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PROFESSOR JÁN JAKUBÍK SEXAGENARIAN

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The development of Mathematics as a scientific discipline in Slovakia started later than in the other European countries. There were not even the minimal conditions for it during the Austro-Hungarian monarchy. Slovakia's economy was underdeveloped and Slovaks had no schools of their own – even the elementary education was in Hungarian. It was only after the World War I that a university with permanent existence was founded in Slovakia; however, its programme included only humanities

and medicine. Thus the development of mathematical research in the true sense of the word started only after the World War II, being favourably affected by close relations to mathematical centres in Prague and Brno, which represented a very high standard. O. Borůvka lectured at Bratislava for many years; occasionally, E. Čech, F. Vyčichlo and others came to help. The Slovak professors J. Hronec and Š. Schwarz succeeded in forming — in a relatively short period of time — a group of young mathematicians who engaged themselves in research. The most successful among them was Ján Jakubík.

Ján Jakubík was born on October 8, 1923 at Dudince as a farmer's son, studied secondary school at Banská Štiavnica and Mathematics and Physics at Bratislava University. In 1949 he joined the Slovak Technical University, Bratislava, but in 1952 he moved to the newly founded Technical University at Košice (Eastern Slovakia). There he was appointed Associated Professor (1956) and Full Professor (1963). In the same year he obtained the Doctor of Science degree. Shortly after he was elected corresponding member of the Slovak (1964) and Czechoslovak (1965) Academies of Sciences. In the year 1977 he was elected ordinary member (Academician) of the Slovak and Czechoslovak Academies.

The scientific work of J. Jakubík is so extensive and comprehensive (more than 100 papers) that it would be hardly possible to give its satisfactory survey. Therefore we shall give only brief information about some of his papers, referring the reader to [e], [f], [i] for further details.

The first works of J. Jakubík (except for three papers from mathematical analysis) dealt with various problems from algebra. His further research was strongly influenced by the study of monographs of G. Birkhoff [a] (Professor Borůvka recommended the study of the first edition to a group of mathematicians in Bratislava) and of L. V. Kantorovič, B. Z. Vulich and A. G. Pinsker [d] (recommended to him by Professors E. Čech and M. Katětov). Problems proposed in Birkhoff's monographs considerably affected the development of the theory of lattices, which started intensively in the post-war years. Among Czechoslovak mathematicians who were associated with these topics, Jakubík was the most successful, having solved the largest number of Birkhoff's problems. The study of the monograph [d] probably played its part in the fact that Jakubík later studied mostly (partially) ordered groups and lattice ordered groups (l-groups). The monograph [d] directly stimulated for example his papers [29], [30], [35], [37]. In some papers, e.g. [80], Jakubík discussed the problem of transferring the theorems on vector lattices ($\mathscr{K}$-lineals in the terminology of [d]) to l-groups.

Since the beginning of his research work, Jakubík paid special attention to decompositions of algebraic structures into direct products. Already his first work [1] concerns the problem of uniqueness of the decomposition of a lattice into the direct product of irreducible factors (provided such a decomposition exists; thus he answered in the affirmative Problem 11 from [a, 2nd ed.]) Further, let us mention only several examples from the number of papers concerning products. In papers [28], [30] he found necessary and sufficient conditions for two direct decompositions of an ordered group to have a common refinement. Simultaneously he proved that for directed ordered groups the decomposition of the corresponding ordered set into a direct product yields a decomposition of the given ordered group into such a product as well.

Let us note that Jakubík found some more cases when the behaviour of an ordered group $(G; +, \leqq)$ depends only on the properties of the ordered set $(G; \leqq)$. This is the case, for instance, with the decomposability of an l-group into a completely semi-direct product of linearly ordered groups [27].

In [19] Jakubík proved that an infinitely distributive complete lattice can be decomposed into the direct product of two lattices such that the first is decomposable into the direct product of directly irreducible factors while the other, provided it contains more than one element, is (nontrivially) directly decomposable and all its direct factors have the same property. Moreover, such a decomposition is unique (up to isomorphisms). In [68] sufficient conditions are found for an l-group to be a direct factor of every l-group in which it forms an l-ideal.

An ordered set P is called discrete if any of its bounded chains is finite; almost discrete, if for every $a, b \in P$, $a < b$, there is a finite sequence $a = a_0 < a_1 < \ldots \ldots < a_n = b$, each interval $[a_{i-1}, a_i]$ of which is directly irreducible. In [58] it is proved that every almost discrete ordered set is a weak product of directly irreducible ordered sets. In particular, every discrete lattice is a weak product of directly irreducible lattices, which implies – as shown in [54] – a necessary and sufficient condition for two discrete modular lattices in order that any isomorphism of their graphs may imply their lattice isomorphism (partial solution of Birkhoff's Problem 8 [a, 2nd ed.]). The proof of existence of isomorphic refinements of any two decompositions of an ordered group in a mixed product with directed factors [50] generalizes results by L. Fuchs [b, Chap. II, Thm. 9] and A. I. Mal'cev [g]. An analogous theorem for lexicographic products of directed groupoids is proved in [38].

A series of Jakubík's papers concerns the problems of embedding and extension of ordered groups. In [35] he proved that every ordered commutative group can be embedded in an ordered commutative divisible group. This result is applied to prove that an Archimedean l-group G can be embedded in a K-space; hence, by virtue of [d, Chap. XIII, Thm. 3.11] G can be represented by real functions. In [83] Jakubík introduced and studied the concept of the generalized Dedekind completion $D_1(G)$ of an l-group G, which is a generalization of the Dedekind completion defined only for Archimedean l-groups. Existence of the Archimedean kernel implies that Archimedean groups form a radical class. In [83] and [84] it is proved that some properties are preserved when passing from G to $D_1(G)$. For instace, if G is the direct product of l-groups G_i, then $D_1(G)$ is the direct product of the l-groups $D_1(G)$.

Two l-groups are said to be lattice isomorphic if the corresponding lattices are isomorphic. For some classes of l-groups (complete, strongly projectable), Jakubík showed [35], [87] that if two l-groups of such classes are lattice isomorphic, then the same holds for their orthogonal completions.

Of the number of topologies on ordered sets, the most natural is the interval topology which, for the ordered set R of real numbers, coincides with the usual topology on R. The properties of this topology depend in a high degree on the properties of the given ordering. Let us consider the following properties of an ordered group G: (t) G is a topological group with respect to the interval topology; (h) the interval topology on G is Hausdorff; (o) the ordering on G is linear. It is easily seen that (o) implies (h). In [37], sufficient conditions are found for an l-group G in order that the converse implication might hold; if the l-group G is commutative, then the converse implication is always true. In [43] it is shown that if an ordered group G is a lexico-extension of its l-ideal S, then G and S either both possess the property (h) or none of them does. Some other cases when (h) implies (o) are also presented. The paper [32] gives a necessary condition for an l-group to have the property (t).

A number of Jakubík's recent works have been devoted to radical and torsion classes of l-groups. There are many important classes of l-groups that are not varieties. J. Martinez [h] introduced the concept of a torsion class of l-groups, which is wider than that of a variety. (Ch. Holland [c] proved that every variety of l-groups is a torion class.) Jakubík [75] proved that the class of torsion l-groups that are not varieties is rich: every ordinal α can be assigned a torsion class T_α, which is not a variety, so that for $\beta < \alpha$, T_β is a proper subclass of the class T_α. In [81] he introduced the concept of a radical class of an l-group, which is wider than the concept of a torsion class, and presented several examples of radical classes which are not torsion classes (e.g. Archimedean l-groups, complete l-groups, completely distributive l-groups; every torsion class is a radical class). He also studied the properties of the collection of radical classes ordered by inclusion, especially as concerns the prime intervals. Further properties of this collection are studied in [93], [94], [95], [98], [100].

In addition to his research papers, Jakubík has been author of a number of popularizing papers, reports and reviews. His many-years teaching activities at the Technical University and the Faculty of Science at Košice have contributed considerably to the mathematical education of young specialists. Today, many creative mathematicians are grateful to him for introducing them in the scientific work. He has also taken part in the organization of science in various committees and boards of the Academies. Prof. Jakubík's modesty, his prudence and tactful approach to all colleagues, students and friends have made him an extraordinarily desirable companion.

On the occasion of sixtieth anniversary of Prof. Jakubík's birthday, we extend our warmest congratulations to him, expressing our hope that for many years to come we shall have opportunity of sharing in the rich results of his creative energy.

LIST OF SCIENTIFIC PAPERS OF J. JAKUBÍK

[1] Uniqueness of decomposition of a lattice in a direct product. (Slovak.) Matem.-fyz. sborník SAV *1*, 1951, 45—50.

[2] О некоторых свойствах пар структур. Чехослов. матем. журн. *4*, 1954, 1—27 (with M. Kolibiar).

[3] О графическом изоморфизме структур. Чехослов. матем. журн. *4*, 1954, 131—141.

[4] On uniform convergence of continuous functions. (Slovak.) Matem.-fyz. časopis *4*, 1954, 154—161.
[5] On the graph isomorphism of semimodular lattices. (Slovak.) Matem.-fyz. časopis *4*, 1954, 162—177.
[6] Система отношений конгруэнтности в структурах. Чехослов. матем. журн. *4*, 1954, 248—273.
[7] О отношениях конгруэнтности на абстрактных алгебрах. Чехослов. матем. журн. *4*, 1954, 314—317.
[8] Congruence relations and weak projectivity in lattices. (Slovak.) Časopis pěst. matem. *80*, 1955, 206—216.
[9] Remark on absolutely convergent series. (Slovak.) Matem.-fyz. časopis *5*, 1955, 133—136.
[10] Прямые разложения единицы в модулярных структурах. Чехослов. матем. журн. *5*, 1955, 399—411.
[11] Прямые разложения вполне дистрибутивных структур. Чехослов. матем. журн. *5*, 1955, 488—491.
[12] On metric lattices. (Slovak.) Matem.-fyz. časopis *5*, 1955, 140—143.
[13] On the Jordan-Dedekind chain condition. Acta scientiarum mathematicarum *16*, 1955, 266—269.
[14] On convergence in linear spaces. (Slovak.) Matem.-fyz. časopis *6*, 1956, 57—67.
[15] On the existence algebras. (Slovak.) Časopis pěst. matem. *81*, 1956, 43—54.
[16] Об аксиомах теории мультиструктур. Чехослов. матем. журн. *6*, 1956, 426—430.
[17] Isomorphism of graphs of multilattices. (Slovak.) Acta Fac. Nat. Univ. Comen. Mathematica *1*, 1956, 255—264.
[18] Remark on the Jordan-Dedekind condition in Booleam algebras. (Slovak.) Časopis pěst. matem. *82*, 1957, 44—46.
[19] The center of infinitely distributive lattices. (Slovak.) Matem.-fyz. časopis *7*, 1957, 116—120.
[20] Note on the endomorphisms of lattices. (Slovak.) Časopis pěst. matem. *83*, 1958, 226—229.
[21] On permutable congruence relations in lattices. (Slovak.) Matem.-fyz. časopis *8*, 1958, 155—162.
[22] On chains in Boolean algebras. (Slovak.) Matem.-fyz. časopis *8*, 1958, 193—202.
[23] Konvexe Ketten in *l*-Gruppen. Časopis pěst. matem. *84*, 1959, 53—63.
[24] Об одном классе структурно упорядоченных групп. Časopis pěst. matem. *84*, 1959, 150—161.
[25] О главных идеалах в структурно упорядоченных группах. Чехослов. матем. журн. *9*, 1959, 528—543.
[26] Convex chains in partially ordered groups. (Slovak.) Matem.-fyz. časopis *9*, 1959, 236—242.
[27] Об одном свойстве структурно упорядоченных групп. Časopis pěst. matem. *86*, 1961, 318—330.
[28] Прямые разложения частично упорядоченных групп. Чехослов. матем. журн. *10*, 1960, 231—243.
[29] К теории частично упорядоченных групп. Časopis pěst. matem. *86*, 1961, 318—330. *11*,
[30] Прямые разложения частично упорядоченных групп, II. Чехослов. матем. журн. 1961, 490—515.
[31] Über eine Klasse von *l*-Gruppen. Acta Fac. Nat. Univ. Comen. Math. *6*, 1961, 267—273.
[32] The interval topology of an *l*-group. Matem.-fyz. časopis *12*, 1962, 209—211.
[33] Über Teilbünde der *l*-Gruppen. Acta scientiarum mathematicarum *23*, 1962, 249—254.
[34] Über ein Problem von Paul Jaffard. Archiv der Mathematik *14*, 1963, 16—21.
[35] Представления и расширения *l*-групп. Чехослов. матем. журн. *13*, 1963, 267—283.
[36] Die Jordan-Dedekindsche Bedingung im direkten Produkt von geordneten Mengen. Acta scientiarum mathematicarum *24*, 1963, 20—23.

[37] Interval topology of an l-group. Colloquium mathematicum *11*, 1963, 65—72.
[38] Лексикографические произведения частично упорядоченных группоидов. Чехослов. матем. журн. *14*, 1964, 281—305.
[39] Über halbgeordnete Gruppen mit verallgemeinerter Jordanscher Zerlegung. Revue Roumaine des Mathem. Pures Appl. *9*, 1964, 187—190.
[40] Über euklidische Verbände. Math. Annalen *155*, 1964, 334—342 (with *M. Kolibiar*).
[41] Über Verbandsgruppen mit zwei Erzeugenden. Czech. Math. J. *14*, 1964, 444—454.
[42] Kompakt erzeugte Verbandsgruppen. Math. Nachrichten *30*, 1965, 193—201.
[43] Intervalltopologie auf einer halbgeordneten Gruppe. Matem.-fyz. časopis *15*, 1965, 257 bis 272.
[44] Die Dedekindschen Schnitte im direkten Produkt von halbgeordneten Gruppen. Matem.-fyz. časopis *16*, 1966, 329—336.
[45] Higher degrees of distributivity in lattices and lattice ordered groups. Czech. Math. J. *18*, 1968, 356—376.
[46] Lattice ordered algebras generated by systems of ideals. Colloquium mathematicum *20*, 1969, 31—44.
[47] Disjoint subsets of a partially ordered group. Archiv der Mathematik *20*, 1969, 572—577.
[48] On some problems concerning disjointness in partially ordered groups. Acta Fac. Rer. Nat. Univ. Comen. Math. *22*, 1969, 47—56.
[49] Partially ordered groups with two disjoint elements. Colloquium mathematicum *21*, 1970, 39—44.
[50] The mixed product decompositions of partially ordered groups. Czech. Math. J. *20*, 1970, 184—206.
[51] L-subgroups of a lattice ordered group. Journ. London Math. Soc. *2*, 1970, 366—368.
[52] M-polars in lattices. Časopis pěst. matem. *95*, 1970, 252—255.
[53] On subgroups of a pseudo lattice ordered group. Pacific J. Math. *34*, 1970, 109—115.
[54] Weak product decompositions of discrete lattices. Czech. Math. J. *21*, 1971, 399—412.
[55] Cardinal properties of lattice ordered groups. Fundamonta Mathematicae *74*, 1972, 85—98.
[56] Distributivity in lattice ordered groups. Czech. Math. J. *22*, 1972, 108—125.
[57] Cantor-Bernstein theorem for lattice ordered groups. Czech. Math. J. *22*, 1972, 159—175.
[58] Weak product decompositions of partially ordered sets. Colloquium mathematicum *25*, 1972, 13—26.
[59] Lattice ordered groups with a basis. Math. Nachrichten 53, 1972, 217—236 (with *O. Dreveňák*).
[60] Homogeneous lattice ordered groups. Czech. Math. J. *22*, 1972, 325—337.
[61] Conditionally α-complete sublattices of a distributive lattice. Algebra universalis *2*, 1972, 255—261.
[62] Lattice ordered groups of finite breadth. Colloquium mathematicum *27*, 1973, 13—20.
[63] Center of a complete lattice. Czech. Math. J. *23*, 1973, 125—138.
[64] On σ-complete lattice ordered groups. Czech. Math. J. *23*, 1973, 164—174.
[65] Lattice ordered groups with complete epimorphic images. Colloquium mathematicum *21*, 1974, 21—28.
[66] Quasiorder on systems of directed sets. Matem. časopis *24*, 1974, 173—177.
[67] Normal prime filters of a lattice ordered group. Czech. Math. J. *24*, 1974, 91—96.
[68] Splitting property of lattice ordered groups. Czech. Math. J. *24*, 1974, 257—269.
[69] Conditionally orthogonally complete l-groups. Math. Nachrichten *65*, 1975, 153—162.
[70] Unoriented graphs of modular lattices. Czech. Math. J. *25*, 1975, 240—246.
[71] Modular lattices of locally finite length. Acta scientiarum mathematicarum *37*, 1975, 79—82.

[72] Center of a bounded lattice. Matem. časopis *25*, 1975, 339—343.
[73] Sublattices with saturated chains. Czech. Math. J. *25*, 1975, 442—444.
[74] Cardinal sums of linearly ordered groups. Czech. Math. J. *25*, 1975, 568—575.
[75] Products of torsion classes of lattice ordered groups. Czech. Math. J. *25*, 1975, 576—585.
[76] Lattice ordered groups with cyclic linearly ordered subgroups. Časopis pěst. matem. *101*, 1976, 88—90.
[77] Principal projection bands of a Riesz space. Colloquium mathematicum *36*, 1976, 195—203.
[78] Pairs of lattices with common congruence relations. Topics in lattice theory, Coll. Math. Soc. J. Bolyai, *14*, 1976, 171—183.
[79] W-isomorphisms of distributive lattices. Czech. Math. J. *26*, 1976, 330—338.
[80] Strongly projectable lattice ordered groups. Czech. Math. J. *26*, 1976, 642—652.
[81] Radical classes and radical mappings of lattice ordered groups. Symposia mathematica 21, Academic Press, New York—London, 1977, 451—477.
[82] Lattices with a third distributive operation. Math. Slovaca *27*, 1977, 287—292 (with *M. Kolibiar*).
[83] Archimedean kernel of a lattice ordered group. Czech. Math. J. *28*, 1978, 140—154.
[84] Generalized Dedekind completion of a lattice ordered group. Czech. Math. J. *28*, 1978, 294—311.
[85] Maximal Dedekind completion of a lattice ordered group. Czech. Math. J. *28*, 1978, 611—631.
[86] Orthogonal hull of a strongly projectable lattice ordered group. Czech. Math. J. *28*, 1978, 484—504.
[87] On algebraic operations of a lattice ordered group. Colloquium mathematicum *61*, 1979, 35—44.
[88] Generalized lattice identities in lattice ordered groups. Czech. Math. J. *30*, 1980, 127—134.
[89] Isometries of lattice ordered groups. Czech. Math. J. *30*, 1980, 142—152.
[90] Weak isomorphisms of Abelian lattice ordered groups. Czech. Math. J. *30*, 1980, 438—444.
[91] Products of radical classes of lattice ordered groups. Acta Math. Univ. Comenianae *39*, 1980, 31—42.
[92] On isometries of non-Abelian lattice ordered groups. Math. Slovaca *31*, 1981, 171—175.
[93] On value selectors and torsion classes of lattice ordered groups. Czech. Math. J. *31*, 1981, 306—313.
[94] Prime selectors and torsion classes of lattice ordered groups. Czech. Math. J. *31*, 1981, 325—337.
[95] On the lattice of torsion classes of lattice ordered groups. Czech. Math. J. *31*, 1981, 510—513.
[96] On linearly ordered subgroups of a lattice ordered group. Časopis pěst. matem. *107*, 1982, 175—179.
[97] Projectable kernel of a lattice ordered group. Universal algebra and applications, Banach Center Publ. Vol. *9*, 1982, 105—112.
[98] Trosion radicals of lattice ordered groups. Czech. Math. J. *32*, 1982, 347—363.
[99] On the lattice of radical classes of linearly ordered groups. Studia scientiarum mathem. Hungarica (to appear).
[100] Distributivity of intervals of torsion radicals. Czech. Math. J. *32*, 1982, 548—555.
[101] On lexico extensions of lattice ordered groups. Math. Slovaca *33*, 1983, 81—84.
[102] On the lattice of semisimple classes of linearly ordered groups. Časopis pěst. matem. (submitted).
[103] On *K*-radical classes of lattice ordered groups. Czech. Math. J. *33*, 1983, 149—163.
[104] Isometries of multilattice groups. Czech. Math. J. (submitted; with *M. Kolibiar*).

References

[a] *G. Birkhoff:* Lattice Theory, Amer. Math. Soc. Colloq. Publ., first edition 1940; second edition 1948; third edition 1967, Providence, R. I.

[b] *L. Fuchs:* Partially ordered algebraic systems, Pergamon Press, Oxford—London—New York—Paris 1963.

[c] *Ch. Holland:* Varieties of *l*-groups are torsion classes. Czech. Math. J. *29* (*104*) (1979), 11—12.

[d] *Л. В. Канторович, Б. З. Вулих, А. Г. Пинскер:* Функциональный анализ в полуупорядоченных пространствах, Москва 1950.

[e] *M. Kolibiar:* Professor Ján Jakubík quinquagenarian. (Slovak) Matem. časopis *23* (1973), 295—296.

[f] *M. Kolibiar:* Academician Ján Jakubík awarded the Klement Gottwald state prize. Czech. Math. J. *29* (*104*) (1979), 664—665.

[g] *А. И. Мальцев:* Об упорядоченных группах. Изв. Акад. Наук СССР, серия матем. *13* (1949), 473—482.

[h] *J. Martinez:* Torsion theory for lattice ordered groups. Czech. Math. J. *25* (*100*) (1975), 284—299.

[i] *F. Šik:* Slovak state prizes awarded. Czech. Math. J. *20* (*5*) (1970), 349—351.