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CORRECTION TO OUR PAPER: PERIODIC SOLUTIONS TO ABSTRACT DIFFERENTIAL EQUATIONS*)

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V. LOVICAR brought our attention to the fact that in Lemma 1.2.1 of our paper [1] the assertion $\mathscr{R}(\mathbf{A})$ is closed is not valid unless another assumption, namely e.g. assumption

$$\inf_{\lambda \in \Lambda \setminus \Lambda_1} \sum_{i,j=1}^{2} |a_{ij}(\lambda)|^2 = \alpha_0 > 0 , \tag{1}$$

where $\Lambda_1 = \{\lambda \in \sigma(A);\ a_{ij}(\lambda) = 0,\ i, j = 1, 2\}$, is added. He suggested a more general version of the lemma and also a simpler proof. Further he pointed out that the assumption $\Lambda = \{\lambda_n\}_{n=1}^{\infty}$ may be omitted. Let us introduce the correct proof of the closedness of $\mathscr{R}(\mathbf{A})$ under the new assumptions.

We suppose that all the assumptions of Lemma 1.2.1 are satisfied except for $\Lambda = \{\lambda_n\}_{n=1}^{\infty}$ and that (1) holds. Let P be as in the proof of Lemma 1.2.1 and denote by Q the orthogonal projection onto $\mathscr{N}(\sum_{i,j=1}^{2} A_{ij}^2) = \bigcap_{i,j=1}^{2} \mathscr{N}(A_{ij}) \subseteq \mathscr{N}(D)$ and $R = P - Q$. First, we see that $\mathscr{R}(\sum_{i,j=1}^{2} A_{ij}^2)$ is closed. This can be deduced from (1) similarly as $\mathscr{R}(D) = \overline{\mathscr{R}(D)}$ was deduced from (1.2.5) of [1]. Second, by elementary calculation, for $[f_1, f_2] \in \mathscr{R}(\mathbf{A})$ we get $A_{22}f_1 - A_{12}f_2,\ -A_{21}f_1 + A_{11}f_2 \in \mathscr{R}(D)$ and hence the same is true for $[f_1, f_2] \in \overline{\mathscr{R}(\mathbf{A})} = \mathscr{N}(\mathbf{A}^*)^{\perp}$. Now, let $[f_1, f_2] \perp \mathscr{N}(\mathbf{A}^*)$. Show that $[f_1, f_2] \in \mathscr{R}(\mathbf{A})$. The solution of the equation $\mathbf{A}(x_1, x_2) = [(I - P) f_1, (I - P) f_2]$ reads

$$y_1 = \int_{R \setminus \Lambda} d(\lambda)^{-1}\, dE(\lambda)\, (A_{22}(I - P) f_1 - A_{12}(I - P) f_2) ,$$

$$y_2 = \int_{R \setminus \Lambda} d(\lambda)^{-1}\, dE(\lambda)\, (-A_{21}(I - P) f_1 + A_{11}(I - P) f_2) .$$

*) Czechoslovak Mathematical Journal, *23* (*98*) 1973, 635—669.

So we have $[(I - P)f_1, (I - P)f_2] \in \mathcal{N}(\mathbf{A}^*)^\perp$ and hence $[Pf_1, Pf_2] \in \mathcal{N}(\mathbf{A}^*)^\perp$. The letter together with $A_{ij}P = A_{ij}R$ imply

$$(2) \qquad A_{22}Rf_1 - A_{12}Rf_2 = -A_{21}Rf_1 + A_{11}Rf_2 = 0\,.$$

Since $Rf_1, Rf_2 \in \mathcal{N}(\sum_{i,j=1}^{2} A_{ij}^2)^\perp$ we can define

$$z_1 = \int_{\sigma(A)\setminus\Lambda_1} a_{11}(\lambda)\,(\sum_{i,j=1}^{2} a_{ij}(\lambda)^2)^{-1}\,\mathrm{d}E(\lambda)\,Rf_1 + \int_{\sigma(A)\setminus\Lambda_1} a_{21}(\lambda)\,(\sum_{i,j=1}^{2} a_{ij}(\lambda)^2)^{-1}\,\mathrm{d}E(\lambda)\,Rf_2\,,$$

$$z_2 = \int_{\sigma(A)\setminus\Lambda_1} a_{12}(\lambda)\,(\sum_{i,j=1}^{2} a_{ij}(\lambda)^2)^{-1}\,\mathrm{d}E(\lambda)\,Rf_1 + \int_{\sigma(A)\setminus\Lambda_1} a_{22}(\lambda)\,(\sum_{i,j=1}^{2} a_{ij}(\lambda)^2)^{-1}\,\mathrm{d}E(\lambda)\,Rf_2$$

and by (2) clearly obtain $\mathbf{A}(z_1, z_2) = [Rf_1, Rf_2]$. As $[Rf_1, Rf_2] \in \mathcal{N}(\mathbf{A}^*)^\perp$ it is $[Qf_1, Qf_2] \in \mathcal{N}(\mathbf{A}^*)^\perp$ too. But on the other hand $Qf_1, Qf_2 \in \bigcap_{i,j=1}^{2} \mathcal{N}(A_{ij})$ which implies $[Qf_1, Qf_2] \in \mathcal{N}(\mathbf{A}^*)$. Hence $Qf_1 = Qf_2 = 0$. Setting $x_j = y_j + z_j$, $j = 1, 2$ we get $\mathbf{A}(x_1, x_2) = [f_1, f_2]$.

Our omission in Lemma 1.2.1 necessitates some further changes in the text.

In the proof of Lemma 3.2.1 there is to set $\psi = -\omega^{-1}P_0 g_1 + \int_m^\infty \mathrm{d}(\lambda)^{-1}\,\mathrm{d}E(\lambda)$ $(A_{21}g_1 - A_{11}g_2)$. The third formula in (4.2.2) and the fourth formula in (4.3.2) and $G_4(\varepsilon)(\mathbf{u})$ on page 663 change to

$$\psi_2 = \tfrac{1}{2}\,\mathrm{sh}^{-1}\,\omega(\alpha - \beta\gamma)\int_0^\omega e^{2\tau(\alpha-\beta\gamma)}\,PF(\tau, u(\tau))\,\mathrm{d}\tau + $$

$$+ 2(D/\mathcal{N}(P))^{-1}\int_0^\omega e^{-(\alpha+\beta A)(\omega-\tau)}\sin(\tau - \tfrac{1}{2}\omega)\,[A + \gamma - (\alpha + \beta A)^2]^{1/2} *$$

$$* \sin\tfrac{1}{2}\omega[A + \gamma - (\alpha + \beta A)^2]^{1/2}\,(I - P)\,F(\tau, u(\tau))\,\mathrm{d}\tau$$

and to

$$\psi_2 = -\int_0^\omega \frac{\tau}{\omega}\,P_0F(\tau, u(\tau))\,\mathrm{d}\tau + 2[A^\varrho D/\mathcal{N}(P)]^{-1}$$

$$\int_0^\omega \sin(\tau - \tfrac{1}{2}\omega)\,(A + \gamma)^{1/2} * \sin\tfrac{1}{2}\omega(A + \gamma)^{1/2} * A^\varrho(I - P_0)\,F(\tau, u(\tau))\,\mathrm{d}\tau$$

and to

$$G_4(\varepsilon)(\mathbf{u}) = u_4 + \varepsilon \int_0^\omega \frac{\tau}{\omega} P_0 F(\tau, u_1(\tau))\, \mathrm{d}\tau - 2\varepsilon [A^\varrho D/\mathcal{N}(P)]^{-1} *$$

$$* \int_0^\omega \sin(\tau - \tfrac{1}{2}\omega)(A + \gamma)^{1/2} * \sin \tfrac{1}{2}\omega (A + \gamma)^{1/2} * A^\varrho (I - P_0)\, \tilde{F}(\tau, u_1(\tau))\, \mathrm{d}\tau$$

respectively.

References

[1] *I. Straškraba, O. Vejvoda:* Periodic solutions to abstract differential equations. Czech. Math. J., *23 (98)* 1973, 635—669.

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