopic. Hence there exists a homotopy $k : Y_d \times I \to Y_d$ such that $k(y, 0) = y$ and $k(y, 1) = f \circ \varphi(y)$.

Finally, the identity $\{(fU)^0 : U \in \alpha\} = X_d$ is obvious from the construction of d and hence Proposition (2.7) is proved.

(2.8) **Proposition.** *Let X be an $AR(\mathscr{S})$ and $Y^1, \ldots, Y^n$ its open subsets. Then there exists a continuous metric d on X such that $X_d \in AR(\mathscr{M})$, $Y^1_d, \ldots, Y^n_d$ are open subsets of X_d and the natural bijection $f : X \to X_d$ as well as all its restrictions $f'_1 : Y^1 \to Y^1_d, \ldots, f'_n : Y^n \to Y^n_d$ are homotopy equivalences. Moreover, if α is an open cover of X then the metric d can be chosen in such a way that $\bigcup\{(fU)^0 : U \in \alpha\} = X_d$.*

(2.9) **Remark.** In what follows we shall refer to d that satisfies the statement of (2.8) as the metric that takes into account the subspaces $Y^1, \ldots, Y^n$ and the cover α.

We prove Proposition (2.8) by induction. In case $n = 1$ it reduces to Proposition (2.7) proved above. Assume, that the statement of (2.8) is proved for $n = k$ and consider the case of $n = k + 1$ open subsets $Y^1, \ldots, Y^k, Y^{k+1} = Y$ of X. According to our assumption there exists a metric d' on X which takes into account the subspaces $Y^1, \ldots, Y^k$ and the cover α. Now construct a metric d on X quite in the same way as in the proof of (2.7) (for an open subset $Y^{k+1} = Y$ of X) but with an additional requirement that $\mathscr{V}_{n+1}$ must also have the following property

(4V) $\mathscr{V}_{n+1}$ is a refinement of $\mathscr{B}'_{n+1} = \{B'_n(x, d') : x \in X\}$ where $B'_n(x, d') = \{z : z \in X, d'(x, z) < 2^{-n-1}\}$. It is easy to notice that the metric d thus obtained takes into account the subspaces $Y^1, \ldots, Y^{k+1}$ and this completes the proof of (2.8).

(2.10) **Remark.** It is obvious from the above proof that if $X \in AR(\mathscr{S})$, $Y^1, \ldots, Y^{n+1}$ are its open subsets, α an open cover of X and the metric d' on X is the one which takes into account the subspaces $Y^1, \ldots, Y^n$ and the cover α then there exists a metric d on X which takes into account the subspaces $Y^1, \ldots, Y^n, Y^{n+1}$, the cover α, such that the natural bijection $f : X_d \to X_{d'}$ is continuous.

Now let A denote the set of all open covers α of X and $\Lambda = \bigcup \Lambda_n$ where $\Lambda_1 = A$ and $\Lambda_{n+1} = \{(\lambda, \lambda') : \lambda, \lambda' \in \Lambda_n\}$ for $n \geqq 1$. We introduce a partial order on the set Λ putting $\lambda' \leqq \lambda$ iff there exists a finite sequence $\lambda_1, \ldots, \lambda_n \in \Lambda$ such that $\lambda' = \lambda_1$, $\lambda = \lambda_n$ and $\lambda_i \in \lambda_{i+1}$ for every $i \leqq n-1$. Let $Y^1, \ldots, Y^m$ be open subsets of $X \in AR(\mathscr{S})$. By induction we shall construct metrics d_λ on X which take into account the subsets $Y^1, \ldots, Y^m$ and all the covers "contained" in λ. If $\lambda = \alpha \in A = \Lambda_1$ we define d_λ as a metric which takes into account the subsets $Y_1, \ldots, Y_m$ and the cover α (see (2.8), (2.9)). The corresponding metric space will be now denoted simply by X_λ. Assume that for all $\lambda \in \Lambda_i$, $i \leqq k$ we have constructed metrics d_λ on X which take into account the subsets $Y^1, \ldots, Y^m$ and all the covers $\alpha \in A$ which satisfy the inequality $\alpha \leqq \lambda$ and such that for $\lambda' \leqq \lambda$ the natural bijection $p_{\lambda\lambda'} : X_\lambda \to X_{\lambda'}$ is continuous. Take $\lambda = \{\lambda_1, \lambda_2\} \in \Lambda_{k+1}$ and construct the metric d_λ on X analogously as in the proof of (2.7) and (2.8) but with an additional requirement for every cover $\mathscr{V}_{n+1}$

(5V) $\mathscr{V}_{n+1}$ refines both $\mathscr{B}^1_n = \{B_n(x, d_{\lambda_1}) : x \in X\}$ and $\mathscr{B}^2_n = \{B_n(x, d_{\lambda_\lambda}) : x \in X\}$.

It is easy to notice that the metric d_λ obtained in this way takes into account subspaces $Y^1, \ldots, Y^m$ and all the covers $\alpha \in A$ which satisfy the inequality $\alpha \leqq \lambda$. Moreover, if $\lambda' \leqq \lambda$ then the natural bijection $p_{\lambda\lambda'} : X_\lambda \to X_{\lambda'}$ is continuous. In such a way we get by induction an inverse system $\{X_\lambda, p_{\lambda\lambda'}, \Lambda\}$; one can easily notice that X is its inverse limit.

Applying the next proposition (2.11) the proof of which is trivial and therefore omitted we can sum up our results given above in the statement of Theorem (2.12).

(2.11) **Proposition.** *Let the mappings $f : X \to Y$, $g : X \to Z$ and $h : Y \to Z$ satisfy the identity $h \circ f = g$ and let $\varphi : Y \to X$, $\psi : Z \to X$ be homotopy inverses for f and g respectively. Then $f \circ \psi$ is the homotopy inverse for h.*

(2.12) **Theorem.** *Let X be an $AR(\mathscr{S})$ and $Y^1, \ldots, Y^m$ its open subsets. Then there exists an inverse system $\{X_\lambda, p_{\lambda\lambda'}, \Lambda\}$ where $X = \varprojlim X_\lambda$, $X_\lambda \in AR(\mathscr{M})$ and all the projections $p_\lambda : X \to X_\lambda$ $p_{\lambda\lambda'} : X_\lambda \to X_{\lambda'}$ as well as their restrictions $p^k_\lambda : Y^k \to Y^k_\lambda$, and $p^k_{\lambda\lambda'} : Y^k_\lambda \to Y^k_{\lambda'}$, $k \leqq m$ are bijections and homotopy equivalences.*

(2.13) **Remark.** In what follows we shall refer to the inverse system $\{X_\lambda, p_{\lambda\lambda'}, \Lambda\}$ which satisfies the statement of (2.12) as the one that is associated with X and takes into account the subspaces $Y^1, \ldots, Y^m$.

(2.14) **Remark.** Let $Y^1, \ldots, Y^m, Y^{m+1}$ be open subsets in $X \in AR(\mathscr{S})$ and $\{X_\lambda^m, p_{\lambda\lambda'}, \Lambda\}$ an inverse system which is associated with X and takes into account the subspaces $Y^1, \ldots, Y^m$. Applying (2.10) and analysing the proof of (2.12) we can easily construct an inverse system $\{X_\lambda^{m+1}, p_{\lambda\lambda'}, \Lambda\}$ which is associated with X and takes into account the subspaces $Y^1, \ldots, Y^m, Y^{m+1}$, such that all the natural bijections $f : X_\lambda^{m+1} \to X_\lambda^m$, are continuous.

(2.15) **Remark.** A theorem analogous to (2.12) but with a weaker assumption that $X \in ANR(\mathscr{S})$ and hence with a weaker assertion that all $X_\lambda \in ANR(\mathscr{M})$ is also true. The proof of this result is in its essence similar to the proof of (2.12) but it is technically more complicated.

We are able now to prove Theorem (2.3). Let X be a stratifiable space and S an $AR(\mathscr{S})$ which contains X as a closed subspace. Consider a decreasing sequence $\{Y^n\}$ of open neighborhoods of X in S satisfying $\bigcap Y^n = X$. By Theorem (2.12) for every n there exists an inverse system $\{S_\lambda^n, \pi_{\lambda\lambda'}^n, \Lambda\}$ which is associated with S and takes into account the subspaces $Y^1, \ldots, Y^n$. Moreover, (2.14) allows us to choose these inverse systems in such a way that for all $n' \leqq n$ and all λ the natural bijections $\pi_\lambda^{nn'} : S_\lambda^n \to S_\lambda^{n'}$ are continuous.

Consider now the set $B = \Lambda \times \mathbb{N}$ and introduce a partial order on it putting for its elements $\beta = (\lambda, n)$ and $\beta' = (\lambda', n')$ $\beta' \leqq \beta$ iff $\lambda' \leqq \lambda$ and $n' \leqq n$. Let $S_\beta = S_\lambda^n$ and define the projection $\pi_{\beta\beta'} : S_\beta \to S_{\beta'}$ by the identity $\pi_{\beta\beta'} = \pi_{\lambda\lambda'}^{n'} \circ \pi_\lambda^{nn'}(\pi_\lambda^{nn'} : S_\beta = S_\lambda^n \to S_\lambda^{n'};\ \pi_{\lambda\lambda'}^{n'} : S_\lambda^{n'} \to S_{\lambda'}^{n'};\ S_{\lambda'}^{n'} = S_{\beta'})$. It is easy to notice that in such a way we obtain an inverse system $\{S_\beta, \pi_{\beta\beta'}, B\}$ which is associated with S and takes into account every Y_n. Let $\pi_\beta : S \to S_\beta$ be the limit projections.

Now, for every $\beta = (\lambda, n) \in B$ let $M_\beta = \pi_\beta(Y_n) \subset S_\beta$, if $\beta' = (\lambda', n') \leqq \beta$ let $p_{\beta\beta'} : M_\beta \to M_{\beta'}$ be the restriction of $\pi_{\beta\beta'}$ to the subset M_β, and consider the inverse system $\{M_\beta, p_{\beta\beta'}, B\}$. Since M_β is an open subset of $S_\beta \in AR(\mathscr{M})$ it is an $ANR(\mathscr{M})$. A trivial checking shows that the limit of this system is X. Moreover, it is clear that $p_{\beta\beta'} = p_{\lambda\lambda'}^{n'} \circ p_\lambda^{nn'}$ where $p_{\lambda\lambda'}^{n'} : M_\lambda^{n'} \to M_{\lambda'}^{n'}$ is the restriction of $\pi_{\lambda\lambda'}^{n'}$ and $p_\lambda^{nn'}$ is the restriction of $\pi_\lambda^{nn'}$. The mapping $p_{\lambda\lambda'}^{n'}$ is obviously a bijection and since both the restrictions $\pi_\lambda^{n'}|_{Y^{n'}} : Y^{n'} \to M_\lambda^{n'}$ and $\pi_\lambda^n|_{Y^n} : Y^n \to M_\lambda^n$ are homotopy equivalences, according to (2.11), we get that it is a homotopy equivalence as well. The mapping $p_\lambda^{nn'}$ can be factorized in a natural way into a composition $p_\lambda^{nn'} = j \circ i$ where $i : M_\lambda^n \to N_\lambda^{n'} = \pi_\lambda^n(Y_{n'}) \subset S_\lambda^n$ is an embedding and $j : N_\lambda^{n'} \to M_\lambda^{n'}$ is the restriction of a bijection $\pi_\lambda^{nn'} : S_\lambda^n \to S_\lambda^{n'}$ to $N_\lambda^{n'}$. Applying (2.11) once again we now easily notice that j is a bijection and a homotopy equivalence. Thus, all projections $p_{\beta\beta'}$ are weak embeddings. To complete the proof we only have to show that the limit projections $p_\beta : X \to M_\beta$ are weak embeddings as well. But this follows immediately from the identity $p_\beta = (\pi_\lambda^n|_{Y^n}) \circ i$ where i is the natural inclusion of the space X into its neighborhood Y^n.

(2.16) **Remark.** Theorem (2.3) has some application in the shape theory, especially for the geometric approach to the theory of shape for stratifiable spaces. This application will be considered in another paper.

References

[1] *A. V. Archangel'skii, V. I. Ponomarev:* Основы общей топологии в задачах и упражнениях. Москва, 1974.

[2] *A. V. Archangel'skii:* Об одном классе пространств, содержащем все метрические и все локально бикомпактные пространства. Матем. сб. *67* (*109*) : 1 (1965), с. 55—85.

[3] *A. V. Archangel'skii:* Отображения и пространства. Успехи Матем. Наук т. *21*, вып. 4 (1966), с. 133—184.

[4] *A. V. Archangel'skii:* Классы топологических групп. Успехи Матем. Наук, т. *36*, вып. 3 (1981), с. 127—146.

[5] *C. J. R. Borges:* On stratifiable spaces. Pacific J. Math., v. *17* (1966) p. 1—16.

[6] *C. J. R. Borges:* A study of absolute extensor spaces. Pacific J. of Math., *31* (1969), p. 609 to 617.

[7] *C. J. R. Borges:* Connectivity of function spaces. Canad. J. Math., v. *23* (1971), p. 759—763.

[8] *K. Borsuk:* Theory of Retracts. Warszawa, PWN, 1967.

[9] *K. Borsuk:* Theory of Shape. Warszawa, PWN 1975.

[10] *Ju. Bregman:* A note about M_1-spaces and stratifiable spaces. Comment. Math. Univ. Carolinae.

[11] *R. Cauty:* Une generalisation du theorem de Borsuk-Whitehead-Hanner aux espaces stratifiables, C. R. Acad Sc. Paris, t. *275* (1972) Ser A. p. 271—274.

[12] *R. Cauty:* Retractions dans les espaces stratifiables. Bull. Soc Math France, t. *102* (1974) p. 129—149.

[13] *J. K. Ceder:* Some generalizations of metric spaces. Pacific J. Math. v. *11* (1961), p. 105—126.

[14] *R. Engelking:* General Topology. Warszawa, PWN, 1977.

[15] *R. H. Fox:* On shape. Fund. Math., *74* (1972) p. 47—71.

[16] *Sze-Tsen Hu:* Theory of Retracts. Detroit, 1965.

[17] *J-iti Nagata:* Some theorems on generalized metric spaces. Theory of Sets and Topology (a Collection of Papers in Honour of Felix Hausdorff) Berlin, VEB, 1972 p. 377—390.

[18] *K. Nagami:* Dimension theory. N.Y., Academic Press, 1970.

[19] *A. Okuyama:* On metrizability of M-spaces. Proc. Japan. Acad. *40* (1964), p. 176—179.

[20] *A. Šostak:* О кружевных абсолютных ретрактах. Сб. научных трудов Латвийского Госуниверситета „Топологические пространства и их отображения", (to appear in 1984.)

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