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News and Notices. In memoriam Professor Svatopluk Fučík

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IN MEMORIAM PROFESSOR SVATOPLUK FUČÍK

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On Friday, May 18, 1979, both the Czechoslovak and the world's Mathematics suffered a severe loss. In the early hours of the day, RNDr. SVATOPLUK FUČÍK CSc., Professor of the Faculty of Mathematics and Physics, Charles University, Prague, succumbed to a heavy and insidious illness. The following lines are intended to recall to those who knew Professor Fučík the immense work he accomplished during his life time, and to offer an account of his personality and activities to those for whom it is no more possible to meet him.

Svatopluk Fučík was born on October 21, 1944 in Prague. He attended elementary and secondary schools in Hradec Králové and studied then mathematical analysis at the Faculty of Mathematics and Physics of Charles University at Prague in the years 1962–1967. The title of his diploma thesis was *Local Degree of Mappings*. The subject of this work led him to further investigations resulting in two papers A[1], A[2] on the basis of which he received his RNDr. degree in 1969. During the years 1967–1969 he was research student (aspirant) in the Department of Mathematical Analysis. This period resulted in his thesis *Solution of Nonlinear Operator Equations*. The beginnings as well as the whole first period of Fučík's more than ten years long scientific career are connected with the name of Professor J. Nečas DrSc. who was his teacher, adviser and collaborator.

Since 1969 till his death Fučík was member of the Department of Mathematical Analysis of the Faculty of Mathematics and Physics, Charles University, as Lecturer and Senior Lecturer. He wrote and defended his habilitation thesis in 1973; in 1977 he was appointed Reader in Mathematics.

Fučík's activity was not restricted merely to his work at University. He collaborated closely with the Mathematical Institute of the Czechoslovak Academy of Sciences and participated in several projects of the National Program of Basic Research. Since 1971 Fučík held various important offices in the Society of Czechoslovak Mathematicians and Physicists. Last but not least, it was thanks to him that, under the modest cover of the Information Bulletin of the Mathematical Scientific Section, the reader could find modern and interesting texts with clear traces of his individual humour.

Let us now give a brief survey of Fučík's scientific work.

It was at the very start of his dazzling scientific career that Professor Fučík manifested his interest in nonlinear functional analysis. In his diploma thesis he deals with a fundamental notion concerning nonlinear mappings in R^n , the degree of the mapping, and he adapts the definition due to E. Heinz. Fučík's interpretation of the degree appears then in the book D[1] which is the result of joint efforts of himself,



J. Nečas, J. Souček and V. Souček. As mentioned above, three other works are concerned with this problem, namely A[1], A[2], A[3]. In A[1] Fučík generalizes Rothe's theorem on fixed point. Paper A[2] deals also with the existence of a fixed point for the mapping $T = B + C$, where B is of contraction type while C has roughly the properties of a totally continuous mapping. Fučík's basic assertion represents a generalization of Kačurovskii-Krasnoselskii-Zabreiko's theorem. The last one of this series, A[3] concerns surjectivity of the operator $h = I + H$ when the norm of H is in a certain sense less than one.

The next period of Fučík's research work covers Fredholm Alternative for the nonlinear operator $\lambda T - S$. Fučík, unlike S. I. Pochožaiev who had introduced this

concept into nonlinear functional analysis together with J. Nečas, investigates operators T and S that map a Banach space X into an arbitrary Banach space Y and not only into X^* . He assumes that T is a (K, L, a) -homeomorphism of X onto Y :

$$L\|x\|_X^a \leq \|T(x)\|_Y \leq K\|x\|_X^a.$$

One of Fučík's versions of Fredholm Alternative reads: *Let T be a a -homogeneous operator, which is a (K, L, a) -homeomorphism, let S be odd, a -homogeneous, totally continuous mapping. Then $\lambda T - S$ is regularly surjective (i.e., the inverse mapping is bounded) if and only if λ is not an eigenvalue of the pair (T, S) .*

This subject is studied also in Fučík's papers A[5], A[6] and the results are included also in the book D[1].

Later on, Fučík's scientific activity expanded in many directions. Perhaps the most important research of this period is that concerning spectrum of the operator $\lambda f' - g'$ where f and g are two even functionals. Fučík together with J. Nečas generalized Lyusternik-Schnirelmann's theory of the existence of critical values and eigenvalues, see A[10]. The generalization concerns smoothness of the functionals f and g , so that the abstract theory can be applied also to spaces of the type L^p , $1 < p < 2$. The principal idea was that of replacing the homotopic deformations obtained the solution of the abstract differential equation by their approximation. Its main result (obtained with J. Nečas, J. Souček and V. Souček) is the assertion on denumerability of critical values of the functional g with respect to the manifold $f(x) = r$ for real analytic functionals f and g . The assertion is based on the work of J. and V. Souček on Morse's theorem for real analytic functions. The results just described as well as those of Fučík's paper A[9], A[11], A[12], A[13], A[15], A[18], A[19] were partially included in the book D[1].

A fundamental part of Fučík's mathematical work is devoted to the study of the range of nonlinearly perturbed noninvertible linear operators in Banach spaces, and to its applications to differential equations. Although his results cover abstract, partial and ordinary differential equations, we shall restrict ourselves in their description, for the sake of simplicity, mostly to the case of ordinary differential equations. Those who knew Svatopluk Fučík and his sense of humour may remember him qualifying ordinary differential equations as partial differential equations of dimension less than $\pi/3$.

Combining the alternative method with Schauder's fixed point theorem, Landesman and Lazer were the first to find in 1970 conditions upon $f \in L^2(0, \pi)$, necessary and sufficient for the Dirichlet problem

$$(1) \quad u'' + n^2u + g(u) = f(x), \quad u(0) = u(\pi) = 0$$

to have at least one solution, provided g is continuous and satisfies the assumption

$$(2) \quad -\infty < g(-\infty) < g(s) < g(+\infty) < +\infty, \quad s \in]-\infty, +\infty[,$$

where $g(\pm\infty)$ denotes the limits

$$\lim_{s \rightarrow \pm\infty} g(s),$$

which are assumed to exist. Motivated by these results and by a corresponding abstract version due to J. Nečas, Fučík applied a similar approach in a joint paper with M. Kučera and J. Nečas A[23] to the case where (2) is replaced by

$$(3) \quad -\infty < g(-\infty) \leq g(s) \leq g(+\infty) < +\infty, \quad s \in]-\infty, +\infty[, \\ g(0) \neq g(\pm\infty),$$

and the case of $g(s) = |s|^p \operatorname{sign} s$, $p \in]0, 1[$. The basic abstract results of this paper deal with operator equations in a Hilbert space H which are of the form

$$(4) \quad A(u) - S(u) = h$$

with $A : D(A) \subset H \rightarrow H$ linear, $h \in H$ and $S : H \rightarrow H$ satisfying a growth condition of the form

$$(5) \quad |S(u)| \leq \mu_1 + \mu_2 |u|^\delta, \quad \delta \in [0, 1[.$$

They are extended in A[21] to the case where $\delta = 1$ and μ_2 is sufficiently small. A systematization and many generalizations of the above results for (4) by a similar approach is given in A[22]. Continuing his investigations of cases left open by the above quoted papers, Fučík in A[20] initiated the use of the method of truncated equations for studying, in the special case if (1) with $n = 1$, the case when g is such that

$$g(+\infty) = g(-\infty) = 0.$$

The corresponding problem for an arbitrary n is considered in A[33] and more general results, with applications to elliptic problems, are given in A[37] (with M. Krbeč), where the useful concept of expansive function is introduced. Equations with expansive nonlinearities are further studied in A[40] (with A. Ambrosetti), where the notion of expansively periodic function is defined and used to prove, by the alternative method together with topological degree, the existence of infinitely many solutions for some equations of the type (4) with the null-space of A odd-dimensional and the Nemyckii operator associated to an expansively periodic nonlinearity.

Because of the growth restriction on g , all the above mentioned results cover, in the special case of (1), boundary value problems of the form

$$(6) \quad u'' + h(u) = f(x), \quad u(0) = u(\pi) = 0,$$

where h is continuous and such that

$$\lim_{u \rightarrow +\infty} (h(u)/u) = \lim_{u \rightarrow -\infty} (h(u)/u) = n^2.$$

In his fundamental paper A[31], Fučík called the function h non jumping if

$$\lim_{u \rightarrow +\infty} (h(u)/u) = \lim_{u \rightarrow -\infty} (h(u)/u)$$

and jumping if those two limits do not coincide. Thus the above mentioned works all deal with the case of non jumping nonlinearities. The jumping case where h does not jump over an eigenvalue of the associated problem linear, i.e. where

$$n^2 < \lim_{u \rightarrow -\infty} (h(u)/u) \neq \lim_{u \rightarrow +\infty} (h(u)/u) < (n + 1)^2$$

has been well known and easy to treat while the case where h jumps from the first to the second eigenvalue of the associated linear problem, i.e.

$$\lim_{u \rightarrow -\infty} (h(u)/u) < 1 < \lim_{u \rightarrow +\infty} (h(u)/u) < 4$$

had been initiated by Ambrosetti and Prodi in 1973. In A[31] the existence problem is considered for the first time when the nonlinearity jumps over one arbitrary eigenvalue, or more than one eigenvalue, or from an eigenvalue to another one, and also when it jumps off an eigenvalue but not to another one. The treatment is based upon a clever use of the Leray-Schauder degree. In A[27] one can find a result of the Ambrosetti-Prodi type for weak solutions, based on the alternative method together with the Banach fixed point theorem, and an abstract treatment of problems with jumping nonlinearities is found in A[30].

As was noticed in 1977 by J. Mawhin, results of the above type hold not only for ordinary and elliptic partial differential equations but also for the time periodic solutions of partial differential equations of evolution type. Fučík immediately contributed to this area and the paper A[38] (with J. Mawhin) covered the case of nonlinear telegraph equations, the papers A[35] and A[42] (with V. Šťastnová) deal with nonlinear heat equations, and the case of nonlinear beam equation is studied in B[16]. One must notice that the "initial conditions" for the investigation of this type of problems in Prague were particularly favourable because of the outstanding work of O. Vejvoda and his group in the study of time-periodic solutions of weakly nonlinear evolution equations. Problems of this type are also considered in A[41] (with P. Hess), which generalizes and completes the results of A[33], A[37], A[38] and A[42].

When A is selfadjoint and S a potential operator, the variational approach gives better results than the topological method in the study of problems of the type (4), as had been shown for the first time in 1976 by Ahmad, Lazer and Paul. Their results were generalized substantially by Fučík in A[34] and A[39], where Fučík could fully manifest his familiarity with the variational methods he had learned in the group of J. Nečas.

Other papers of S. Fučík cover problems of the type (4) where S does not satisfy a growth condition of the type (5) with $\delta \in [0, 1]$. The corresponding abstract back-

ground is still far from being unified and many problems have remained open. In the case of ordinary differential equations, existence of periodic solutions of the equation

$$x'' + g(x) = f(t)$$

when $g(u)/u \rightarrow +\infty$ for $u \rightarrow \infty$ is considered in A[25] (with V. Lovicar) by means of the shooting method and Brouwer degree; existence of periodic solutions of higher order equations of the form

$$x^{(2k)} + \sum_{j=1}^{2k-1} a_j x^{(2k-j)} + g(x) + f(x) x' = f(t)$$

is treated in A[29] using the Schauder fixed point theorem and a corresponding vector equation is studied in A[26] (with J. Mawhin) via the coincidence degree. In the case of partial differential equations the resonance problem at the first eigenvalue

$$\begin{aligned} -\Delta u - \lambda_1 u + g(u) &= f(x), \quad x \in \Omega, \\ u(x) &= 0, \quad x \in \partial\Omega \end{aligned}$$

is considered in A[36] provided g is superlinear, continuous and nondecreasing, by the alternative method combined with the theory of monotone operators.

Even the short description given above reveals that each Fučík's paper contains significant new results and contributes to this field of nonlinear functional analysis and differential equations by opening new prospects of research as well as by improving substantially former results. However, this is only one face of Fučík's basic contribution to this domain of mathematics. His personality and activity influenced considerably the work of the group of Prague mathematicians oriented in this direction and tens of mathematicians have continued his work, developing further his ideas and results.

In a number of survey papers (B[9], B[11], B[12]) based on lectures at various conferences, and especially D[5] which can be considered his scientific testament and whose final version was completed by Fučík in the hospital, he gave a beautiful account of the state of art in this field. He also listed many open problems, most of which have remained still unsolved, and traced in this way the main stream of research of the last years in the study of nonlinear problems at resonance. There is no doubt that Fučík's work will continue for a long time to be the best guide for everybody interested in the important unsolved problems in this area.

In his extensive and many-sided educational work Professor Fučík manifested his unceasing activity which had its source in his deep knowledge, in his outstanding teaching qualities and, last but not least, his organizing abilities. In the last period of his life Professor Fučík took an active part in all stages of education of students in mathematical analysis.

By comparing the chronological course given by the list of Fučík's publications with his work as a teacher, we find out a remarkable unity of both scientific and educational work in all their aspects: optional lectures or seminars for graduated students transformed gradually into advanced research seminars, the whole process being permeated by the sense for team work which he always stimulated.

Both his extensive scientific and educational work was appreciated and awarded by the Faculty, The Society of Czechoslovak Mathematicians and Physicists, and in 1979 by the bestowal of the Prize of the Minister of Education.

The life work left to us by Svatopluk Fučík is unique and extraordinary. Even a brief account of his activity, compared with the short time granted to him by Nature, confirms the prominence of his personality. Everyone who met him knew his love for Mathematics, his devotion to his work and students. His students remember his high demands and exactness but, on the other hand, his understanding and tact. We all remember his frankness, his individual humour that helped to overcome many difficulties and to reach successful results. We cannot end the account of Fučík's career without emphasizing that, since 1973, he did know that his time is strictly limited. His reaction was the tremendous mathematical activity just described as well as a thirst for human contacts, revealed in particular by the number of joint papers he contributed and the number of meetings he attended.

During his life time Fučík delivered many beautiful lectures, by his life he gave us an unforgettable lesson. And everything was done in all simplicity! His death is a severe loss for Mathematics and for the University which we realize at present but which we shall feel even more seriously in the future.

LIST OF PAPERS OF PROFESSOR SVATOPLUK FUČÍK

A. Original papers

- [1] Fixed point theorems based on Leray-Schauder degree, *Comment. Math. Univ. Carolin.* 8 (1967), 683–690.
- [2] Fixed point theorems for sum of nonlinear mappings, *Comment. Math. Univ. Carolin.* 9 (1968), 133–143.
- [3] Solving of nonlinear operator's equations in Banach spaces, *Comment. Math. Univ. Carolin.* 10 (1969), 177–188.
- [4] Remarks on monotone operators, *Comment. Math. Univ. Carolin.* 11 (1970), 271–284.
- [5] Note on the Fredholm alternative for nonlinear operators, *Comment. Math. Univ. Carolin.* 12 (1971), 213–226.
- [6] Fredholm alternative for nonlinear operators in Banach spaces and its applications to the differential and integral equations, *Časopis pěst. mat.* 96 (1971), 371–390.
- [7] On the convergence of sequences of linear operators and adjoint operators, *Comment. Math. Carolin.* 12 (1971), 753–763 (with *J. Milota*).
- [8] On the existence of Schauder bases in Sobolev spaces, *Comment. Math. Univ. Carolin.* 13 (1972), 163–175 (with *O. John*, and *J. Nečas*).
- [9] Strengthening upper bound for the number of critical levels of nonlinear functionals, *Comment. Math. Univ. Carolin.* 13 (1972), 297–310 (with *J. Nečas*, *J. Souček* and *V. Souček*).

- [10] Ljusternik-Schnirelmann theorem and nonlinear eigenvalue problems, *Math. Nachr.* 53 (1972), 277–289 (with *J. Nečas*).
- [11] Upper bound for the number of critical levels for nonlinear operators in Banach spaces of the type of second order nonlinear partial differential operators, *Journal Functional Analysis* 11 (1972), 314–333 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [12] New infinite dimensional versions of Morse-Sard theorem, *Boll. Unione Mat. Ital.* 6 (1972), 317–322 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [13] Upper bound for the number of eigenvalues for nonlinear operators, *Ann. Scuola Norm. Sup. Pisa* 27 (1973), 53–71 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [14] Note to nonlinear spectral theory: Application to boundary value problems for ordinary integrodifferential equations, *Comment. Math. Univ. Carolin.* 14 (1973), 583–608 (with *Tran Dien Hien*).
- [15] Note to nonlinear spectral theory: Application to the nonlinear integral equations of the Lichtenstein type, *Math. Nachr.* 58 (1973), 257–267 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [16] Existence of solutions of nonlinear boundary value problems, *Acta Polytechnica* 4 (1973), 17–24 (Czech).
- [17] Kačanov-Galerkin method, *Comment. Math. Univ. Carolin.* 14 (1973), 651–659 (with *A. Kratochvíl* and *J. Nečas*).
- [18] Krasnoselskii's main bifurcation theorem, *Arch. Rat. Mech. Anal.* 54 (1974), 328–339 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [19] Morse-Sard theorem in infinite dimensional Banach spaces and investigation of the set of all critical levels, *Časopis pěst. mat.* 99 (1974), 217–243 (with *M. Kučera*, *J. Nečas*, *J. Souček* and *V. Souček*).
- [20] Further remark on a theorem by E. M. Landesman and A. C. Lazer, *Comment. Math. Univ. Carolin.* 15 (1974), 259–271.
- [21] Surjectivity of operators involving linear noninvertible part and nonlinear compact perturbation, *Funkcial. Ekvac.* 17 (1974), 73–83.
- [22] Nonlinear equations with noninvertible linear part, *Czechoslovak Math. J.* 24(1974), 467–495.
- [23] Ranges of nonlinear asymptotically linear operators, *J. Differential Equations* 17 (1975), 375–394 (with *M. Kučera* and *J. Nečas*).
- [24] Kačanov's method and its application, *Rev. Roumaine Math. Pures Appl.* 20 (1975), 907–916 (with *A. Kratochvíl* and *J. Nečas*).
- [25] Periodic solutions of the equation $x''(t) + g(x(t)) = p(t)$, *Časopis pěst. mat.* 100 (1975), 160–175 (with *V. Lovičar*).
- [26] Periodic solutions of some nonlinear differential equations of higher order, *Časopis pěst. mat.* 100 (1975), 276–283 (with *J. Mawhin*).
- [27] Remarks on a result by A. Ambrosetti and G. Prodi, *Boll. Un. Mat. Ital.* 11 (1975), 259–267.
- [28] Linear and nonlinear variational inequalities on halfspaces, *Comment. Math. Univ. Carolin.* 16 (1975), 663–682 (with *J. Milota*).
- [29] Periodic solutions of generalized Liénard equation with forcing term, *Colloquia Math. Soc. J. Bolyai*, 15, *Diff. Equat.*, Keszthely 1975, pp. 155–169.
- [30] Remarks on the solvability and nonsolvability of weakly nonlinear equations, *Comment. Math. Univ. Carolin.* 17 (1976), 61–70.
- [31] Boundary value problems with jumping nonlinearities, *Časopis pěst. mat.* 101 (1976), 69–87.
- [32] Ranges of operators involving linear noninvertible part and nonlinear perturbation, *Beiträge Anal.* 9 (1976), 19–21.
- [33] Remarks on some nonlinear boundary value problems, *Comment. Math. Univ. Carolin.* 17 (1976), 721–730.

- [34] Nonlinear equations with linear part of resonance: Variational approach, *Comment. Math. Univ. Carolin.* *18* (1977), 723–734.
- [35] Note to periodic solvability of the boundary value problem for nonlinear heat equation, *Comment. Math. Univ. Carolin.* *18* (1977), 735–740 (with *V. Štátnová*).
- [36] Remarks on superlinear boundary value problems, *Bull. Austral. Math. Soc.* *16* (1977), 181–188.
- [37] Boundary value problems with bounded nonlinearity and general null-space of the linear part, *Math. Z.* *155* (1977), 129–138 (with *M. Krbeč*).
- [38] Generalized periodic solutions of nonlinear telegraph equations, *Nonlinear Anal.* *2* (1978), 609–617 (with *J. Mawhin*).
- [39] Nonlinear potential equations with linear parts at resonance, *Časopis pěst. mat.* *103* (1978), 78–94.
- [40] Nonlinear equations with expansive nonlinearities (with *A. Ambrosetti*), *Ann. Univ. Ferrara, Sez. VII N.S.* *24* (1978), 209–219.
- [41] Nonlinear perturbations of linear operators having null-space with strong unique continuous property (with *P. Hess*), *Nonlinear Anal.* *3* (1979), 271–277.
- [42] Weak periodic solutions of the boundary value problem for nonlinear heat equation (with *V. Štátnová*), *Aplikace mat.* *24* (1979), 284–303.

B. Preliminary reports and conference talks

- [1] Fredholm alternative for nonlinear operators in Banach spaces and its applications to the differential and integral equations, *Comment. Math. Univ. Carolin.* *11* (1970), 271–284.
- [2] Upper bound for the number of eigenvalues for nonlinear operators, *Comment. Math. Univ. Carolin.* *13* (1972), 191–195 (with *J. Nečas*, *J. Souček* and *V. Souček*).
- [3] Spectral theory of nonlinear operators, *Proc. of Equadiff III, Czechoslovak Conference on Differential Equations and Their Applications, Brno 1972*, 163–174 (with *J. Nečas*).
- [4] Topics on nonlinear spectral theory, *Theory of Nonlinear Operators, Proc. of a Summer School held in October 1972 at Neuendorf, Hiddensee, GDR, Akademie Verlag, Berlin 1974*, pp. 57–73 (with *M. Kučera*, *J. Souček* and *V. Souček*).
- [5] A more up-to-date and effective approach to differential calculus of functions of more variable. *Sborník referátů na pedagogické konferenci MFF UK (Czech)*.
- [6] Kačanov-Galerkin method and its application, *Acta Univ. Carolinae, Math.-Physica 15* (1974), 31–33, *Proc. of the Third Conference on Basic Problems of Numerical Mathematics* (with *A. Kratochvíl* and *J. Nečas*).
- [7] Boundary value and periodic problem for the equation $x''(t) + g(x(t)) = p(t)$, *Comment. Math. Univ. Carolin.* *15* (1974), 351–355 (with *V. Lovicar*).
- [8] Spektral'nyj analiz nelinejnych operatorov, *Časopis pěst. mat.* *100* (1975), 179–192.
- [9] Solvability and nonsolvability of weakly nonlinear equations, *Proceedings Int. Summer School "Theory of Nonlinear Operators. Constructive Aspects"* held in September 1975 at Berlin. *Abhandlungen der Akademie der Wissenschaften der DDR, Jahrgang 1977*, 57–68.
- [10] Solvability of nonlinear equations, *Proceedings of Conference at Kühlungsborn, DDR*.
- [11] Nonlinear noncoercive boundary value problems, *Proc. of Conference, Equadiff IV, Praha 1977, Lecture Notes in Mathematics, No 703, Springer-Verlag 1979*, 99–109.
- [12] Open problems in the solvability of nonlinear equations, *Proc. of Conference, Oberwolfach 1977. Lecture Notes in Mathematics No 703, Springer-Verlag 1979*, 99–109.
- [13] Generalized periodic solutions of nonlinear telegraph equations, *Comment. Math. Univ. Carolin.* *18* (1977), 813–816 (with *J. Mawhin*).
- [14] Variational noncoercive nonlinear operators, *Proc. Int. Summer School "Theory of Nonlinear Operators. Constructive Aspects"*, held in September 1977 in Berlin. *Abhandlungen der Akademie der Wissenschaften der DDR, Jahrgang 1978, Nr. 6 N*, 61–69.

- [15] Nonlinear perturbations of linear operators having null-space with strong unique continuation property, *Comment. Math. Univ. Carolin.* 19 (1978), 403–407.
- [16] Nonlinear noncoercive problems: Generalized periodic solutions of nonlinear beam equation, 3° seminario di Analisi Funzionale ed Applicazioni S.A.F.A. III, Bari 1978, pp. 52.

D. Books

- [1] Spectral Analysis of Nonlinear Operators, Springer Verlag 1973, Lecture Notes in Mathematics No 346 (with *J. Nečas*, *J. Souček* and *V. Souček*), pp. 287.
- [2] Function Spaces, Nordhoff and Academia 1977 (with *O. John* and *A. Kufner*), pp. 454.
- [3] Einführung in die Variationsrechnung, Teubner-Texte zur Mathematik, Teubner Verlag, Leipzig 1977 (with *J. Nečas*, *J. Souček* and *V. Souček*), pp. 175.
- [4] Nonlinear Differential Equations, SNTL Praha 1978 (with *A. Kufner*), pp. 244 (Czech).
- [5] Solvability of Nonlinear Equations and Boundary Value Problems D. Reidel Publishing Company 1980, pp. 490.
- [6] Nonlinear analysis, function spaces and applications, Proceedings of a Springer School, Teubner-Verlag, Leipzig 1979 (Editor with *A. Kufner*), pp. 224.
- [7] Nonlinear Differential Equations, Elsevier, Amsterdam and SNTL, Praha 1980 (with *A. Kufner*), pp. 359.

Note. A more detailed list of lecture notes and other publications (mostly in Czech) is published in *Časopis pro pěst. mat.* 105 (1980), 91–101.