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W-ISOMORPHISMS OF DISTRIBUTIVE LATTICES

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The notion of weak isomorphism was introduced by A. GOETZ and E. MARCZEWSKI ([2], [6], [7]). Weak isomorphisms and weak automorphisms of universal algebras and of special types of algebraic structures were investigated by J. DUDEK and E. PŁONKA [1], T. TRACZYK [10], R. SENFT [8] and J. SICHLER [9]. In this note a generalization of the notion of weak automorphism (called W-isomorphism) will be dealt with.

Let $A = (M; F)$ be an algebra with the underlying set M and with the set F of fundamental operations. The operations e_j^n on A of the form $e_j^n(x_1, \ldots, x_n) = x_j$ are called trivial. The smallest family of operations in A containing all trivial and fundamental operations, and closed with respect to superposition is called the family of algebraic operations and denoted by $\alpha(A)$. By $\alpha_n(A)$ we denote the family of all algebraic n-ary operations. Let $n, m \geqq 0$ be integers and let $f \in \alpha_{n+m}(A)$. Let $a_1, \ldots, a_m \in M$. The n-ary operation $f(x_1, \ldots, x_n, a_1, \ldots, a_m)$ will be called a polynomial in A and the family of all polynomials in A will be denoted by $\beta(A)$.

Let be given two algebras $A_1 = (M_1; F_1)$ and $A_2 = (M_2; F_2)$ and let φ be a one-to-one mapping of the set M_1 onto M_2. For each n-ary operation $f \in F_1$ we define an n-ary operation f^* on the set M_2 by putting

$$f^*(c_1, \ldots, c_n) = \varphi(f(\varphi^{-1}(c_1), \ldots, \varphi^{-1}(c_n)))$$

for each n-tuple $(c_1, \ldots, c_n)$ of elements of M_2. Analogously, for each n-ary operation $g \in F_2$ there exists a uniquely defined n-ary operation g^* on M_1 such that

$$g^*(d_1, \ldots, d_n) = \varphi^{-1}(g(\varphi(d_1), \ldots, \varphi(d_n))) \tag{1}$$

for each n-tuple $(d_1, \ldots, d_n)$ of elements of M_1.

The mapping φ is called a weak isomorphism of A_1 onto A_2 if for each $f \in F_1$ and each $g \in F_2$ the operation f^* belongs to $\alpha(A_2)$ and the operation g^* belongs to $\alpha(A_1)$. (Cf. GOETZ [2].)

The mapping φ will be called a W-isomorphism of A_1 onto A_2 if for each $f \in F_1$ and each $g \in F_2$ the operation f^* belongs to $\beta(A_2)$ and the operation g^* belongs to $\beta(A_1)$.

In this note we shall investigate W-isomorphisms of a distributive lattice $L_1 = (M_1; \wedge, \vee)$ onto a lattice $L_2 = (M_2, \cap, \cup)$. Denote $g_1(x_1, x_2) = x_1 \cap x_2$, $g_2(x_1, x_2) = x_1 \cup x_2$.

Let us remark that if φ is a weak isomorphism of L_1 onto L_2 then, because of distributivity of L_1, the operation $g_1^*(x_1, x_2)$ – being algebraic in L_1 – must be a join of some of the expressions

$$x_1, x_2, x_1 \wedge x_2,$$

hence either $g_1^*(x_1, x_2) = x_1 \vee x_2$ or $g_1^*(x_1, x_2) = x_1 \wedge x_2$. Therefore φ is either an isomorphism or a dual isomorphism.

In what follows we assume (unless otherwise stated) that $L_1 = (M_1; \wedge, \vee)$ is a distributive lattice, $L_2 = (M_2; \cap, \cup)$ is a lattice and that φ is a one-to-one mapping of M_1 onto M_2. Further we suppose that g_1^* and g_2^* belong to $\beta(L_1)$.

It will be shown that L_2 is distributive and that, if L_1 is not bounded, then φ is either an isomorphism or a dual isomorphism. There are lattices P and Q such that L_1 is isomorphic to the direct product $P \times Q$ and L_2 is isomorphic to $P \times Q'$, where Q' is a lattice dual to Q. Moreover, if L_1 is bounded, then g_1^* and g_2^* necessarily have a very special form; namely, there exist elements $u, v \in M_1$ such that u is a complement of v in L_1, and for each pair $d_1, d_2 \in M_1$ we have

$$g_1^*(d_1, d_2) = (d_1 \wedge d_2 \wedge v) \vee ((d_1 \vee d_2) \wedge u),$$

$$g_2^*(d_1, d_2) = ((d_1 \vee d_2) \wedge v) \vee (d_1 \wedge d_2 \wedge u).$$

For analogous results concerning Boolean algebras cf. TRACZYK [10] and GOETZ [3].

Let us define the operations $\cap$ and $\cup$ on M_1 by putting

$$x \cap y = g_1^*(x, y), \quad x \cup y = g_2^*(x, y) \tag{2}$$

for each pair of elements $x, y \in M_1$. Then according to (1), $L_1^* = (M_1; \cap, \cup)$ is a lattice and φ is an isomorphism of L_1^* onto L_2. The partial order in L_1 or L_1^* will be denoted by $\leqq$ or $\preceq$, respectively.

From the fact that both g_1^* and g_2^* belong to $\beta(L_1)$ it follows immediately:

(∗) If R is a congruence relation on the lattice L_1, then R is also a congruence relation on L_1^*.

For any congruence relation R on L_1 and any $c \in M_1$ we denote by $c(R)$ the class of R containing the element c. The set $c(R)$ is a sublattice in both lattices L_1 and L_1^*. If we view this set as a sublattice of L_1 or L_1^*, then we denote it respectively by $c(R, L_1)$ or $c(R, L_1^*)$. The symbols R^0 and R^1 denote respectively the least and the greatest congruence relation on L_1.

The following result has been proved in [5]:

(A) Let $L_1 = (M_1; \wedge, \vee, \leqq)$ and $L_1^0 = (M_1; \cap, \cup, \preceq)$ be any pait of lattices that are defined on the same underlying set M_1. Assume that if R is a congruence relation on the lattice L_1, then R is also a congruence relation on L_1^0. Further assume that the lattice L_1 is distributive. Then the following assertions hold:

(α) The lattice L_1^0 is distributive.

(β) For $x, y \in M_1$ put xR_1y (xR_2y) if $x \wedge y \preceq x \vee y$ (respectively, $x \wedge y \succeq x \vee y$). Then R_1 and R_2 are permutable congruence relations on L_1, $R_1 \wedge R_2 = R^0$, $R_1 \vee R_2 = R^1$.

(γ) Let $c_0 \in M_1$. Then $c_0(R_1, L_1)$ coincides with the lattice $c_0(R_1, L_1^0)$, and $c_0(R_2, L_1)$ is dual to the lattice $c_0(R_2, L_1^0)$.

(δ) For each $z \in M_1$ let us denote by $\psi_1(z)$ or $\psi_2(z)$ the unique element contained respectively in $c_0(R_1) \cap z(R_2)$ or in $c_0(R_2) \cap z(R_1)$. Then the mapping

$$\psi(z) = (\psi_1(z), \psi_2(z))$$

is an isomorphism of the lattice L_1 onto the direct product $c_0(R_1, L_1) \times c_0(R_2, L_1)$. At the same time, ψ is an isomorphism of L_1^0 onto $c_0(R_1, L_1^0) \times c_0(R_2, L_1^0)$.

In what follows we shall use the same notation as in (A) with $L_1^0 = L_1^*$. According to (*) and (A), the assertions (α)–(δ) are valid for lattices L_1, L_1^*. Since L_2 is isomorphic with L_1^*, by putting $P = c_0(R_1, L_1)$, $Q = c_0(R_2, L_1)$ we obtain

Theorem 1. *Let L_1 be a distributive lattice and let L_2 be a lattice W-isomorphic to L_1. Then L_2 is distributive and there are lattices P, Q such that L_1 is isomorphic to $P \times Q$ and L_2 is isomorphic to $P \times Q'$, where Q' is a lattice dual to Q.*

As above, let c_0 be a fixed element of M_1.

Lemma 1. *Suppose that $c_0(R_1) = \{c_0\}$ (or $c_0(R_2) = \{c_0\}$). Then φ is a dual isomorphism (respectively, an isomorphism).*

Proof. Let $c_0(R_1) = \{c_0\}$. Then by (β), $c_0(R_2) = M_1$ and hence $c_0(R_2, L_1) = L_1$, $c_0(R_2, L_1^*) = L_1^*$. Therefore according to (γ), the lattice L_1^* is dual to L_1. Since φ is an isomorphism of L_1^* onto L_2, we obtain that φ is a dual isomorphism of L_1 onto L_2. The other assertion can be verified analogously.

Lemma 2. *Suppose that $c_0(R_1) \neq \{c_0\} \neq c_0(R_2)$. Then the lattice $c_0(R_2, L_1)$ possesses a greatest element.*

Proof. Because L_1 is distributive, the polynomial $g_1^*(x, y)$ can be written as a join of some of the expressions

$$a, b \wedge x, \quad c \wedge y, \quad d \wedge x \wedge y, \quad x, y, x \wedge y,$$

where a, b, c, d are fixed elements of the set M_1.

If $x_0, y_0 \in c_0(R_1)$ or $x_0, y_0 \in c_0(R_2)$, then according to (2) and (γ) we have

$$(3) \qquad g_1^*(x_0, y_0) = x_0 \cap y_0 = x_0 \wedge y_0 \text{ (respectively, } g_1^*(x_0, y_0) = x_0 \vee y_0) .$$

(a) Suppose that $g_1^*(x, y) = x \vee y \vee D$, where D is either empty or D is a join of some of the expressions

$$a, b \wedge x, \quad c \wedge y, \quad d \wedge x \wedge y, \quad x \wedge y .$$

Then

$$(4) \qquad g_1^*(x, y) = a \vee x \vee y \quad \text{or} \quad g_1^*(x, y) = x \vee y .$$

Choose $x_0, y_0 \in c_0(R_1)$, $x_0 \neq y_0$. According to (3) and (4) we have

$$x_0 \wedge y_0 = a \vee x_0 \vee y_0 \quad \text{or} \quad x_0 \wedge y_0 = x_0 \vee y_0 .$$

Thus $x_0 = y_0$, which is a contradiction. Hence without loss of generality we may assume that $g_1^*(x, y)$ is a join of some of the expressions

$$a, b \wedge x, \quad c \wedge y, \quad d \wedge x \wedge y, \quad x, x \wedge y .$$

(b) Suppose that $g_1^*(x, y) = x \vee D$, where D is a join of some of the expressions $a, b \wedge x, c \wedge y, d \wedge x \wedge y, x \wedge y$. Then

$$(5) \qquad g_1^*(x, y) = x \vee a \vee (c \wedge y) \quad \text{or} \quad g_1^*(x, y) = x \vee (c \wedge y)$$

(the relations $g_1^*(x, y) = x$, $g_1^*(x, y) = x \vee a$ being obviously impossible).

Put $\psi_1(a) = a_1$, $\psi_1(c) = c_1 = y_0$ and choose $x_0 \in c_0(R_1)$, $x_0 \neq y_0$. Then (because $\psi_1(z) = z$ for each $z \in c_0(R_1)$) from (5) we obtain

$$(6) \qquad g_1^*(x_0, y_0) = x_0 \vee a_1 \vee (c_1 \wedge y_0) = x_0 \vee a_1 \vee y_0 ,$$
$$\text{or} \quad g_1^*(x_0, y_0) = x_0 \vee y_0 .$$

From (6) and (3) we conclude, analogously as in (a), that $x_0 = y_0$, which is a contradiction. Therefore $g_1^*(x, y)$ is a join of some of the expressions

$$a, b \wedge x, \quad c \wedge y, \quad d \wedge x \wedge y, \quad x \wedge y .$$

(c) Suppose that the lattice $c_0(R_2, L_1)$ has no greatest element. Then there are distinct elements $x_0, y_0 \in c_0(R_2)$ such that

$$(7) \qquad x_0 \wedge y_0 > \psi_2(a) \vee \psi_2(b) \vee \psi_2(c) \vee \psi_2(d) = a_0 .$$

The element $g_1^*(x_0, y_0)$ is a join of some of the elements

$$\psi_2(a), \psi_2(b) \wedge x_0, \psi_2(c) \wedge y_0, \psi_2(d) \wedge x_0 \wedge y_0, x_0 \wedge y_0$$

(because $\psi_2(z) = z$ for each $z \in c_0(R_2)$). Hence by (7),

$$g_1^*(x_0, y_0) \leqq x_0 \wedge y_0 . \tag{8}$$

From (8) and from (3) we get $x_0 \vee y_0 \leqq x_0 \wedge y_0$, thus $x_0 = y_0$, which is a contradiction. Therefore the lattice $c_0(R_2, L_1)$ possesses a greatest element.

Lemma 3. *Let $c_0(R_1) \neq \{c_0\} \neq c_0(R_2)$. Then the lattice $c_0(R_1, L_1)$ has a greatest element.*

The proof is analogous to that of Lemma 2 with the distinction that we consider the polynomial $g_2^*(x, y)$ instead of $g_1^*(x, y)$.

Lemma 4. *Let $c_0(R_1) \neq \{c_0\} \neq c_0(R_2)$. Then both lattices $c_0(R_1, L_1)$ and $c_0(R_2, L_1)$ have least elements.*

The proofs are dual to the proofs of Lemma 2 and Lemma 3.

A lattice will be called bounded if it has a least as well as a greatest element.

Lemma 5. *Let $c_0(R_1) \neq \{c_0\} \neq c_0(R_2)$. Then both lattices L_1 and L_2 are bounded.*

Proof. The assertion for L_1 follows from Lemmas 2, 3, 4 and from (δ). Similarly, from Lemmas 2, 3, 4, from (γ) and (δ) we obtain that L_1^* has a least and a greatest element; because L_2 is isomorphic to L_1^*, the same holds for L_2.

Theorem 2. *Let L_1 be a distributive lattice and let φ be a W-isomorphism of L_1 onto a lattice L_2. Suppose that either L_1 or L_2 is not bounded. Then φ is either an isomorphism or a dual isomorphism.*

This follows from Lemma 5 and Lemma 1.

Now let us consider the case when the lattice L_1 is bounded.

Lemma 6. *Let $L_1 = (M_1; \wedge, \vee)$ be a bounded distributive lattice. Let $u, v \in M_1$ such that u is a complement of v. Define on M_1 binary operations $\cap, \cup$ by the rules*

$$x \cap y = (x \wedge y \wedge v) \vee ((x \vee y) \wedge u), \tag{9}$$

$$x \cup y = ((x \vee y) \wedge v) \vee (x \wedge y \wedge u). \tag{10}$$

Then (i) *$L = (M_1; \cap, \cup)$ is a distributive lattice with the least element u and the greatest element v;* (ii) *for each $x, y \in M_1$,*

$$x \wedge y = (x \cap y \cap b) \cup ((x \cup y) \cap a), \tag{9'}$$

$$x \vee y = ((x \cup y) \cap b) \cup (x \cap y \cap a) \tag{10'}$$

is valid, where a and b are respectively the least and the greatest element of L_1.

Proof. Let a and b be respectively the least and the greatest element of L_1. Denote

$$X_1 = \{x \in M_1 : a \leqq x \leqq u\}, \quad X_2 = \{x \in M_1 : a \leqq x \leqq v\}.$$

Since $u \wedge v = a$, $u \vee v = b$ and since L_1 is distributive, the mapping

$$\psi(x) = (x \wedge u, x \wedge v)$$

is an isomorphism of the lattice L_1 onto the direct product of lattices $(X_1, \wedge, \vee)$, $(X_2, \wedge, \vee)$, and for any $x_1 \in X_1$, $x_2 \in X_2$ we have

$$\psi^{-1}((x_1, x_2)) = x_1 \vee x_2 .$$

Thus, in particular, ψ is a one-to-one mapping of the set M_1 onto the set $X_1 \times X_2$. Let us define binary operations $\cap$, $\cup$ on X_1 and on X_2 in such a way that $(X_1, \cap, \cup)$ is a lattice dual to $(X_1, \wedge, \vee)$, and $(X_2, \cap, \cup)$ coincides with $(X_2, \wedge, \vee)$. Then it follows from (9) and (10) that ψ is an isomorphism of the algebra $(M_1, \cap, \cup)$ onto the direct product $(X_1, \cap, \cup) \times (X_2, \cap, \cup)$. Therefore $(M_1, \cap, \cup)$ is a distributive lattice.

Two lattices P and Q defined on the same underlying set M will be said to fulfil the condition (D) if there exist lattices A_1, A_2 (defined respectively on the set A_1 and A_2) and a mapping ψ of M onto $A_1 \times A_2$ such that ψ is an isomorphism of P onto $A_1 \times A_2$ and, at the same time, ψ is an isomorphism of Q onto $A_1^* \times A_2$, where A_1^* is the lattice dual to A_1.

We have verified that the lattices $(M_1, \wedge, \vee)$ and $(M_1, \cap, \cup)$ fulfil the condition (D). Because both these lattices are distributive, according to [4] they fulfil also the condition (E), namely, there exist elements t and t' in M_1 such that t' is a complement of t in $(M_1, \cap, \cup)$ and the relations

$$x \wedge y = (x \cap y) \cup (y \cap t) \cup (t \cap x),$$
$$x \vee y = (x \cap y) \cup (y \cap t') \cup (t' \cap x),$$

hold for each pair $x, y \in M_1$. Since $(M_1, \cap, \cup)$ is distributive, we have

$$(x \cap y) \cup (y \cap t) \cup (t \cap x) = [(x \cap y) \cap (t \cup t')] \cup [(x \cup y) \cap t] =$$
$$= [(x \cap y) \cap t] \cup [(x \cap y) \cap t'] \cup [(x \cup y) \cap t] = [(x \cup y) \cap t] \cup [x \cap y \cap t'] .$$

Hence

$$x \wedge y = [(x \cup y) \cap t] \cup [x \cap y \cap t'] .$$

Analogously we can verify that

$$x \vee y = [x \cap y \cap t] \cup [(x \cup y) \cap t'] .$$

In particular,

$$x \wedge t = [(x \cup t) \cap t] \cup [x \cap t \cap t'] = t\,,$$

$$x \vee t' = [x \cap t' \cap t] \cup [(x \cup t') \cap t'] = t'$$

for each $x \in M_1$. Hence $t = a$, $t' = b$. Thus (9′) and (10′) hold.

Theorem 3. *Let $L_1 = (M_1;\ \wedge, \vee)$ be a bounded distributive lattice. Let $u, v \in M_1$ such that u is a complement of v. Let M_2 be a set with two binary operations $\cap$ and $\cup$, and let φ be a one-to-one mapping of M_1 onto M_2 such that for each pair $x', y' \in M_2$ we have*

(9″) $$\varphi^{-1}(x' \cap y') = (x \wedge y \wedge v) \vee ((x \vee y) \wedge u)\,,$$

(10″) $$\varphi^{-1}(x' \cup y') = ((x \vee y) \wedge v) \vee (x \wedge y \wedge u)\,,$$

where $x = \varphi^{-1}(x')$, $y = \varphi^{-1}(y')$. Then $L_2 = (M_2; \cap, \cup)$ is a lattice and φ is a W-isomorphism of L_1 onto L_2.

Proof. From the assertion (i) of Lemma 6 and from (9″), (10″) it follows that L_2 is a distributive lattice. For any $x, y \in M_1$ denote $h_1(x, y) = x \wedge y$, $h_2(x, y) = x \vee y$, $\varphi(x) = x'$, $\varphi(y) = y'$, $g_1(x', y') = x' \cap y'$, $g_2(x', y') = x' \cup y'$. Then (using the same notation as in the introduction) we infer from (9″) that

$$g_1^*(x, y) = (x \wedge y \wedge v) \vee ((x \vee y) \wedge u)\,,$$

hence $g_1^* \in \beta(L_1)$. Analogously, from (10″) we obtain $g_2^* \in \beta(L_1)$. Further we have

$$h_1^*(x', y') = \varphi(\varphi^{-1}(x') \wedge \varphi^{-1}(y')) = \varphi(x \wedge y)\,.$$

Denote $x \cap y = g_1^*(x, y)$, $x \cup y = g_2^*(x, y)$. The assertion (ii) of Lemma 6 (cf. (9′)) implies

$$h_1^*(x', y') = \varphi((x \cap y \cap b) \cup ((x \cup y) \cap a))\,.$$

The mapping φ is obviously an isomorphism with respect to both operations $\cap$ and $\cup$; thus

$$h_1^*(x', y') = (x' \cap y' \cap b') \cup ((x' \cup y') \cap a')\,.$$

Hence $h_1^* \in \beta(L_2)$. Similarly we can verify that $h_2^* \in \beta(L_2)$. Therefore φ is a W-isomorphism of L_1 onto L_2.

We shall show that if L_1 is a bounded distributive lattice, then each W-siomorphism of L_1 onto a lattice L_2 has the form described in Thm. 3.

The following statement was established in [5].

(B) Let L_1 and L_1^0 be as in (A). Suppose that a and b are respectively the least and the greatest element of L_1. Put $u = a \cap b$, $v = a \cup b$. Then u and v are respectively the least and the greatest element in L_1^0, u is a complement of v and for each pair $x, y \in M_1$ the relations (9) and (10) are valid.

In view of (*), the statement of Thm. (B) holds for the pair of lattices L_1 and $L_1^0 = L_1^*$.

Theorem 4. *Let $L_1 = (M_1; \wedge, \vee)$ be a distributive lattice and let φ be a W-isomorphism of L_1 onto a lattice $L_2 = (M_2; \cap, \cup)$. Let a and b be respectively the least and the greatest element of L_1. Then*

(i) *L_2 is bounded (the least and the greatest element of L_2 will be denoted by u_2 and v_2, respectively, and we put $\varphi^{-1}(u_2) = u$, $\varphi^{-1}(v_2) = v$);*

(ii) *if $x_2, y_2 \in M_2$ and $x = \varphi^{-1}(x_2)$, $y = \varphi^{-1}(y_2)$, then*

(11) $$x_2 \cap y_2 = \varphi((x \wedge y \wedge v) \vee ((x \vee y) \wedge u)),$$

(12) $$x_2 \cup y_2 = \varphi(((x \vee y) \wedge v) \vee (x \wedge y \wedge u));$$

(iii) $$u \wedge v = a, \quad u \vee v = b,$$

Proof. Let u, v be as in (B). Because φ is an isomorphism of L_1^0 onto L_2, $\varphi(u)$ and $\varphi(v)$ are respectively the least and the greatest element of L_2. The assertions (ii) and (iii) are immediate consequences of (B).

Remark. The relations (11) and (12) are clearly equivalent with the relations

$$g_1^*(x, y) = (x \wedge y \wedge u) \vee ((x \vee y) \wedge v),$$
$$g_2^*(x, y) = ((x \vee y) \wedge u) \vee (x \wedge y \wedge v).$$

If $u = a$, then $v = b$ and hence from (11) and (12) we obtain

$$x_2 \cap y_2 = \varphi(x \wedge y), \quad x_2 \cup y_2 = \varphi(x \vee y);$$

thus φ is an isomorphism of L_1 onto L_2. If $u = b$, then $v = a$, and by (11) and (12),

$$x_2 \cap y_2 = \varphi(x \vee y), \quad x_2 \cup y_2 = \varphi(x \wedge y),$$

and hence φ is a dual isomorphism of L_1 onto L_2. Therefore we have

Corollary 1. *Let L_1, L_2, a, b, u, v be as in Thm. 4. If $u = a$ (or $u = b$), then φ is an isomorphism (a dual isomorphism, respectively).*

For an analogous result concerning Boolean algebras cf. TRACZYK [10].

Since L_1 is distributive and v is a complement of u, the element v is uniquely determined by u. Thus from Thm. 4 we conclude

Corollary 2. *Let L_1, L_2, a, b, u_2 be as in Thm. 4. Then L_2 is determined up to an isomorphism by L_1 and by the element $u = \varphi^{-1}(u_2)$.*

References

[1] *J. Dudek, E. Płonka:* Weak automorphisms of linear spaces and of some other abstract algebras, Coll. Math. *22* (1971), 201—208.

[2] *A. Goetz:* On weak automorphisms and weak homomorphisms of abstract algebras, Coll. Math. *14* (1966), 163—167.

[3] *A. Goetz:* On various Boolean structures in a given Boolean algebra, Publ. Mathem. *18* (1971), 103—108.

[4] *J. Jakubík, M. Kolibiar:* O nekotorych svojstvach par struktur, Czechoslov. Math. J. *4* (1954), 1—27.

[5] *J. Jakubík:* Pairs of lattices with common congruence relations (to appear).

[6] *E. Marczewski:* A general scheme of the notion of independence in mathematics, Bull. Acad. Polon. Sci. Sér. Math. Phys. Astron. *6* (1958), 731—736.

[7] *E. Marczewski:* Independence in abstract algebras. Results and problems, Colloq. Math. *14* (1966), 169—188.

[8] *R. Senft:* On weak automorphisms of universal algebras, Dissertationes Math. *74* (1970).

[9] *J. Sichler:* Weak automorphisms of universal algebras, Alg. Univ. *3* (1973), 1—7.

[10] *T. Traczyk:* Weak isomorphisms of Boolean and Post algebras. Coll. Math. *13* (1965), 159—164.

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