Czechoslovak Mathematical Journal

Ján Regenda

On the oscillation of solutions of a class of linear fourth order differential equations

Czechoslovak Mathematical Journal, Vol. 33 (1983), No. 1, 141–148

Persistent URL: `http://dml.cz/dmlcz/101865`

Czechoslovak Mathematical Journal, 33 (108) 1983, Praha

ON THE OSCILLATION OF SOLUTIONS OF A CLASS OF LINEAR FOURTH ORDER DIFFERENTIAL EQUATIONS

JÁN REGENDA, Bratislava

(Received February 18, 1982)

1. INTRODUCTION

In the present paper we shall consider the differential equation

$$L[y] = y^{(4)} + P(t)\, y'' + R(t)\, y' + Q(t)\, y = 0\,, \tag{R}$$

where $P(t), R(t), Q(t)$ are real-valued continuous functions on the interval $I = \langle a, \infty), -\infty < a < \infty$.

In order to prove both the preparatory and the main results of this paper, we shall use the following assumptions

$$P(t) \leqq 0\,, \quad R^2(t) \leqq 2\, P(t)\, Q(t) \quad \text{for all} \quad t \in I\,, \tag{A}$$

$$P(t) \leqq 0\,, \quad R(t) \leqq 0\,, \quad R^2(t) \leqq 2\, P(t)\, Q(t) \quad \text{for all} \quad t \in I \tag{B}$$

and $Q(t)$ not identically zero in any subinterval of I.

One can verify easily that the above assumptions are satisfied if $P(t) \leqq R(t) \leqq 0$, $2\, Q(t) \leqq R(t)$ for all $t \in I$.

Simple examples show that under the assumptions (A), (B) the equation (R) includes oscillatory equations as well as nonoscillatory ones.

The study of the oscillatory behaviour of solutions of linear fourth order differential equations goes to Leighton and Nehari [2] and has received a great deal of attention up to present. For typical results on the subject we refer to the papers [1, 3, 5, 6].

A necessary and sufficient condition is given for the oscillation of the differential equation (R) in terms of the behaviour of nonoscillatory solutions. At the same time necessary and sufficient condition is derived for the nonoscillation of the equation (R).

The results of this paper are obtained by methods similar to those of the paper [4].

A nontrivial solution of a differential equation of the n-th order is called *oscillatory* if its set of zeros is not bounded from above. Otherwise, it is called *nonoscillatory*. A differential equation of the n-th order will be called *nonoscillatory*, when all its solutions are nonoscillatory; *oscillatory*, when at least one of its solutions is oscillatory. A differential equation of the n-th order is said to be *disconjugate* in an interval I iff every nontrivial solution has at most $n - 1$ zeros in I.

Let $C^n(I)$ denote the set of all real-valued functions such that its n-th derivatives are continuous in I.

2. PRELIMINARIES

We begin by formulating preparatory results which are needed in proving the main theorem in Section 5.

Lemma 1 [4]. *Let $A(t, s)$ be a nonnegative and continuous function for $t_0 \leqq s \leqq t$ (nonpositive for $a \leqq t \leqq s \leqq t_0$). If $g(t)$, $\varphi(t)$ $(\psi(t))$ are continuous functions in the interval $\langle t_0, \infty)$ $(\langle a, t_0\rangle)$ and*

$$\varphi(t) \leqq g(t) + \int_{t_0}^{t} A(t, s)\, \varphi(s)\, \mathrm{d}s \quad \textit{for} \quad t \in \langle t_0, \infty)$$

$$\left(\psi(t) \geqq g(t) + \int_{t_0}^{t} A(t, s)\, \psi(s)\, \mathrm{d}s\,, \quad \textit{for} \quad t \in \langle a, t_0\rangle\right),$$

then every solution $y(t)$ of the integral equation

$$y(t) = g(t) + \int_{t_0}^{t} A(t, s)\, y(s)\, \mathrm{d}s \tag{1}$$

satisfies the inequality

$$y(t) \geqq \varphi(t) \ \textit{in}\ \langle t_0, \infty) \quad (y(t) \leqq \psi(t) \ \textit{in}\ \langle a, t_0\rangle)\,.$$

If we suppose in addition that $g(t) \geqq 0$ for $t \in \langle t_0, \infty)$ $(g(t) \leqq 0$ for $t \in \langle a, t_0\rangle)$, then the solution $y(t)$ of (1) satisfies the inequality

$$y(t) \geqq g(t) \geqq 0 \ \text{for}\ t \in \langle t_0, \infty) \quad (y(t) \leqq g(t) \leqq 0 \ \text{for}\ t \in \langle a, t_0\rangle)\,.$$

Lemma 2. *Suppose that* (A) *holds and let $y(t)$ be a nontrivial solution of* (R) *satisfying the initial conditions*

$$y(t_0) = y_0 \geqq 0\,, \quad y'(t_0) = y'_0 = 0\,, \quad y''(t_0) = y''_0 \geqq 0\,, \quad y'''(t_0) = y'''_0 \geqq 0$$

$(t_0 \in I$ arbitrary). Then $y^{(i)}(t) > 0$ for all $t > t_0$, $i = 0, 1, 2, 3$.

Under the additional assumption $R(t) \leqq 0$, we can replace $y'(t_0) = y'_0 = 0$ by $y'(t_0) = y'_0 \geqq 0$ and the conclusion of the Lemma is valid.

Proof. The initial-value problem

$$L[y] = 0\,, \quad y(t_0) = y_0\,, \quad y'(t_0) = y'_0\,, \quad y''(t_0) = y''_0\,, \quad y'''(t_0) = y'''_0$$

is equivalent to the following Volterra's integral equation

$$y'''(t) = g(t) + \int_{t_0}^{t} A(t, s)\, y'''(s)\, \mathrm{d}s\,,$$

where

$$g(t) = y_0''' - y_0'' \int_{t_0}^{t} \left[P(s) + (s - t_0) R(s) + \frac{(s - t_0)^2}{2} Q(s) \right] \mathrm{d}s -$$

$$- y_0' \int_{t_0}^{t} [R(s) + (s - t_0) Q(s)] \, \mathrm{d}s - y_0 \int_{t_0}^{t} Q(s) \, \mathrm{d}s$$

and

$$A(t, s) = - \int_{s}^{t} \left[P(\xi) + (\xi - s) R(\xi) + \frac{(\xi - s)^2}{2} Q(\xi) \right] \mathrm{d}\xi \, .$$

The hypotheses of the Lemma imply that $g(t) > 0$ and $A(t, s) \geqq 0$ for $t \in (t_0, \infty)$. Then by Lemma 1, $y'''(t) \geqq g(t) > 0$ for all $t \in (t_0, \infty)$. Hence the assertion of the Lemma follows.

Lemma 3. *Suppose that* (A) *holds and let* $y(t)$ *be a nontrivial solution of* (R) *satisfying the initial conditions*

$$y(t_0) = y_0 \geqq 0 \, , \quad y'(t_0) = y_0' = 0 \, ,$$

$$y''(t_0) = y_0'' \geqq 0 \, , \quad y'''(t_0) = y_0''' \leqq 0 \, , \quad t_0 \in I \, .$$

Then

$$(-1)^i \, y^{(i)}(t) > 0 \quad \textit{for all} \quad t \in \langle a, t_0) \, , \quad i = 0, 1, 2, 3 \, .$$

The proof is similar to that of Lemma 2 and will be omitted.

Note that if y is a solution of (R), then so is $-y$. Hence it follows from Lemma 2 that $y(t_0) \leqq 0$, $y'(t_0) = 0$, $y''(t_0) \leqq 0$, $y'''(t_0) \leqq 0$ (but not all zero) implies $y(t) < 0$, $y'(t) < 0$, $y''(t) < 0$, $y'''(t) < 0$ for all $t > t_0$. Similarly, it follows from Lemma 3 that if y is a nontrivial solution such that $y(t_0) \leqq 0$, $y'(t_0) = 0$, $y''(t_0) \leqq 0$ and $y'''(t_0) \geqq 0$ (but not all zero), then $y(t) < 0$, $y'(t) > 0$, $y''(t) < 0$ and $y'''(t) > 0$ for all $t \in \langle a, t_0)$.

Lemma 4. *Suppose that* (A) *holds. Then for every nontrivial nonoscillatory solution* $u(t)$ *of the equation* (R) *there exists a number* $\tau \geqq a$ *such that either*

$$u(t) \, u'(t) > 0 \quad \textit{for} \quad t > \tau$$

or

$$u(t) \, u'(t) < 0 \quad \textit{for} \quad t > \tau \, .$$

Proof. Let $u(t)$ be a nontrivial nonoscillatory solution of (R). Then there exists a number $b \geqq a$ such that $u(t) \neq 0$ in $\langle b, \infty)$. Assume, without loss of generality, that $u(t) > 0$ in $\langle b, \infty)$. In order to prove the Lemma we will first show that $u'(t)$ can change from negative to positive values at most twice in the interval $\langle b, \infty)$. Let s_1 and s_2 $(b \leqq s_1 < s_2)$ be any two consecutive points at which $u'(t)$ changes from negative to positive values. Then the solution $u(t)$ satisfies the following conditions $u(s_i) > 0$, $u'(s_i) = 0$ and $u''(s_i) \geqq 0$, $i = 1, 2$. If $u'''(s_2) \geqq 0$ then by Lemma 2 we have $u'(t) > 0$ for $t > s_2$. If $u'''(s_2) \leqq 0$ then by Lemma 3 we hawe $u'(t) < 0$ for $t \in \langle a, s_2)$, which contradicts the assumption $u'(s_1) = 0$. This establishes our above assertion.

Hence there exists a number $\tau_0 \geqq s_1 \geqq b$ such that either $u'(t) \geqq 0$ or $u'(t) \leqq 0$ for all $t \geqq \tau_0$.

We note that $u'(t)$ is not identically zero in any subinterval of I since $u(t)$ = constant is not a solution of (R).

We will now show that there exists at most one point $\tau > \tau_0$ such that $u'(\tau) = 0$. In fact, if $u'(t) \geqq 0$ for $t \geqq \tau_0$ and $u'(t)$ has a zero at some point $\tau > \tau_0$ then $u''(\tau) = 0$ and $u'''(\tau) \geqq 0$. Hence the solution $u(t)$ satisfies the conditions $u(\tau) > 0$, $u'(\tau) = 0$, $u''(\tau) = 0$ and $u'''(\tau) \geqq 0$, so that $u(t) > 0$ and $u'(t) > 0$ for $t > \tau$ by Lemma 2. In order to prove this assertion in the case if $u'(t) \leqq 0$ for $t \geqq \tau_0$, suppose that $u'(t)$ has two zeros τ_0, τ_1, $\tau_0 < \tau < \tau_1$. Then it follows that $u''(\tau_1) = 0$ and $u'''(\tau_1) \leqq 0$. Hence $u(t)$ satisfies the conditions $u(\tau_1) > 0$, $u'(\tau_1) = 0$, $u''(\tau_1) = 0$, $u'''(\tau_1) \leqq 0$, so that $u(t) > 0$ and $u'(t) < 0$ for $t \in \langle a, \tau_1)$ by Lemma 3 which contradicts $u'(\tau) = 0$. This completes the proof of the Lemma.

Lemma 5. *Let there be functions* $w_i(t) \in C^4\langle t_0, \infty)$, $i = 1, 2, 3$, $t_0 \in I$ *with the properties*

$$w_2 > 0\,, \quad w_3 > 0\,,$$
$$W(w_1, w_2; t) > 0\,, \quad W(w_1, w_3; t) > 0\,, \quad W(w_2, w_3; t) > 0\,,$$
$$W(w_1, w_2, w_3; t) > 0 \quad \textit{for} \quad t \in (t_0, \infty)$$

and

$$L[w_1] \leqq 0\,, \quad L[w_2] \geqq 0\,, \quad L[w_3] \leqq 0 \quad \textit{for} \quad t \in (t_0, \infty)\,,$$

where $W(w_1, w_2, w_3; t)$, $W(w_i, w_\kappa; t)$ *denote the Wronskian determinants. Then the equation* (R) *is disconjugate in the interval* $\langle t_0, \infty)$ ([3], pp. 77–80).

Lemma 6 [6]. *Let* $c(t), f(t)$ *be functions of class* $C\langle t_0, \infty)$, *assume that the differential equation*

$$w'' + c(t)\, w = 0$$

is nonoscillatory and $f(t)$ *does not change its sign in* $\langle t_0, \infty)$. *Then also the differential equation*

$$w'' + c(t)\, w = f(t)$$

is nonoscillatory in $\langle t_0, \infty)$.

3. THE EXISTENCE OF MONOTONIC SOLUTIONS

Throughout the remainder of this paper let z_0, z_1, z_2 and z_3 denote the solutions of (R) defined on I by the initial conditions

$$z_i^{(j)}(a) = \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \quad \text{for} \quad i, j = 0, 1, 2, 3\,.$$

Theorem 1. *Suppose that* (A) *holds. There exists a solution* $y(t)$ *of* (R) *such that* $y(t) > 0$, $y'(t) > 0$, $y''(t) > 0$, $y'''(t) > 0$ *for all* $t > a$.

Proof. Let $y(t)$ be a solution of (R) which satisfies the initial conditions $y(a) > 0$, $y'(a) = 0$, $y''(a) > 0$ and $y'''(a) > 0$. Then by Lemma 2, $y(t) > 0, y'(t) > 0$, $y''(t) > > 0$, $y'''(t) > 0$ for all $t > a$.

Theorem 2. *Suppose that* (A) *holds. There exists a solution* $z(t)$ *of* (R) *such that* $z(t) > 0$, $z'(t) < 0$, $z''(t) > 0$ *and* $z'''(t) \leqq 0$ *for all* $t \in I$.

Proof. For each natural number $n > a$, let c_{0n}, c_{1n}, c_{2n} and c_{3n} be numbers satisfying

$$(2) \qquad \sum_{i=0}^{3} c_{in}\, z_i^{(j)}(n) = a_j\,, \quad j = 0, \ldots, 3\,; \quad a_1 = a_2 = a_3 = 0\,, \quad a_4 < 0\,.$$

Let $z_n(t) = c_{0n}\, z_0(t) + c_{1n}\, z_1(t) + c_{2n}\, z_2(t) + c_{3n}\, z_3(t)$. The existence of numbers c_{0n}, c_{1n}, c_{2n} and c_{3n}, satisfying the above conditions, is easy to verify.

Since z_0, z_1, z_2 and z_3 are linearly independent, $z_n(t)$ is a nontrivial solution of (R). Since for each natuıal number n, the sequences $\{c_{in}\}$, $i = 0, 1, 2, 3$ are bounded, there exists a sequence of integers $\{n_j\}$ such that the subsequences $\{c_{in_j}\}$ converge to numbers c_i, $i = 0, 1, 2, 3$. From (2) we see that $c_0^2 + c_1^2 + c_2^2 + c_3^2 = 1$. The sequences $\{z_{n_j}(t)\}$, $\{z'_{n_j}(t)\}$, $\{z''_{n_j}(t)\}$ and $\{z'''_{n_j}(t)\}$ converge uniformly on any finite subinterval of I to the functions $z(t)$, $z'(t)$, $z''(t)$ and $z'''(t)$, respectively, where $z(t)$ is a nontrivial solution of (R). By Lemma, 3 $(-1)^i\, z^{(i)}(t) \geqq 0$ for all $t \in I$ and $i = = 0, 1, 2, 3$. Since $z(t)$ is a nontrivial solution of (R), it is easy to show by the Uniqueness Theorem that there is no number $\tau \in I$ such that $z(\tau) = 0$. Further, we will show that there is no number $\tau \in I$ such that $z'(\tau) = 0$ or $z''(\tau) = 0$. In fact, if $z'(t)$ vanished at a point $\tau \in I$, it would then follow that $z'(t) = z''(t) = z'''(t) = 0$ for all $t \geqq \tau$, since $z'(t) \leqq 0$, $z''(t) \geqq 0$ and $z'''(t) \leqq 0$ on I. Since $z(t) > 0$ for all $t \in I$, $z'(t) > 0$ for all $t > \tau$ by Lemma 2, which would contradict the fact that $z'(t) \leqq 0$ for all $t \in I$. If $z''(t)$ vanished at a point $\tau \in I$, then by the same argument as used above, $z''(t) = z'''(t) = 0$ for all $t \geqq \tau$, so that $z'(t)$ would be equal to a negative constant for all $t \geqq \tau$. It follows that $z(t)$ would eventually become negative in $\langle \tau, \infty)$, which would be a contradiction. Therefore, it follows that $z(t)$ satisfies the requirements of the Theorem.

4. CONDITIONS FOR DISCONJUGATION

The following two theorems are proved from Lemma 5 in the same way as Theorems 4 and 5 were proved from Lemma 6 in [4]. Therefore the proofs will be omitted.

Theorem 3. *Let there be functions* $w_i(t) \in C^4\langle t_0, \infty)$, $i = 1, 2, 3$, $t_0 \in I$ *such that*

$$(4) \qquad \begin{aligned} &w_1(t) > 0\,, \quad w_1'(t) < 0\,, \quad w_1''(t) > 0 \quad \textit{for} \quad t \in \langle t_0, \infty)\,,\\ &w_2(t) > 0\,, \quad w_2'(t) > 0\,, \quad w_2''(t) \leqq 0 \quad \textit{for} \quad t \in \langle t_0, \infty)\,,\\ &w_3(t) > 0\,, \quad w_3'(t) > 0\,, \quad w_3''(t) > 0 \quad \textit{for} \quad t \in (t_0, \infty)\,, \quad w_3(t_0) = 0 \end{aligned}$$

and

$$L[w_1] \leqq 0\,, \quad L[w_2] \geqq 0\,, \quad L[w_3] \leqq 0 \quad \textit{for} \quad t \in (t_0, \infty)\,.$$

Then the equation (R) *is disconjugate on* $\langle t_0, \infty)$.

Theorem 4. *Let there be functions* $w_i(t) \in C^4\langle t_0, \infty)$, $i = 1, 2, 3$, $t_0 \in I$ *such that*

$$(5) \qquad \begin{aligned} &w_1(t) > 0\,, \quad w_1'(t) < 0\,, \quad w_1''(t) > 0\,, \quad w_1'''(t) \leqq 0 \quad \textit{for} \quad t \in \langle t_0, \infty)\,,\\ &w_2(t) > 0\,, \quad w_2'(t) > 0\,, \quad w_2''(t) > 0\,, \quad w_2'''(t) \leqq 0 \quad \textit{for} \quad t \in \langle t_0, \infty)\,,\\ &w_3(t) > 0\,, \quad w_3'(t) > 0\,, \quad w_3''(t) > 0\,, \quad w_3'''(t) > 0 \quad \textit{for} \quad t \in (t_0, \infty)\,,\\ &\qquad\qquad w_3(t_0) = w_3'(t_0) = 0 \end{aligned}$$

and

$$L[w_1] \leqq 0\,, \quad L[w_2] \geqq 0\,, \quad L[w_3] \leqq 0 \quad \textit{for} \quad t \in (t_0, \infty)\,.$$

Then the equation (R) *is disconjugate on* $\langle t_0, \infty)$.

The following consequences follow from Theorems 3 and 4.

Corollary 1. *Let* (R) *have solutions* w_1, w_2, w_3 *satisfying* (4).

Then the equation (R) *is disconjugate on* $\langle t_0, \infty)$.

Corollary 2. *Let* (R) *have solutions* w_1, w_2 *and* w_3 *satistying* (5).

Then the equation (R) *is disconjugate on* $\langle t_0, \infty)$.

The following sufficient conditions for (R) to be disconjugate are simple consequences of Theorems 1, 2, 3 and 4.

Corollary 3. *Suppose that* (A) *holds and let there be a function* $w \in C^4\langle t_0, \infty)$, $t_0 \in I$ *such that either* $w > 0$, $w' > 0$, $w'' \leqq 0$, $L[w] \geqq 0$ *or* $w > 0$, $w' > 0$, $w'' > 0$, $w''' \leqq 0$ *and* $L[w] \geqq 0$ *on* (t_0, ∞). *Then* (R) *is disconjugate on* $\langle t_0, \infty)$.

5. NECESSARY AND SUFFICIENT CONDITIONS FOR OSCILLATORY AND NONOSCILLATORY EQUATIONS

Theorem 5. *Suppose that* (B) *holds. Then the equation* (R) *is oscillatory if and only if for every nonoscillatory solution* $y(t)$ *of* (R) *we hawe either*

$$(6) \qquad y(t)\, y'(t) > 0\,, \quad y(t)\, y''(t) > 0\,, \quad y(t)\, y'''(t) > 0$$

on $\langle t_0, \infty)$ *for some* $t_0 \in I$, *or*

$$(6') \qquad y(t)\, y'(t) < 0$$

on I.

Proof. Suppose that (R) is oscillatory and let $y(t)$ be a nonoscillatory solution of (R). Then by Lemma 4 there exists a number $t_1 \in I$ such that either $y(t)\, y'(t) > 0$ or $y(t)\, y'(t) < 0$ for all $t \geqq t_1$. There is no loss of generality in assuming that $y(t) > 0$ for all $t \geqq t_1$. Substitution $y''(t) = u(t)$ into (R) leads to the following differential equation for u

$$(7) \qquad u'' + P(t)\, u = -R(t)\, y' - Q(t)\, y\,.$$

If $y'(t) > 0$ for all $t \geqq t_1$, then $-R(t)\, y' - Q(t)\, y$ does not change the sign in $\langle t_1, \infty)$. Since the equation $u'' + P(t)\, u = 0$ is nonoscillatory in $\langle t_1, \infty)$, it follows that equation (7) is nonoscillatory in $\langle t_1, \infty)$, by Lemma 6. Hence there exists a number $t_2 \geqq t_1$ such that $u(t) \neq 0$, i.e. $y''(t) \neq 0$ in $\langle t_2, \infty)$. From this it follows further that either $y''(t) > 0$ or $y''(t) < 0$ in $\langle t_2, \infty)$. We note that if $y''(t) > 0$, it then follows from (R) that $y^{(4)}(t) \geqq 0$ (not identically zero in any subinterval). Hence the following cases are possible

(a) $\qquad y(t) > 0\,, \quad y'(t) > 0\,, \quad y''(t) > 0\,, \quad y'''(t) > 0\,,$

(b) $\qquad y(t) > 0\,, \quad y'(t) > 0\,, \quad y''(t) > 0\,, \quad y'''(t) < 0\,,$

(c) $\qquad y(t) > 0\,, \quad y'(t) > 0\,, \quad y''(t) < 0\,,$

(d) $\qquad y(t) > 0\,, \quad y'(t) < 0$

for all $t \geqq t_0$, where t_0 is some number greater than or equal to t_2. Suppose that $y(t)$ does not satisfy the conditions (6), (6'). Then either (b) or (c) holds. If a solution satisfying condition (b) or (c) existed, then the equation (R) would be nonoscillatory by Corollary 3, contrary to the hypothesis. This completes the proof of the first half of Theorem 5.

The proof that (6) and (6') are sufficient for (R) to be oscillatory is the same as that of Theorem 3 ([1], p. 293) and will be omitted.

Remark 1. *If* (R) *is oscillatory, then it has three linearly independent oscillatory solutions.*

The proof of this is virtually the same as that of Theorem 4 ([1], p. 294).

Remark 2. We note that in view of Theorem 5 Remark 1, the conditions (6), (6') are equivalent to the existence of three linearly independent oscillatory solutions.

Theorem 6. *Suppose that* (B) *holds. Then the equation* (R) *is nonoscillatory on* I *if and only if there exists a number* $t_0 \in I$ *and a solution* $y(t)$ *of* (R) *such that either*

$$y(t) > 0\,, \quad y'(t) > 0\,, \quad y''(t) < 0\,,$$

or

$$y(t) > 0\,, \quad y'(t) > 0\,, \quad y''(t) > 0\,, \quad y'''(t) < 0$$

for all $t \geqq t_0$.

Proof. The sufficient condition follows from Corollary 3. It is easy to show that the existence of such a solution is also necessary. Indeed, if (R) is nonoscillatory there must exist a nonoscillatory solution $y(t)$ which does not satisfy the conditions (6), (6'). It then follows from the proof of Theorem 5 that there exists a number $t_0 \in I$ such that either $y(t) > 0$, $y'(t) > 0$, $y''(t) < 0$ or $y(t) > 0$, $y'(t) > 0$, $y''(t) > 0$, $y'''(t) < 0$ for all $t \geqq t_0$.

The following theorem is proved from Theorem 6 and Corollary 3 in the same way as Theorem 8 was proved from Theorem 7 and Corollaries 3 and 4 in [4]. The proof will be omitted.

Theorem 7. *Suppose that* (B) *holds. Then* (R) *is nonoscillatory on* I *if and only if there exists a function* $w(t) \in C^4\langle t_0, \infty)$, $t_0 \in I$, *such that either*

$$w(t) > 0\,, \quad w'(t) > 0\,, \quad w''(t) < 0\,, \quad L[w] \geqq 0$$

or

$$w(t) > 0\,, \quad w'(t) > 0\,, \quad w''(t) > 0\,, \quad w'''(t) < 0\,, \quad L[w] \geqq 0\,.$$

Theorem 8. *Suppose that* (B) *holds. Then* (R) *is nonoscillatory on* I *if and only if there exists a number* $t_0 \in I$ *such that* (R) *is disconjugate on* $\langle t_0, \infty)$.

The proof of this theorem is similar to that of Theorem 9 [4] and is omitted.

References

[1] *Ahmad Shair:* On the oscillation of solutions of a class of linear fourth order differential equations. Pac. J. Math., *34*, 1970.

[2] *Leighton W.* and *Nehari Z.:* On the oscillation of solutions of self-adjoint linear differential equations of the fourth order. Trans. Amer. Math. Soc., *89*, 1958.

[3] *Levin A. Ju.:* Neoscil'acija rešenij uravnenija $x^{(n)} + p_1(t)\, x^{(n-1)} + \ldots + p_n(t)\, x = 0$. Uspechi matem. nauk, T. *XXIV*, vyp. 2 (146), 1969.

[4] *Regenda J.:* Oscillatory and nonoscillatory properties of solutions of the differential equation $y^{(4)} + P(t)\, y'' + Q(t)\, y = 0$. Math. Slov., *28*, 1978, No 4.

[5] *Swanson C. A.:* Comparison and Oscillation Theory of Linear Differential Equations. New York and London 1968.

[6] *Švec M.:* On various properties of the solutions of third and fourth order linear differential equations. Proceedings of the conference held in Prague in September 1962.

Author's address: 812 31 Bratislava, Gottwaldovo nám. 17, ČSSR (Katedra matematiky a deskriptívnej geometrie Strojníckej fakulty SVŠT).