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ON E-SEQUENTIALLY REGULAR SPACES

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The purpose of the present paper is to define and study classes of E-sequentially regular and E-sequentially complete convergence, resp. sequential, spaces, E being a subspace of the real line. In the first section we prove and generalize some results concerning the property p of convergence spaces defined in [2]. As a main result we prove that for each $E \subset R$ the E-sequential regularity (completeness) is equivalent either to $[0, 1]$-sequential or to $\{0, 1\}$-sequential regularity (completeness). The second section is devoted to equality of E-sequential envelopes. In the third section we apply the results of the previous two sections to sequential spaces.

Throughout the paper we make a blanket assumption that all spaces have unique sequential limits and all convergence spaces satisfy axioms $(\mathscr{L}_0) - (\mathscr{L}_3)$. The definitions and basic properties of convergence spaces can be found in [7], [8], [2], [5], and those of sequential spaces in [1], [3]. Recall that if (L, u) is a topological space, then in the associated convergence space (L, λ) a sequence $\langle x_n \rangle$ converges to a point x whenever each u-neighborhood of x contains all but finitely many x_n. By λ^{ω_1} we shall denote the topological modification of λ and by sL the sequential modification (L, λ^{ω_1}) of (L, u). If (L, u) is a sequential space, then we have $u = \lambda^{\omega_1}$. We shall use the following notation: R denotes the real line, E a subspace of R, N natural numbers, $[0, 1]$ the closed unit interval, and $\{0, 1\}$ the two-point isolated space. If (L, u) is a space, then $C = C(L)$ denotes the set of all continuous functions on L, $C_E \subset C$ the set of all continuous functions on L into E, and C_0 a subset of C.

1.

Definition 1.1. We say that a convergence space (L, λ) has *the property p with respect to C_0* if

(p) For each two sequences $\langle x_n \rangle, \langle y_n \rangle$ of points of L such that $(\lambda \bigcup(x_n)) \cap \cap (\lambda \bigcup(y_n)) = \emptyset$ there is a function $f \in C_0$ such that $\lim f(x_n) = \lim f(y_n)$ does not hold.

Notice that if in (p) we have $y_n = y$, $n \in N$, then we obtain the definition of the C_0-sequential regularity for convergence spaces (cf. [8]).

Definition 1.2. A C_0-sequentially regular convergence space (L, λ) is called C_0-*sequentially complete* if (L, λ) is a closed subspace of each sequentially regular convergence space in which it is C_0-embedded.

Theorem 1.3. *A C_0-sequentially regular convergence space (L, λ) has the property p with respect to C_0 iff it is C_0-sequentially complete.*

The proof of this theorem will appear in [4].

Corollary 1.4. *A convergence space (S, σ) is a C_0-sequential envelope of a C_0-sequentially regular convergence space (L, λ) iff*

(i) (L, λ) *is a sequentially dense* (i.e. λ^{ω_1}-*dense*) C_0-*embedded subspace of* (S, σ).

(ii) (S, σ) *has the property p with respect to* $\bar{C}_0(S) = \{f \in C(S) : f/L \in C_0\}$.

Corollary 1.5. *A C_0-sequentially regular space has the property p with respect to C_0 iff it is a C_0-sequential envelope of itself.*

Notation 1.6. If $C_0 = C_E$, $E \subset R$, then we speak of E-sequentially regular (complete) convergence spaces and if $E = R$, then we simply speak of sequentially regular (complete) spaces. Similarly, we speak of the property p_E, resp. p, and E-sequential envelope $\sigma_E(L)$, resp. sequential envelope $\sigma(L)$.

Lemma 1.7. *Let (L, λ) be a convergence space having the property p. Then for each two sequences $\langle x_n \rangle$, $\langle y_n \rangle$ such that $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$ there are subsequences $\langle x'_n \rangle$ of $\langle x_n \rangle$ and $\langle y'_n \rangle$ of $\langle y_n \rangle$ and a function $f \in C_{[0,1]}$ such that for each $n \in N$ we have $f(x'_n) = 0$, $f(y'_n) = 1$.*

Proof. By the assumption, if $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$, then there is a function $g \in C$ such that $\lim g(x_n) = \lim g(y_n)$ does not hold. Consequently, there are subsequences $\langle x_n \rangle$ of $\langle x_n \rangle$ and $\langle y'_n \rangle$ of $\langle y_n \rangle$ such that $\overline{\bigcup(g(x'_n))} \cap \overline{\bigcup(g(y'_n))} = \emptyset$. From the normality of R it follows that there is a function $h \in C_{[0,1]}(R)$ such that for each $n \in N$ we have $h(g(x'_n)) = 0$, $h(g(y'_n)) = 1$. Since $f = g \circ h \in C_{[0,1]}(L)$, the proof is finished.

Let (L, λ) be a convergence space having the property p with respect to C_1 and let $C_1 \subset C_2 \subset C$. It follows immediately that (L, λ) has the property p with respect to C_2.

Corollary 1.8. *Let $E \subset R$ contain an interval. Then the properties p and p_E are equivalent.*

Lemma 1.9. *Let $E \subset R$ do not contain any interval and let (L, λ) be a convergence space having the property p_E. Then for each two sequences $\langle x_n \rangle$, $\langle y_n \rangle$ such that $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$ there are subsequences $\langle x'_n \rangle$ of $\langle x_n \rangle$ and $\langle y'_n \rangle$ of $\langle y_n \rangle$ and a function $f \in C_{\{0,1\}}$ such that for each $n \in N$ we have $f(x'_n) = 0, f(y'_n) = 1$.*

Proof. By the assumption, if $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$, then there is a function $g \in C_E$ such that $\lim g(x_n) = \lim g(y_n)$ does not hold. There are two possibilities. I. One of the sequences $\langle g(x_n) \rangle$, $\langle g(y_n) \rangle$, say $\langle g(x_n) \rangle$, is unbounded. Then there is a subsequence $\langle x''_n \rangle$ of $\langle x_n \rangle$ such that the sequence $\langle g(x''_n) \rangle$ is strictly monotone, say increasing, and has no limit point in R. Then there is a sequence $\langle r_n \rangle$ in $R - E$ such that for each $n \in N$ we have $g(x''_n) < r_n < g(x''_{n+1})$. Denote by $E_1 = E \cap (\bigcup(r_{2n-1}, r_{2n}))$ and by $E_2 = E - E_1$. Since $E = E_1 \cup E_2$, there is a subsequence $\langle y''_n \rangle$ of $\langle y_n \rangle$ such that $\langle g(y''_n) \rangle$ is contained in E_i, $i \in \{1, 2\}$. Define a function h on E as follows:

$h(z) = 0$ for $z \in E_i$ and

$h(z) = 1$ for $z \in E - E_i$.

II. Both sequences $\langle g(x_n) \rangle$ and $\langle g(y_n) \rangle$ are bounded. Then there are subsequences $\langle x''_n \rangle$ of $\langle x_n \rangle$ and $\langle y''_n \rangle$ of $\langle y_n \rangle$ and numbers $a, b \in R$, $a \neq b$, such that $a = \lim g(x''_n)$, $b = \lim g(y''_n)$. Consequently, there are numbers $p, q \in R - E$ such that $a \in (p, q)$, $b \notin (p, q)$. Define a function h on E as follows:

$h(z) = 0$ for $z \in E \cap (p, q)$ and

$h(z) = 1$ for $z \in E - (p, q)$.

In both cases I and II the function h is continuous on E, $f = g \circ h \in C_{\{0,1\}}(L)$, and for some subsequences $\langle x'_n \rangle$ of $\langle x''_n \rangle$ and $\langle y'_n \rangle$ of $\langle y''_n \rangle$ we have $f(x'_n) = 0$, $f(y'_n) = 1$, $n \in N$. This completes the proof.

Corollary 1.10. *Let $E \subset R$ contain at least two points and do not contain any interval. Then the properties p_E and $p_{\{0,1\}}$ are equivalent.*

Example 1.11. The real line R has not the property $p_{\{0,1\}}$. For, if $x, y \in R$, $x \neq y$, then there is no continuous function $f: R \to \{0, 1\}$ such that $f(x) = 0$ and $f(y) = 1$. On the other hand, R has the property p.

Denote by ϱ the relation defined on the set of all subsets of R containing at least two points as follows: $E \varrho F$ if the properties p_E and p_F are equivalent. Then ϱ is clearly an equivalence relation. From the above considerations it follows that there are only two equivalence classes. One contains $[0, 1]$ and the other $\{0, 1\}$. Moreover, it is easy to see that $E \varrho F$ iff E-sequential and F-sequential regularities are equivalent. To sum up we have the following

Theorem 1.12. *Let (L, λ) be an E-sequentially regular (complete) convergence space. If E contains an interval, then (L, λ) is $[0, 1]$-sequentially regular (complete).*

If E does not contain any interval, then (L, λ) is $\{0, 1\}$-sequentially regular (complete).

In [7] it was proved that the sequential and $\{0, 1\}$-sequential regularities are convergence productive and hereditary properties. It was also proved that a convergence space (L, λ) is $[0, 1]$-sequentially ($\{0, 1\}$-sequentially) regular iff it is homeomorphic with a subspace of some convergence power $[0, 1]^m$ of $[0, 1]$ ($\{0, 1\}^m$ of $\{0, 1\}$). In the same way it can be proved

Theorem 1.13. *A convergence space is E-sequentially regular iff it is homeomorphic with a subspace of some convergence power E^m of E.*

We shall prove similar representation theorems for E-sequentially complete spaces. First we prove a generalization of Theorem 12 in [5].

Theorem 1.14. *Let (L, u) be a normal topological space. Then the associated convergence space (L, λ) has the property p.*

Proof. Let $\langle x_n \rangle$ and $\langle y_n \rangle$ be sequences such that

$(*)$ $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$.

There are two possibilities: I. $(u \bigcup(x_n)) \cap (u \bigcup(y_n)) = \emptyset$. Since (L, u) is normal, there is a function $f \in C(L, u) \subset C(L, \lambda)$ such that for each $n \in N$ we have $f(x_n) = 0$, $f(y_n) = 1$.

II. There is $x \in (u \bigcup(x_n)) \cap (u \bigcup(y_n))$. From $(*)$ it follows that there are an open u-neighborhood U of x and a closed u-neighborhood $V \subset U$ of x and subsequences $\langle x'_n \rangle$ of $\langle x_n \rangle$ and $\langle y'_n \rangle$ of $\langle y_n \rangle$ such that $\bigcup(x'_n) \subset V$ and $\bigcup(y'_n) \subset L - U$. It follows from the normality of (L, u) that there is a function $f \in C$ such that $f[V] = 0$, $f[L - U] = 1$. Thus for each $n \in N$ we have $f(x'_n) = 0$ and $f(y'_n) = 1$.

In both cases I and II $\lim f(x_n) = \lim f(y_n)$ does not hold. This completes the proof.

Lemma 1.15. *Let E be a subspace of the real line. Then E has the property p_E.*

Proof. From Theorem 1.14 it follows that E has the property p. If E contains an interval, then, by Corollary 1.8, E has the property p_E. If E consists of a single point, then the theorem is trivial. Finally, let E contain at least two points u and v and do not contain any interval. If $\langle x_n \rangle$, $\langle y_n \rangle$ are sequences in E such that $\overline{\bigcup(x_n)} \cap \overline{\bigcup(y_n)} = \emptyset$, where the closure is taken in E, then there are three possibilities. I. One of the sequences contains a strictly monotone unbounded subsequence.

II. There are numbers $a, b \in R$, $a \neq b$, and subsequences $\langle x'_n \rangle$ of $\langle x_n \rangle$ and $\langle y'_n \rangle$ of $\langle y_n \rangle$ such that $a = \lim x'_n$, $b = \lim y'_n$.

III. There is a point $a \in R - E$ such that $\lim x_n = a = \lim y_n$.

In all three cases a function $h \in C_{\{u,v\}}(E) \subset C_E(E)$ can be constructed in a similar way as in the proof of Lemma 1.9 such that $\lim h(x_n) = \lim h(y_n)$ does not hold.

Lemma 1.16. *The property p_E is convergence productive.*

Proof. Let $(L_\alpha, \lambda_\alpha)$, $\alpha \in I$, be convergence spaces having the property p_E and let (L, λ) be their convergence product. If $\langle x_n \rangle$ and $\langle y_n \rangle$ are sequences of points $x_n = \langle x_n^\alpha, \alpha \in I \rangle$, $y_n = \langle y_n^\alpha, \alpha \in I \rangle$, such that $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$, then there is an index $\beta \in I$ such that $\lim x_n^\beta = \lim y_n^\beta$ does not hold in L_β. Thus, there are subsequences $\langle x_{n_i}^\beta \rangle$ of $\langle x_n^\beta \rangle$ and $\langle y_{n_i}^\beta \rangle$ of $\langle y_n^\beta \rangle$ such that $(\lambda_\beta \bigcup(x_{n_i}^\beta)) \cap (\lambda_\beta \bigcup(y_{n_i}^\beta)) = \emptyset$. Then, by the assumption, there is a function $g \in C_E(L_\beta)$ such that $\lim g(x_{n_i}^\beta) = \lim g(y_{n_i}^\beta)$ does not hold. Since the function f defined on L by $f(\langle x^\alpha, \alpha \in I \rangle) = g(x^\beta)$ belongs to $C_E(L)$, the proof is finished.

Lemma 1.17. *Let (L, λ) be a convergence space having the property p_E and let (M, μ) be a closed subspace of (L, λ). Then (M, μ) has the property p_E.*

Proof. Let $\langle x_n \rangle$, $\langle y_n \rangle$ be sequences in M such that $(\mu \bigcup(x_n)) \cap (\mu \bigcup(y_n)) = \emptyset$. Since M is closed in (L, λ), we have $(\lambda \bigcup(x_n)) \cap (\lambda \bigcup(y_n)) = \emptyset$. Thus $\lim g(x_n) = \lim g(y_n)$ does not hold for some $g \in C_E(L)$. But $f = g/M \in C_E(M)$ and the proof is complete.

Corollary 1.18. *Let (L, λ) be a convergence product space of convergence spaces $(L_\alpha, \lambda_\alpha)$, $\alpha \in I$. If (L, λ) has the property p_E, then for each $\alpha \in I$ the factor space $(L_\alpha, \lambda_\alpha)$ has the property p_E.*

Theorem 1.19. *A convergence space (L, λ) has the property p_E iff it is homeomorphic with a closed subspace of some convergence power E^m of E.*

Proof. I. Let (L, λ) have the property p_E. From Corollary 1.5 it follows that (L, λ) is an E-sequential envelope of itself. By Theorem 3 in [8] $\varphi : L \to R^m$, where $\varphi(x) = \langle \varphi_f(x), f \in C_E \rangle$, $\varphi_f(x) = f(x)$, and m is the cardinal number of $C_E(L)$, is a homeomorphism of L onto a closed subspace $\varphi[L]$ of the convergence power R^m. It is easy to see that $\varphi[L]$ is actually a closed subspace of the convergence power E^m,

II. From Lemma 1.15, Lemma 1.16, and Lemma 1.17 it follows immediately that a closed subspace of the convergence power E^m has the property p_E.

From Corollary 1.8, Theorem 1.13, and Theorem 1.19 we obtain

Corollary 1.20. *Let $E \subset R$ contain an interval. Then a convergence space (L, λ) is E-sequentially regular (complete) iff it is homeomorphic with a (closed) subspace of some convergence power $[0, 1]^m$ of $[0, 1]$.*

From Corollary 1.10, Theorem 1.13, and Theorem 1.19 we obtain

Corollary 1.21. *Let $E \subset R$ do not contain any interval. Then a convergence space (L, λ) is E-sequentially regular (complete) iff it is homeomorphic with a (closed) subspace of some convergence power $\{0, 1\}^m$ of $\{0, 1\}$.*

We conclude this section with a result announced in [2].

Theorem 1.22. *Let (L, u) be a realcompact space. Then the associated convergence space (L, λ) has the property p.*

Proof. Let φ be the evaluation mapping of (L, u) into the topological power R^m of R, where m is the cardinal number of $C(L)$. Then φ is a homeomorphism and $\varphi[L]$ is closed in R^m. It can be easily proved that φ is also a homeomorphism of (L, λ) into the convergence power R^m of R. Since $\varphi[L]$ is sequentially closed in R^m, the assertion follows from Theorem 1.19.

2.

In [2] we announced that C^*-sequential and C-sequential envelopes of a sequentially regular convergence space are homeomorphic and the homeomorphism leaves the original space pointwise fixed. In this section we shall prove more general results.

Notation 2.1. Let (L, λ) be both E-sequentially and F-sequentially regular convergence space and let $\sigma_E(L)$, resp. $\sigma_F(L)$, be an E-sequential, resp. F-sequential, envelope of (L, λ). By $\sigma_E(L) = \sigma_F(L)$ we mean that there is a homeomorphism of $\sigma_E(L)$ onto $\sigma_F(L)$ that leaves L pointwise fixed.

Lemma 2.2. *Let (L, λ) be an E-sequentially regular convergence space and let $(S, \sigma) = \sigma_E(L)$. Then*

(i) *$\bar{C}_E(S) = C_E(S)$, where $\bar{C}_E(S) = \{g \in C(S) : g/L \in C_E(L)\}$.*

(ii) *$\sigma_E(L)$ has the property p_E.*

Proof. (i) By Corollary 1.4, (S, σ) has the property p with respect to $\bar{C}_E(S)$. We shall prove that $\bar{C}_E(S) = C_E(S)$. From the Extension theorem in [4] it follows that if f is a continuous mapping of (L, λ) into an F-sequentially complete convergence space (M, μ) and for each $h \in C_F(M)$ the composition $f \circ h$ belongs to $C_E(L)$, then f can be extended to a continuous mapping g of $\sigma_E(L)$ into (M, μ). From Lemma 1.15 and Theorem 1.3 it follows that E is E-sequentially complete. If we put $(M, \mu) = E$, then each $f \in C_E(L)$ can be extended to a continuous function g of $\sigma_E(L)$ into E. Hence $\bar{C}_E(S) \subset C_E(S)$. Since clearly $C_E(S) \subset \bar{C}_E(S)$, we have $\bar{C}_E(S) = C_E(S)$.

(ii) follows immediately from (i).

Theorem 2.3. *Let (L, λ) be an E-sequentially regular convergence space. Then*

(i) *If (L, λ) is not $\{0, 1\}$-sequentially regular, then $\sigma_E(L) = \sigma(L)$.*

(ii) *If* (L, λ) *is* $\{0, 1\}$*-sequentially regular and* E *contains at least two different points* $a, b \in R$ *but not an interval, then* $\sigma_E(L) = \sigma_{\{0,1\}}(L)$.

Proof. (i). From $C_E \subset C$ it easily follows that (L, λ) is C_E-embedded in $\sigma(L)$. Since $\sigma(L)$ has the property p it has, by Theorem 1.12 and Corollary 1.8, the property p_E. Since clearly $C_E(\sigma(L)) \subset \bar{C}_E(\sigma(L))$, by Corollary 1.4, $\sigma(L)$ is an E-sequential envelope of (L, λ). Thus, by Theorem 5 in [8], we have $\sigma_E(L) = \sigma(L)$.

(ii) Using Lemma 2.2 we can prove the second statement in the same way.

Corollary 2.4. (i) *A sequentially regular convergence space* (L, λ) *which is not* $\{0, 1\}$*-sequentially regular has a unique E-sequential envelope.*

(ii) *A* $\{0, 1\}$*-sequentially regular convergence space* (L, λ) *has at most two different E-sequential envelopes*: $\sigma(L)$ *and* $\sigma_{\{0,1\}}(L)$.

Problem 2.5. Is there a $\{0, 1\}$-sequentially regular convergence space (L, λ) such that $\sigma(L) \neq \sigma_{\{0,1\}}(L)$?

3.

In the sequel we shall frequently use the simple statement that if (M, μ) is the convergence space associated with a topological space (M, v), then for each $L \subset M$ the convergence space $(L, \mu/L)$ is associated with $(L, v/L)$. Recall also that for L open or closed in M we have $(\mu/L)^{\omega_1} = (\mu^{\omega_1})/L$ and therefore $(L, (\mu/L)^{\omega_1})$ is a subspace of (M, μ^{ω_1}).

Definition 3.1. A topological space X is said to be C_0*-sequentially regular* if the convergence of sequences in X is projectively generated by $C_0 \subset C(X)$, i.e. $\langle x_n \rangle$ converges to x in X whenever for each $f \in C_0$ we have $f(x) = \lim f(x_n)$.

This is a generalization of Definition 2 in [3].

Definition 3.2. A C_0-sequentially regular sequential space is said to be C_0*-sequentially complete* if it is a closed subspace of each sequentially regular sequential space in which it is C_0-embedded.

As a rule, if $C_0 = C_E$, $E \subset R$, then we speak of E-sequential regularity, resp. completeness, and if $E = R$, then we omit the letter E.

Theorem 3.3. *Let* (L, u) *be a sequential space and* (L, λ) *the associated convergence space. Then* (L, u) *is* C_0*-sequentially regular iff* (L, λ) *is* C_0*-sequentially regular.*

Proof. The convergence of sequences in (L, u) and in (L, λ) is the same and $C(L, u) = C(L, \lambda)$.

Corollary 3.4. *A sequential space X is E-sequentially regular iff it is homeomorphic with the sequential modification sY of a subspace Y of some topological power E^m of E, or equivalently iff X is homeomorphic with the topological modification of a subspace of some convergence power E^m of E.*

Corollary 3.5. *Let $E \subset R$ do not contain any interval. Then a sequential space X is E-sequentially regular iff it is homeomorphic with the sequential modification sY of a subspace Y of some topological power $\{0, 1\}^m$ of $\{0, 1\}$, or equivalently iff X is homeomorphic with the topological modification of a subspace of some convergence power $\{0, 1\}^m$ of $\{0, 1\}$.*

Theorem 3.6. *A C_0-sequentially regular sequential space (L, u) is C_0-sequentially complete iff the associated convergence space (L, λ) is C_0-sequentially complete.*

Proof. I. Let (L, u) be C_0-sequentially complete. Suppose that, on the contrary, (L, λ) is not C_0-sequentially complete. Let (S, σ) be a C_0-sequential envelope of (L, λ). From Theorem 1.3 and Corollary 1.5 it follows that there is a point $x \in \sigma L - L$. Then (L, λ) is a C_0-embedded sequentially dense subspace of a sequentially regular convergence space (M, μ), where $M = L \cup (x)$ and $\mu = \sigma/M$. Since $\lambda = \mu/L$ and L is open in M, we have $\lambda^{\omega_1} = (\mu/L)^{\omega_1} = \mu^{\omega_1}/L$ and (L, λ^{ω_1}) is a dense subspace of (M, μ^{ω_1}). From $C(L, \lambda) = C(L, \lambda^{\omega_1})$ it follows that (L, λ^{ω_1}) is C_0-embedded in (M, μ^{ω_1}) which is, by Theorem 3.3, sequentially regular. This is a contradiction.

II. Let (L, u) be not C_0-sequentially complete, i.e. (L, u) is a proper dense C_0-embedded subspace of a sequentially regular sequential space (M, v). Let (M, μ) be the sequentially regular convergence space associated with (M, v). Then $(L, \mu/L)$ is associated with (L, u) and hence $\mu/L = \lambda$. Since $C(L, \lambda) = C(L, \lambda^{\omega_1})$ and $vL = \mu^{\omega_1}L = M$, it follows that (L, λ) is a C_0-embedded sequentially dense proper subspace of a sequentially regular convergence space (M, μ). Thus (L, λ) is not C_0-sequentially complete.

Corollary 3.7. *A sequential space X is E-sequentially complete iff it is homeomorphic with a closed subspace of the sequential modification sE^m of some topological power E^m of E, or equivalently iff X is homeomorphic with a closed subspace of the topological modification of some convergence power E^m of E.*

Corollary 3.8. *Let $E \subset R$ do not contain any interval. Then a sequential space X is E-sequentially complete iff it is homeomorphic with a closed subspace of the sequential modification $s\{0, 1\}^m$ of some topological power $\{0, 1\}^m$ of $\{0,1\}$, or equivalently iff X is homeomorphic with a closed subspace of the topological modification of some convergence power $\{0, 1\}^m$ of $\{0, 1\}$.*

Using the example of a sequentially regular convergence space L_{11} given in [7] for which $\sigma(L_{11}) \neq L_{11}$, we can construct a sequentially regular convergence space

(L, λ) such that if (S, σ) is its sequential envelope, then $S - L$ consists of a convergent sequence $\langle x_m \rangle \to x$, L contains a double sequence $\langle x_{mn} \rangle$ such that for each $m \in N$ we have $\langle x_{mn} \rangle \to x_m$, and no sequence in L converges to x. Since (S, σ) is also a sequential envelope of $(M, \sigma/M)$, $M = L \cup (x)$, and $(M, (\sigma/M)^{\omega_1})$ is not a sequential subspace of (S, σ^{ω_1}), it follows that the theory of sequential envelopes cannot be applied to define a sequential envelope in the category of sequentially regular sequential spaces. However, using the theory of multisequences, developed by P. Kratochvíl in [6], this is possible for some subcategory of sequentially regular sequential spaces. This will be done in a forthcoming paper.

References

[1] *S. P. Franklin:* Spaces in which sequences suffice. Fund. Math. *57* (1965), 107—115.

[2] *R. Frič:* Sequential envelope and subspaces of the Čech-Stone compactification. General Topology and its Relation to Modern Analysis and Algebra, III (Proc. Third Prague Topological Sympos., 1971). Academia, Prague, 1972, 123—126.

[3] *R. Frič:* Regularity and extension of mappings in sequential spaces. Comment. Math. Univ. Carolinae *15* (1974), 161—171.

[4] *R. Frič - V. Koutník:* On sequentially complete spaces. To appear.

[5] *V. Koutník:* On sequentially regular convergence spaces. Czechoslovak Math. J. *17* (*92*) 1967, 232—247.

[6] *P. Kratochvíl:* Convergence of multisequences and its application to probability measure theory. (Czech). Ph. D. Thesis, MÚ ČSAV, Praha, Czechoslovakia.

[7] *J. Novák:* On convergence spaces and their sequential envelopes. Czechoslovak Math. J. *15* (*90*) 1965, 74—100.

[8] *J. Novák:* On sequential envelopes defined by means of certain classes of continuous functions. Czechoslovak Math. J. *18* (*93*) 1968, 450—465.

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