Jaroslav Kurzweil

To the sixtieth anniversary of birthday of Professor Marko Švec, DrSc.

Czechoslovak Mathematical Journal, Vol. 30 (1980), No. 1, 163–170

Persistent URL: http://dml.cz/dmlcz/101666

Czechoslovak Mathematical Journal, 30 (105) 1980, Praha

TO THE SIXTIETH ANNIVERSARY OF BIRTHDAY OF PROFESSOR MARKO ŠVEC, DRSC.

JAROSLAV KURZWEIL, Praha

On October 10, 1979, Professor MARKO ŠVEC, DrSc., a prominent specialist in the theory of differential equations and a university teacher who has educated and trained in mathematics whole generations of engineers, scientists and mathematicians, reached sixty years of age.

Marko Švec was born at Kmeťovo, district Nové Zámky (Slovakia). He attended secondary schools at Nové Zámky and Šurany and then studied mathematics and physics at the Faculty of Science of the Slovak University at Bratislava. He graduated from the university in 1944 and became teacher at secondary schools at Šurany and Bratislava. In 1949 he joined the Faculty of Electrical Engineering of the Slovak Technical University at Bratislava. He worked as lecturer till 1955, reader till 1966 and full professor during the years 1966–1968. Since 1968 he has been professor at the Department of Mathematical Analysis at Komenský University at Bratislava. In the years 1969–1972 and in 1974 he worked as UNESCO expert at the university at Bahia, Brazil. He was granted the RNDr. degree by the Faculty of Science of the Slovak University at Bratislava in 1949, the scientific degree of Candidate of Physico-Mathematical Sciences by the Faculty of Science of J. E. Purkyně University at Brno in 1957 and the scientific degree of Doctor of Physico-Mathematical Sciences by the Scientific Board of J. E. Purkyně University at Brno in 1965.

In his scientific work Marko Švec has dealt with a wide range of problems from the field of ordinary differential equations. He devoted himself with great effort to the investigation of asymptotic and oscillatory properties of differential equations of order higher than two, both linear and nonlinear; these difficult problems attracted him from the very beginning of his scientific career. He proved already in [2] that the equation

$$x^{(4)} + Q(t)\,x = 0 \tag{1}$$

with $Q(t) \geqq 0$ for $t \in \mathsf{R}$ has the following property:

(E) *Every nontrivial solution has at most one double zero point.*

Further he found a number of properties of the general fourth order linear differential equation which follow from the property (E). A result which is extraordinarily important and interesting is included in [5]: *If* $Q(t) \geqq 0$ *for* $t \geqq a$, *then*

(F) *all solutions of the equation* (1) *have the same character*

(i.e., they either are all oscillatory or none of them is).

M. Biernacki formulated a conjecture that under certain assumptions on Q there exist at least two linearly independent solutions of the equation (1) which tend to zero for $t \to \infty$ and that there exist solutions which are unbounded for $t \to \infty$. M. Švec proved in [6] that the conjecture on existence of two linearly independent solutions which tend to zero is correct under essentially weaker assumptions on Q. For the proof of existence of unbounded solutions he required the condition $0 < m \leqq Q(t) \leqq M < \infty$. The method used by Švec is worth special attention:

Let $Q(t) \geqq 0$ for $t \in \mathsf{R}$ and let the function Q be identically equal to zero on no open interval. Let us further assume that all solutions of the equation (1) oscillate for $t \to \infty$. Let S be the set of such solutions u of (1) that

$$\dot{u}(\varrho)\, \ddot{u}(\varrho)\, \dddot{u}(\varrho) \neq 0\,, \quad \operatorname{sgn} \dot{u}(\varrho) \neq \operatorname{sgn} \ddot{u}(\varrho) \neq \operatorname{sgn} \dddot{u}(\varrho)\,.$$

Švec proved a number of results concerning the set S; let us mention several of them:

(2) $S \neq \emptyset$ and there exist two linearly independent solutions which belong to S.

(3) If $u \in S$, then u is a bounded function for $t \to \infty$ and

$$\int^{\infty} Q u^2 \, \mathrm{d}t < \infty \,, \quad \int^{\infty} u^2 \, \mathrm{d}t < \infty \,.$$

(4) Let w be the trivial solution of the equation (1). Then $S \cup \{w\}$ is the set of solutions whose first derivative is bounded for $t \to \infty$. $S \cup \{w\}$ is a linear space of dimension 2.

(5) Let $0 < m \leqq Q(t)$. Then every solution $u \in S$ satisfies $\lim_{t \to \infty} u(t) = 0 = \lim_{t \to \infty} \dot{u}(t)$.

In the papers [10] and [11] the linear differential equation of the third order is studied in connection with the properties

(V_1) If a solution u has a double zero point ϱ, then $u(t) \neq 0$ for $t < \varrho$.

(V_2) If a solution u has a double zero point ϱ, then $u(t) \neq 0$ for $t > \varrho$.

It is proved that a third order equation has the properties (V_1), (V_2) if and only if each its solution has at most two zero points (or one double zero), and this is equivalent to the possibility of expressing the corresponding differential operator as a superposition of three differential operators of the first order. Sufficient conditions are found for the third order equation to have the property (V_1) or (V_2), and the relations between these properties, the properties of coefficients and the asymptotic and oscillatory properties of the solutions are established. Asymptotic formulae for solutions of the equation (1) (as well as for solutions of the analogous equation of the third order) are found in [7]. It is assumed that Q is a smooth function, $Q(t) > 0$ for $t \geqq a$, and that a certain integral diverges while some others converge. The paper [7] is thus an interesting supplement of the paper [6].

The paper [16] studies the relation between the oscillatory properties of the linear and nonlinear differential equation of the second order. Conditions are found which guarantee that the solutions of the nonlinear differential equation have analogous properties as those of the linear equation. In [20] it is proved that the equation

$$\ddot{y} = g(t, y)$$

has a T-periodic solution; it is assumed that the function $g : \mathrm{R}^2 \to \mathrm{R}$ is continuous, T-periodic with respect to the variable t and

$$\int_0^y g(t, u) \, \mathrm{d}u \geqq \alpha^2 y^2 + C \,, \quad \alpha \neq 0 \,.$$

This result is obtained by the variational method; the author uses the Ritz method to evaluate the maximum of the functional

$$\int_0^T [-y^2 - G(t, y)] \, dt \,,$$

where $G(t, y) = \int_0^y g(t, u) \, du$, and proves that the sequence of approximate solutions contains a subsequence with good convergence properties.

The paper [9] deals with oscillatory properties of solutions of the equation

$$y^{(n)} + f(t) \, y^\alpha = 0 \,,$$

the papers [3] and [4] extend the notion of dispersion introduced for second order linear differential equations by O. Borůvka and apply it to the study of properties of the equation (1) and of analogous equations of higher orders; certain boundary value problems are also investigated and existence of a family of eigenfunctions is established. The paper [1] is devoted to the multipoint boundary value problem for differential equations and systems of equations; very general conditions for existence of solutions are found.

The papers [12]–[15], [17]–[19] form a unity consistent in both the subject and the method. The oldest one is (according to the "received by the editors" date) the paper [15]. Here it is proved that the equation

$$y^{(n)} + Q(t) \, y = 0 \tag{6}$$

has a solution u which satisfies the conditions

$$(-1)^i \, u^{(i)}(t) > 0 \,, \quad i = 0, 1, 2, \dots, n - 1 \,, \tag{7}$$

$$\lim_{t \to \infty} u^{(i)}(t) = 0 \,, \qquad i = 1, 2, \dots, n - 1 \,, \tag{8}$$

$$\lim_{t \to \infty} u(t) = 1 \tag{9}$$

provided the function Q is nonnegative, not identically equal to zero on any interval and

$$\int^\infty t^{n-1} \, Q(t) \, dt = \infty \,.$$

This result is extended to the equation

$$y^{(n)} + B(t, y, y, \dots, y^{(n-1)}) \, y = 0 \,. \tag{10}$$

Here the function B is majorized in a suitable way. The proof makes use of the preceding result on the equation (6): To a given function v there exists a single solution u of the equation

$$x^{(n)} + B(t, v, v, \dots, v^{(n-1)}) \, x = 0$$

with the properties (7)–(9). We put $Tv = u$ and look for a fixed point of the mapping T; it is quite natural that the corresponding integral equations are introduced and the Schauder Fixed Point Theorem is applied. Also the other papers of this group concern the equation (10), namely the existence of its solutions that satisfy certain limit conditions for $t \to \infty$ and possibly also some initial conditions for $t = 0$. In all cases a fixed point for the corresponding integral operator is to be found. If the Schauder Theorem is used directly then it is necessary to show that certain families of functions defined on an unbounded interval are compact. Švec introduces the notion of q-convergence which is a certain form of point convergence. He makes use of the fact that the operators derived from the problem considered for the equation (10) are – on certain suitable sets of functions – continuous with respect to the q-convergence, and map these sets onto sets which are q-compact. This idea enabled the author to avoid considerable technical difficulties.

The equations

$$\dot{x} = Ax + f(t, x)\,, \tag{11}$$

$$\dot{x} = Ax \tag{12}$$

will be called equivalent if to each solution u of one of them there is a solution v of the other such that

$$u(t) - v(t) \to 0 \quad \text{for} \quad t \to \infty\,. \tag{13}$$

To find conditions for equivalence of the equations (11), (12) was the aim of Švec's papers [21], [22]. As an illustration let us present the following result:

Let the matrix A have the Jordan canonical form, let p be the maximum of orders of such blocks that the corresponding eigenvalue λ satisfies $\operatorname{Re} \lambda = 0$, and let us put $p = 1$ if no such block exists. Let

$$\|f(t, x)\| \leqq F(t, \|x\|)\,,$$

where F is continuous, nonincreasing with respect to the second variable, and let

$$\int^{\infty} t^{p-1} F(t, c)\, \mathrm{d}t < \infty \quad \text{for every} \quad c \in \mathrm{R}^{+}\,.$$

Then to every bounded solution u of the equation (11) there exists a solution v of the equation (12) such that (13) holds. An analogous argument is used to find a general sufficient condition for the asymptotic equivalence of the equations (11), (12). In [25], the problem of asymptotic equivalence is connected with the asymptotic properties of solutions, and sufficient conditions for asymptotic equivalence of general nonlinear n-th order differential equations are found. The results are extended also to functional differential equations. The properties of functional differential equations are studied in [23], [24]. The author investigates existence of $\lim_{t \to T-0} x(t)$, where x is a solution of a functional differential equation whose right hand side is defined for $t < T$, and solves a number of problems (dependence of the limit on the initial condition, existence of solution with a prescribed limit value).

Even this brief description of the scientific publications of Professor Švec shows the richness of his work both in subject and methods. He has developed a number of original ideas and procedures. His works are often quoted and highly appreciated by specialists both in Czechoslovakia and abroad.

Professor Švec has given definitive solution of numerous problems; on the other hand, he initiated further research by many authors. In the theory of ordinary differential equations a special role is played by various technical and often sofisticated tricks (the use of identities, inequalities, estimates etc.); M. Švec is an expert and master in using such technical devices, nevertheless, he has at the same time a rare ability of discovering general formulations and dealing with them. It is this unity of almost contradictory abilities which leads to results which are extremely valuable and interesting. Let us recall in this connection Marko Švec's study of the properties (E), (V_1), (V_2), the assertion (F) or his investigation of properties of the set S in the paper [6], the definition and application of the q-convergence.

Professor Švec has led a seminar on ordinary and functional differential equations for more than 20 years. This seminar has been regularly attended by research workers, university teachers and pre-doctoral students not only from Bratislava but also from other centres of mathematical research. Professor Švec initiated numerous investigations and his advice and ideas affected many scientists in this field. He guided and educated a number of pre-doctoral students; many of them have achieved remarkable results of international significance.

Professor Švec devotes himself with enthusiasm to the education of university students. He has educated a number of students of engineering and science, exciting in them a genuine interest for mathematics and its applications in engineering and science. He is co-author of an extensive textbook Mathematics I, II which includes those branches of mathematics which are most useful in applications and which are lectured at technical universities. Repeated editions of the book give clear evidence of how successfully it filled up the palpable gap in Czechoslovak mathematical literature.

Professor Švec has held a number of important offices in the organization of education and research in Czechoslovakia. He was Vice-Dean of the Faculty of Electrical Engineering of the Slovak Technical University in 1956–58. He is member of the Scientific Board of the Faculty of Science of Komenský University, member of editorial boards of the journals Acta Mathematica (Faculty of Science, Komenský Univ.) and Aplikace matematiky. Further, he is chairman or member of committees for scientific degrees in mathematical analysis, applications of mathematics and theory of mathematical education.

All those who have known Professor Marko Švec, and particularly those who had an opportunity of collaborating with and learning from him, offer their sincere congratulations on the occasion of the sixtieth anniversary of his birthday, and wish him good health and many further successes in both his scientific and educational activities.

LIST OF ORIGINAL PAPERS OF MARKO ŠVEC

[1] To the problem of uniqueness of integrals of the system of linear differential equations. (Slovak), Mat.-fyz. sborník SAV, 1952, 3—22

[2] Über einige neue Eigenschaften der oszillatorischen Lösungen der linearen homogenen Differentialgleichung vierter Ordnung. Czech. Math. J. *4* (*79*), 1954, 75—94

[3] Sur les dispersions des intégrales de l'équation $y^{(4)} + Q(x) y = 0$. Czech. Math. J., *5* (*80*), 1955, 26—60

[4] Eine Eigenwertaufgabe der Differentialgleichung $y^{(n)} + Q(x, \lambda) y = 0$. Czech. Math. J., *6* (*81*), 1956, 46—71

[5] Sur une propriété des intégrales de l'équation $y^{(n)} + Q(x) y = 0$, $n = 3, 4$. Czech. Math. J., *7* (*82*), 1957, 450—461

[6] Sur le comportement asymptotique des intégrales de l'équation $y^{(4)} + Q(x) y = 0$. Czech. Math. J., *8* (*83*), 1958, 230—244

[7] Asymptotische Darstellung der Lösungen der Differentialgleichung $y^{(n)} + Q(x) y = 0$, $n = 3, 4$. Czech. Math. J., *12* (*87*), 1962, 572—581

[8] On various properties of the solutions of third- and fourth-order linear differential equations. Equadiff 1962. Proceedings of the Conference held in Prague in September 1962, Differential Equations and Their Applications, 187—198

[9] Le caractère oscillatoire des solutions de l'équation $y^{(n)} + f(x) y^{\alpha} = 0$, $n > 1$. Czech. Math. J., *13* (*88*), 1963, 481—491 (with *I. Ličko*)

[10] Neskol'ko zamečanij o linejnom differencialnom uravnenii tretego porjadka. Czech. Math. J., *15* (*90*), 1965, 42—49

[11] Einige asymptotische und oszillatorische Eigenschaften der Differentialgleichung $y + A(x) y' + B(x) y = 0$. Czech. Math. J., *15* (*90*), 1965, 378—393

[12] Fixpunktsatz und monotone Lösungen der Differentialgleichung $y^{(n)} + B(x, y, \dot{y}, \dots, y^{(n-1)}) y = 0$. Archivum mathematicum (Brno), *2*, 1966, 43—55

[13] L'existence globale et les propriétes asymptotiques d'une équation différentielle nonlinéaire d'ordre *n*. Archivum mathematicum (Brno), *2*, 1966, 141—151

[14] Les propriétes asymptotiques des solutions d'une équation différentielle nonlinéaire d'ordre *n*. Czech. Math. J., *17* (*92*), 1967, 550—557

[15] Monotone solutions of some differential equations. Colloquium mathematicum, *XVIII*, 1967, 7—21

[16] Some oscillatory properties of second order non-linear differential equations. Ann. Mat. Pura ed Appl. (IV), vol. *LXXVII*, 179—192, 1967

[17] Investigation of the solutions of differential equations on an infinite interval and the fixed point theorems. Proc. of Equadiff. II (1966), Acta FRNUC, Mathematica 1967, 143—153

[18] Remark on the asymptotic behaviour of the solutions of the differential equations. Acta FRNUC, Mathematica *XXII*, 1969, 11—18

[19] Sur un problème aux limites. Czech. Math. J., *19* (*94*), 1969, 17—26

[20] Existence of periodic solutions of differential equations of second order. G.E.O. Giacaglia (ed.), Periodic Orbits, Stability and Resonances, D. Reidel Publishing Company, Dordrecht-Holland 1970, 168—175

[21] Some remarks on the asymptotic equivalence. Proc. of Equadiff 3, Brno 1972, 155—160

[22] Asymptotic relationship between solutions of two systems of differential equations. Czech. Math. J., *24* (*99*), 1974, 44—58

[23] Some properties of functional differential equations. Bolletino U.M.I. (4) 11 Suppl. fasc. *3* (1975), 467—477

[24] Some problems concerning the functional differential equations, Proceeding of Equadiff IV, Praha 1977, 405—414

[25] Asymptotic equivalence and oscillatory properties of ordinary differential equations. Equazioni differentiali ordinarie en equazioni funzionali convegno internazionale, Firenze 1978, 213—222

[26] Behaviour of nonoscillatory solutions of some nonlinear differential equations, Acta facultatis RNUC, in print

Main books

[1] *Kluvánek-Mišík-Švec:* Matematika I. Slov. vyd. techn. lit., str. 728, 1959 — 1. vydání, 1971 — 4. vydání

[2] *Kluvánek-Mišík-Švec:* Matematika II. Slov. vyd. techn. lit., str. 856, 1961 — 1. vydání, 1970 — 3. vydání

Biographical

M. Kolibiar, M. Švec: Academician Jur. Hronec (Slovak). Mat. fyz. čas. SAV, *X.* 1960, 123—131