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SEMI-CONTINUITY AND WEAK-CONTINUITY

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INTRODUCTION

In 1961, N. Levine [8] defined a function f of a topological space X into a topological space Y to be *weakly-continuous* if for each $x \in X$ and each open neighborhood V of $f(x)$ there exists an open neighborhood U of x such that $f(U) \subset \mathrm{Cl}(V)$, where $\mathrm{Cl}(V)$ denotes the closure of V. A subset S of a topological space X is said to be *semi-open* if there exists an open set U of X such that $U \subset S \subset \mathrm{Cl}(U)$. The family of all semi-open sets in X is denoted by $\mathrm{SO}(X)$. In 1963, N. Levine also defined a function $f : X \to Y$ to be *semi-continuous* [9] if $f^{-1}(V) \in \mathrm{SO}(X)$ for every open set V of Y. It has been known that the semi-continuity is equivalent to the quasi-continuity [10, Theorem 1.1]. In 1969, N. Biswas [2] defined a function $f : X \to Y$ to be *semi-open* if $f(U) \in \mathrm{SO}(Y)$ for every open set U of X. In 1972, S. G. Crossley and S. K. Hildebrand [5] defined a function $f : X \to Y$ to be *irresolute* (resp. *pre-semi-open*) if for each $V \in \mathrm{SO}(Y)$ (resp. $U \in \mathrm{SO}(X)$), $f^{-1}(V) \in \mathrm{SO}(X)$ (resp. $f(U) \in \mathrm{SO}(Y)$). The purpose of the present paper is to investigate the interrelation among the weak-continuity, the semi-continuity and some weak forms of open functions. The main results of this paper, which contain two improvements of the results due to T. Neubrunn [11], are the following: (1) A semi-continuous function is irresolute if it is either weakly-open injective or almost-open in the sense of Singal. (2) A semi-open function is pre-semi-open if it is either weakly-continuous or almost-continuous in the sense of Husain. (3) A semi-continuous function is weakly-continuous if the domain is extremally disconnected.

1. IRRESOLUTE FUNCTIONS

Definition 1.1. A function $f : X \to Y$ is said to be *weakly-open* [17] if $f(U) \subset \subset \mathrm{Int}\,(f(\mathrm{Cl}(U)))$ for every open set U of X.

Definition 1.2. A function $f : X \to Y$ is said to be *almost-open* in the sense of Singal (simply a.o.S.) [18] of for every regular open set U of X, $f(U)$ is open in Y.

Definition 1.3. A function $f: X \to Y$ is said to be *almost-open* in the sense of Wilansky (simply a.o.W.) [20] if $f^{-1}(\mathrm{Cl}(V)) \subset \mathrm{Cl}(f^{-1}(V))$ for every open set V of Y, where f is not always injective.

We shall begin by investigating the relationships between semi-openness and the weak forms of openness defined above.

Lemma 1.4. *If $f: X \to Y$ is an a.o.S. function, then it is weakly-open.*

Proof. Let U be an open set of X. Since f is a.o.S., $f(\mathrm{Int}(\mathrm{Cl}(U)))$ is open in Y and hence $f(U) \subset f(\mathrm{Int}(\mathrm{Cl}(U))) \subset \mathrm{Int}\,(f(\mathrm{Cl}(U)))$.

The converse to Lemma 1.4 is not necessarily true as the following example shows.

Example 1.5. Let $X = \{a, b, c, d\}$ and $\sigma = \{X, \{a, b, d\}, \{a, b\}, \{d\}, \emptyset\}$. Let $Y = \{x, y, z\}$ and $\tau = \{Y, \{x, y\}, \{y, z\}, \{y\}, \{z\}, \emptyset\}$. Let $f: (X, \sigma) \to (Y, \tau)$ be a function defined as follows: $f(a) = x$, $f(b) = z$ and $f(c) = f(d) = y$. Then f is weakly-open but it is not a.o.S.

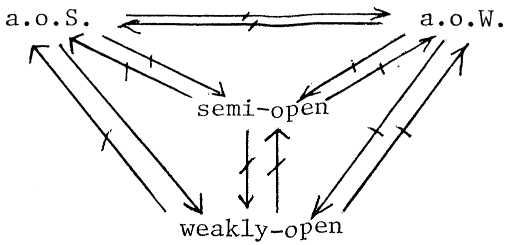
Example 1.6. Let $X = \{a, b, c, d\}$ and $\sigma = \{X, \{a, b, c\}, \{a, c, d\}, \{a, b\}, \{a, c\}, \{a\}, \{c\}, \emptyset\}$. Let $Y = \{x, y, z\}$ and $\tau = \{Y, \{x, y\}, \{z\}, \emptyset\}$. Consider a function $f: (X, \sigma) \to (Y, \tau)$ defined as follows: $f(a) = f(c) = y$, $f(b) = x$ and $f(d) = z$. Then f is a.o.W. but it is neither a.o.S. nor weakly-open.

Example 1.7. Let X be the real numbers with the cocountable topology σ, $Y = \{a, b\}$ with the topology $\tau = \{Y, \{a\}, \emptyset\}$ and $f: (X, \sigma) \to (Y, \tau)$ a function defined as follows: $f(x) = a$ if x is rational; $f(x) = b$ if x is irrational. Then f is a.o.S. but it is not a.o.W.

Example 1.8. Let $X = \{a, b, c, d\}$ and $\sigma = \{X, \{a, b, c\}, \{a, c, d\}, \{a, b\}, \{a, c\}, \{c, d\}, \{a\}, \{c\}, \emptyset\}$. Let $Y = \{x, y, z\}$ and $\tau = \{Y, \{x, y\}, \{z\}, \emptyset\}$. Define a function $f: (X, \sigma) \to (Y, \tau)$ as follows: $f(a) = x$, $f(b) = y$ and $f(c) = f(d) = z$. Then f is continuous, a.o.S. and a.o.W. but it is not semi-open.

Example 1.9. Let $X = Y = \{a, b, c\}$, $\sigma = \{X, \{b, c\}, \{a\}, \emptyset\}$ and $\tau = \{Y, \{a, b\}, \{a\}, \{b\}, \emptyset\}$. Let $f: (X, \sigma) \to (Y, \tau)$ be the identity function. Then f is semi-open (in fact, pre-semi-open) but it is neither a.o.W. nor weakly-open.

By Lemma 1.4 and the previous five examples, we obtain the following diagram, where $A \nrightarrow B$ means that A does not necessarily imply B.



In 1967, D. R. Anderson and J. A. Jensen [1] showed that every open and continuous function is irresolute. In 1977, T. Neubrunn proved that every open and somewhat continuous injection is irresolute [11, Theorem 3]. We shall show that the condition "open" in this result can be replaced by "weakly-open".

Definition 1.10. A function $f : X \to Y$ is said to be *somewhat continuous* [6] if, for each open set V of Y with $f^{-1}(V) \neq \emptyset$, there exists an open set U of X such that $\emptyset \neq U \subset f^{-1}(V)$.

Theorem 1.11. *If $f : X \to Y$ is a weakly-open somewhat continuous injection, then it is irresolute.*

Proof. Let $V \in \mathrm{SO}(Y)$ and $x \in f^{-1}(V)$. Put $y = f(x)$ and let U be any open neighborhood of x. Since f is weakly-open, we have

$$y \in f(U) \cap V \subset \mathrm{Int}\,(f(\mathrm{Cl}(U))) \cap V \in \mathrm{SO}(Y)\,.$$

By Lemma 4 of [13], there exists an open set G such that $\emptyset \neq G \subset \mathrm{Int}\,(f(\mathrm{Cl}(U))) \cap \cap V$. Since f is somewhat continuous and $f^{-1}(G) \neq \emptyset$, there exists an open set W of X such that $\emptyset \neq W \subset f^{-1}(G)$. Therefore, we obtain $W \subset \mathrm{Cl}(U) \cap f^{-1}(V)$ and hence $W \subset \mathrm{Cl}(U) \cap \mathrm{Int}\,(f^{-1}(V))$ because f is injective. Thus, we have $\emptyset \neq \mathrm{Cl}(U) \cap \cap \mathrm{Int}\,(f^{-1}(V))$ and hence $\emptyset \neq U \cap \mathrm{Int}\,(f^{-1}(V))$. This shows that $x \in \mathrm{Cl}(\mathrm{Int}\,(f^{-1}(V)))$ and $f^{-1}(V) \in \mathrm{SO}(X)$.

In 1976, the author showed that every a.o.W. semi-continuous function is irresolute [14, Theorem 1]. Although a.o.S. and a.o.W. are independent of each other, we have

Theorem 1.12. *If a function $f : X \to Y$ is a.o.S. and semi-continuous, then it is irresolute.*

Proof. Let $V \in \mathrm{SO}(Y)$. Then there exists an open set G of Y such that $G \subset V \subset \mathrm{Cl}(G)$; hence $f^{-1}(G) \subset f^{-1}(V) \subset f^{-1}(\mathrm{Cl}(G))$. Since f is semi-continuous, $f^{-1}(G) \in \in \mathrm{SO}(X)$ and hence $f^{-1}(G) \subset \mathrm{Cl}(\mathrm{Int}\,(f^{-1}(G)))$. Now, put

$$F = Y - f(X - \mathrm{Cl}(\mathrm{Int}\,(f^{-1}(G))))\,.$$

Then F is closed in Y because f is a.o.S. and $\mathrm{Cl}(\mathrm{Int}\,(f^{-1}(G)))$ is regular closed in X. By a straightforward calculation we obtain $G \subset F$ and $f^{-1}(F) \subset \mathrm{Cl}(\mathrm{Int}\,(f^{-1}(G)))$. Therefore, we have $f^{-1}(\mathrm{Cl}(G)) \subset \mathrm{Cl}(f^{-1}(G))$. Since $f^{-1}(G) \in \mathrm{SO}(X)$, we obtain $f^{-1}(V) \in \mathrm{SO}(X)$ by Theorem 3 of [9].

In Example 1 of [11], it was shown that an open somewhat continuous function is not necessarily irresolute. Therefore, the condition "semi-continuous" in Theorem 1.12 cannot be replaced by "somewhat continuous". On the other hand, it has been known that a semi-open and semi-continuous function is not necessarily irresolute [15, Example 11]. Thus, the condition "a.o.S." in Theorem 1.12 cannot be replaced by "semi-open". However, every semi-open semi-continuous function is necessarily irresolute if the range is extremally disconnected. To prove this fact we recall some definitions. Let S be a subset of a topological space X. A subset S is said to be *semi-closed* [3] if $X - S$ is semi-open in X. The intersection of all semi-closed sets containing S is called the *semi-closure* of S and denoted by $\underline{S}$ [3]. A topological space X is said to be *extremally disconnected* if the closure of every open set in X is open in X.

Lemma 1.13. *If a topological space X is extremally disconnected, then* $\mathrm{Cl}(U) = \underline{U}$ *for every* $U \in \mathrm{SO}(X)$.

Proof. In general, we have $\underline{S} \subset \mathrm{Cl}(S)$ for every subset S of X. Thus, we shall show that $\underline{U} \supset \mathrm{Cl}(U)$ for each $U \in \mathrm{SO}(X)$. Let $\emptyset \neq \underline{U} \in \mathrm{SO}(X)$ and $x \notin \underline{U}$, then there exists a $V \in \mathrm{SO}(X)$ such that $x \in V$ and $V \cap U = \emptyset$; hence $\mathrm{Int}(V) \cap \mathrm{Int}(U) = \emptyset$. Since X is extremally disconnected, we have $\mathrm{Cl}(\mathrm{Int}(V)) \cap \mathrm{Cl}(\mathrm{Int}(U)) = \emptyset$. Therefore, we have $x \notin \mathrm{Cl}(\mathrm{Int}(U)) = \mathrm{Cl}(U)$ [13, Lemma 2].

Theorem 1.14. *If a topological space Y is extremally disconnected and a function $f: X \to Y$ is semi-open semi-continuous, then f is irresolute.*

Proof. Let $V \in \mathrm{SO}(Y)$. There exists an open set G of Y such that $G \subset V \subset \mathrm{Cl}(G)$; hence $f^{-1}(G) \subset f^{-1}(V) \subset f^{-1}(\mathrm{Cl}(G))$. Since Y is extremally disconnected, we have $\underline{G} = \mathrm{Cl}(G)$ by Lemma 1.13. Since f is semi-open, it follows from Theorem 2 of [12] that $f^{-1}(\underline{G}) \subset \mathrm{Cl}(f^{-1}(G))$. Therefore, we obtain $f^{-1}(\mathrm{Cl}(G)) \subset \mathrm{Cl}(f^{-1}(G))$. Since f is semi-continuous, $f^{-1}(G) \in \mathrm{SO}(X)$ and hence $f^{-1}(V) \in \mathrm{SO}(X)$.

It may be noted that a semi-open continuous function is not necessarily irresolute if the range is not extremally disconnected [15, Example 19].

2. PRE-SEMI-OPEN FUNCTIONS

Definition 2.1. A function $f: X \to Y$ is said to be *almost-continuous* [7] if, for each $x \in X$ and each neighborhood V of $f(x)$, $\mathrm{Cl}(f^{-1}(V))$ is a neighborhood of x, where the topological spaces X and Y are not necessarily Hausdorff.

Definition 2.2. A function $f: X \to Y$ is said to be *somewhat open* [6] if, for each nonempty open set U of X, there exists an open set V of Y such that $\emptyset \neq V \subset f(U)$.

By Example 1 of [17], D. A. Rose showed that the almost-continuity is independent of the weak-continuity. In [10], A. Neubrunnová showed that almost-continuity and semi-continuity are independent of each other. A. Prakash and P. Srivastava [16] stated in Theorem 4 of [16] that the somewhat continuity is independent of the weak-continuity. Although the result is true, the reason is false. It follows from Example 3 of [16] that the weak-continuity does not necessarily imply the somewhat continuity. However, the function f in Example 4 of [16] is not almost-continuous in the sense of Singal [18] but it is weakly-continuous. We recall Example 1.9 here and notice that the inverse function f^{-1} is irresolute but it is not weakly-continuous. Therefore, the semi-continuity is independent of the weak-continuity.

In 1963, N. Levine showed that every open continuous function is pre-semi-open [9, Theorem 9]. In 1969, N. Biswas showed that every semi-open continuous function is pre-semi-open [2, Theorem 11]. Moreover, in 1977 T. Neubrunn improved the result as follows: Every somewhat open continuous function is pre-semi-open [11, Theorem 4]. We shall show that the condition "continuous" in the last result can be replaced by "weakly-continuous".

Theorem 2.3. *If a function $f : X \to Y$ is weakly-continuous somewhat open, then it is pre-semi-open.*

Proof. Let $A \in \mathrm{SO}(X)$ and $y \in f(A)$. Let V be any open neighborhood of y. There exists $x \in A$ such that $y = f(x)$. Since f is weakly-continuous, there exists an open neighborhood U of x such that $f(U) \subset \mathrm{Cl}(V)$. Since $x \in U \cap A \in \mathrm{SO}(X)$, there exists an open set W of X such that $\emptyset \neq W \subset U \cap A$. Moreover, since f is somewhat open, there exists an open set G of Y such that $\emptyset \neq G \subset f(W)$; hence $G \subset \mathrm{Cl}(V) \cap \cap f(A)$. Therefore, we have $G \subset \mathrm{Cl}(V) \cap \mathrm{Int}\,(f(A))$ and hence $V \cap \mathrm{Int}\,(f(A)) \neq \emptyset$. This shows that $y \in \mathrm{Cl}(\mathrm{Int}\,(f(A)))$ and hence $f(A) \subset \mathrm{Cl}(\mathrm{Int}\,(f(A)))$. Consequently, we obtain $f(A) \in \mathrm{SO}(Y)$.

Corollary 2.4. *Every weakly-continuous semi-open function is pre-semi-open.*

Proof. Since every semi-open function is somewhat open, this is an immediate consequence of Theorem 2.3.

The following theorem shows that the condition "continuous" in Theorem 11 of [2] can be replaced by "almost-continuous".

Theorem 2.5. *If a function $f : X \to Y$ is almost-continuous semi-open, then it is pre-semi-open.*

Proof. Let $U \in \mathrm{SO}(X)$. There exists an open set G of X such that $G \subset U \subset \mathrm{Cl}(G)$. Since f is almost-continuous, we have $f(\mathrm{Cl}(G)) \subset \mathrm{Cl}(f(G))$ by Theorem 10 of [17] and hence $f(G) \subset f(U) \subset \mathrm{Cl}(f(G))$. Since f is semi-open, we obtain $f(G) \in \mathrm{SO}(Y)$ and $f(U) \in \mathrm{SO}(Y)$.

Theorem 2.6. *If a topological space X is extremally disconnected and a function $f : X \to Y$ is semi-continuous semi-open, then f is pre-semi-open.*

Proof. Let $U \in \mathrm{SO}(X)$. There exists an open set G of X such that $G \subset U \subset \mathrm{Cl}(G)$. Since X is extremally disconnected, we have $\mathrm{Cl}(G) = \underline{G}$ by Lemma 1.13. Since f is semi-continuous, we obtain $f(\underline{G}) \subset \mathrm{Cl}(f(G))$ by Theorem 1.16 of [4] and hence

$f(G) \subset f(U) \subset \mathrm{Cl}(f(G))$. Since f is semi-open, we have $f(G) \in \mathrm{SO}(Y)$ and $f(U) \in \in \mathrm{SO}(Y)$.

By virtue of the following example due to Z. Piotrowski [15], we may notice that the condition "extremally disconnected" on X in Theorem 2.6 cannot be removed and also a semi-continuous open function is not necessarily pre-semi-open.

Example 2.7. Let $X = Y = \{a, b, c, d\}$, $\sigma = \{X, \{a, b\}, \{a\}, \{b\}, \emptyset\}$ and $\tau = \{Y, \{b, c, d\}, \{a, b\}, \{a\}, \{b\}, \emptyset\}$. Let $f : (X, \sigma) \to (Y, \tau)$ be the identity function. Then f is open and semi-continuous but it is not pre-semi-open. Moreover, X is not extremally disconnected.

3. WEAKLY-CONTINUOUS FUNCTIONS

As we have already noted, the semi-continuity is independent of the weak-continuity. In this section we shall give two sufficient conditions for a semi-continuous function to be weakly-continuous. For this purpose we need the following lemma.

Lemma 3.1. (Rose, [17]). *A function $f : X \to Y$ is weakly-continuous if and only if $\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$ for every open set V of Y.*

Theorem 3.2. *If a topological space X is extremally disconnected and a function $f : X \to Y$ is semi-continuous, then f is weakly-continuous.*

Proof. Let V be any open set of Y. Since f is semi-continuous, $f^{-1}(V) \in \mathrm{SO}(X)$ and $f^{-1}(V) \subset f^{-1}(\mathrm{Cl}(V))$ by Theorem 1.17 of [4]. Since X is extremally disconnected, it follows from Lemma 1.13 that $\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$. Thus, by Lemma 3.1 we obtain that f is weakly-continuous.

In Example 1.9, the topological space (Y, τ) is not extremally disconnected and the inverse function $f^{-1} : (Y, \tau) \to (X, \sigma)$ of f is semi-continuous but not weakly-continuous. Therefore, the condition "extremally disconnected" on X in Theorem 3.2 cannot be removed. A topological space X is aid to be *S-closed* [19] if for every semi-open cover $\{U_\alpha \mid \alpha \in \nabla\}$ of X there exists a finite subset ∇_0 of ∇ such that $X = \bigcup\{\mathrm{Cl}(U_\alpha) \mid \alpha \in \nabla_0\}$.

Corollary 3.3. *Let X be an S-closed regular space and Y a Hausdorff space. If a function $f : X \to Y$ is semi-continuous, then it is closed.*

Proof. Since X is S-closed regular, by Theorem 6 of [19] X is extremally disconnected and hence f is weakly-continuous by Theorem 3.2. Let F be any closed set of X. Every S-closed regular space is compact. Therefore, F is compact in X and hence $f(F)$ is H-closed in Y. Since Y is Hasudorff, $f(F)$ is closed in Y. This completes the proof.

A topological space X is said to be *dense in itself* [15] if, for each $x \in X$, the singleton $\{x\}$ is not open in X.

Theorem 3.4. *If a topological space Y is dense in itself and a function $f : X \to Y$ is pre-semi-open semi-continuous, then f is weakly-continuous.*

Proof. Assume that f is not weakly-continuous. By Lemma 3.1, there exists an open set V in Y such that $\mathrm{Cl}(f^{-1}(V)) \not\subset f^{-1}(\mathrm{Cl}(V))$. Hence, there exists $x \in \mathrm{Cl}(f^{-1}(V))$ such that $x \notin f^{-1}(\mathrm{Cl}(V))$. Since f is semi-continuous, $f^{-1}(V) \in \mathrm{SO}(X)$ and hence $f^{-1}(V) \cup \{x\} \in \mathrm{SO}(X)$. Since f is pre-semi-open, $H = f(f^{-1}(V) \cup \{x\}) \in \mathrm{SO}(Y)$. On the other hand, since $f(x) \notin \mathrm{Cl}(V)$, there exists an open neighborhood G of $f(x)$ such that $G \cap V = \emptyset$. Therefore, we have

$$f(x) \in G \cap H \subset G \cap (V \cup \{f(x)\}) = \{f(x)\} .$$

Thus, $\{f(x)\} = G \cap H \in \mathrm{SO}(Y)$. It follows from Lemma 4 of [13] that $\{f(x)\}$ is open in Y. This contradicts the assumption that Y is dense in itself. Therefore, f is weakly-continuous.

Corollary 3.5 (Piotrowski, [15]). *Let a topological space Y be regular and dense in itself. If a function $f : X \to Y$ is pre-semi-open semi-continuous, then it is continuous.*

Proof. This follows immediately from the result that a function $f : X \to Y$ is continuous if f is weakly-continuous and Y is regular [8, Theorem 2].

Corollary 3.6 (Anderson and Jensen, [1]). *Let a metric space Y be dense in itself. If a function $f : X \to Y$ is pre-semi-open semi-continuous, then it is continuous.*

Proof. Since a metric space is regular, this is an immediate consequence of Corollary 3.5.

References

[1] *D. R. Anderson* and *J. A. Jensen:* Semi-continuity on topological spaces, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) *42* (1967), 782—783.

[2] *N. Biswas:* On some mappings in topological spaces, Bull. Calcutta Math. Soc. *61* (1969), 127—135.

[3] *S. Gene Crossley* and *S. K. Hildebrand:* Semi-closure, Texas J. Sci. *22* (1971), 99—112.

[4] *S. Gene Crossley* and *S. K. Hildebrand:* Semi-closed sets and semi-continuity in topological spaces, Texas J. Sci. *22* (1971), 123—126.

[5] *S. Gene Crossley* and *S. K. Hildebrand:* Semi-topological properties, Fund. Math. *74* (1972), 233—254.

[6] *K. R. Gentry* and *H. B. Hoyle, III*: Somewhat continuous functions, Czech. Math. J. *21* (*96*) (1971), 5—12.

[7] *T. Husain:* Almost continuous mappings, Prace Mat. *10* (1966), 1—7.

[8] *N. Levine:* A decomposition of continuity in topological spaces, Amer. Math. Monthly *68* (1961), 44—46.
[9] *N. Levine:* Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly *70* (1963), 36—41.
[10] *A. Neubrunnová:* On certain generalizations of the notion of continuity, Mat. Časopis *23* (1973), 374—380.
[11] *T. Neubrunn:* On semihomeomorphisms and related mappings, Acta Fac. Rerum Natur. Univ. Comenian. Math. *33* (1977), 133—137.
[12] *T. Noiri:* Remarks on semi-open mappings, Bull. Calcutta Math. Soc. *65* (1973), 197—201.
[13] *T. Noiri:* On semi-continuous mappings, Atti Accad. Nat. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) *54* (1973), 210—214.
[14] *T. Noiri:* On semi-T_2 spaces, Ann. Soc. Sci. Bruxelles *90* (1976), 215—220.
[15] *Z. Piotrowski:* On semi-homeomorphisms, Boll. Un. Mat. Ital. (5) 16-A (1979), 501—509.
[16] *A. Prakash* and *P. Srivastava:* Somewhat continuity and some other generalisations of continuity, Math. Slovaca *27* (1977), 243—248.
[17] *D. A. Rose:* Weak continuity and almost continuity (unpublished).
[18] *M. K. Singal* and *A. R. Singal:* Almost-continuous mappings, Yokohama Math. J. *16* (1968), 63—73.
[19] *T. Thompson:* *S*-closed spaces, Proc. Amer. Math. Soc. *60* (1976), 335—338.
[20] *A. Wilansky:* Topics in Functional Analysis, Lecture Notes in Math., Vol. 45, Springer-Verlag, Berlin, 1967.

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